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SEX PRIORA EUCLIDIS ELEMENTA,

Quibus acceſſerunt Undecimum, & Duodecimum

A HIEMYNIANO RONDELLO

In Bononienſi Archigymnaſio Hydrometriæ Profeſſore in gratiam ſtudioſæ Juventutis iterum expoſita,

E T

Illuſtriſsimis, atque Excelſis

BONONIÆ SENATUS PATRIBUS

Scientiarum omnium Mœcenatibus humilimè Dicata.

Labente Anno gratiæ MDCCXIX.

Bononiæ, Typis Longi. *Superiorum permiſſu.*

191. di S. Pietro

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Hist. of Science
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Illuſtriſsimis, atque Excelſis
BONONIÆ
SENATUS
PATRIBUS,

Hiemynianus Rondellus F.

CUm multis ante Annis Euclidis Elementa expoſuiſſem, eaque edenda curarem: quibus ſtudioſam Juventutem publicè erudire (quod mihi munus ſumma Veſtra, atque immortali beneficentia demandatum fuerat) & ad Mathematicarum diſciplinarum Scientiam dirigere poſſem: non dubitavi eadem Vobis

MVP. 3-23-40

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inſcribere ILLUSTRISSIMI, atque EXCELSI PATRES; viſum eſt enim non aliis eum Librum conſecrandum eſſe, præterquam iis, quorum ſuffragiis factum erat, ut eo Libro ad erudiendos optimos adoleſcentes uti poſſem. Id vero feci non minus audacter, quam feliciter; nam illud, quidquid erat opuſculi, libenter adeò, atque humaniter excepiſtis, ut jam me puderet tantæ virtuti tam puſillum opus conſecraſſe: ac cum multis aliis, propeque innumerabilibus argumentis ſummam Veſtram in bonas Artes, ac Studia omnia propenſionem, liberalitatemque oſtenderitis; quippequi, & eos, qui ſtudia diligenter excolunt, mirum in modum diligitis, & Præceptores publicos ex omni genere multos alitis, nec ullam occaſionem præterire ſinitis, quin Scientias, atque Artes omnes promoveatis, augeatis, amplificetis, quo nomine univerſa Vobis Civitas tantum debet, quantum vix credi poteſt; eamdem iſtam propenſionem Veſtram mihi peculiari etiam benignitate ſignificaviſtis, qua Libellum illum meum, qui qualiſcumque eſſet, commodo certè ſtudioſæ Juventuti futurus erat, libentiſsimè accepiſtis. Ea re igitur factum eſt, ut cum illa priora exemplaria, propter illorum in Scholis penè omnibus uſum, rariſsima facta ſint; ego autem vel ipſa docendi conſuetudine cognoverim, quantum commodi afferat ſtudioſis Juuentutibus Librum publicè editum in manu habere; eadem Eucli-

dis

dis Elementa, ſi non meliori, certè breviori, & clariori modo rurſus exponere, eaque vobis ILLVSTRISSIMI, atque EXCELSI PATRES dicare, atque majorem in modum commendare ſtatuerim. Ego quidem quantulum illud ſit, quod vobis offero, planè intelligo; ſed ſimul etiam & humanitatem, & clementiam, & benignitatem veſtram non intelligo ſolum, ſed cum expertus ſim ſemper, nunc ita experior, ut cum maximè. Facile igitur confido fore, ut libellum hunc meum, quem Vobis offero, ea humanitate accipiatis, quæ Dignitatem Veſtram, & Sapientiam decet. Id ut ne fruſtra confidam Veſtrum erit facere. Ego interim enixe, ea quantum poſsum, ut id faciatis etiam, atque etiam rogo. Valete & Patriæ, cujus Parentes eſtis, & bonis Artibus, quarum eſtis cultores, & Vobis ipſis.

Bononiæ Die 12. Decembris 1719.

Vi-

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Vidit D. Sebaſtianus Giribaldi Cleric. Regul. Sanĉti Pauli in Eccl. Metrop. Bononiæ Pœnetentiarius pro Eminentiſſimo, & Reverendiſſimo Domino, D. Jacobo Cardinali Boncompagno Archiepiſcopo, & S. R. I. P.

Die 28. Junij 1719.

IMPRIMATUR

Fr. Joannes Dominicus Liboni Vicarius Generalis Sanĉti Officii Bononiæ.

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AD LECTOREM.

QUod Navigantibus evenire ſolet, ut alii quidem, qui minoribns Navigijs utuntur, vix a Portu diſcedere, & Littus legere audeant, alii vero, quibus Vela majora ſunt, non modo Portui valedicere, ſed & Oceanum, & Maria omnia traiicere non dubitent, novaſque Terras, & Regiones perluſtrare ſtudeant, atque deſcribere; idem quoque accidit Mathematicis. Horum enim alii, quibus non ita excellens, ad novas res, atque abditas eruendas, Ingenium a Natura datum eſt, Mathematicarum diſciplinarum principia attingunt, ulterius progredi pertimeſcunt; alii vero pauci, *Quos Æquus amavit Jupiter*, quibuſque ſingularis quædam eſt intelligendi vis, non modo principia aſſequi, ſed amplius in interiores Geometriæ partes pervadere, novaſque ſemper res, & in profundo demerſas in lucem proferre nituntur: quorum exempla habemus plurima apud veteres, multa etiam apud recentiores;

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illi enim & concoidem, & cyſſoidem, & ſpiralem, & quadratricem, aliaſque curuas lineas ſumma quadam ingenii vi penetrarunt, deſcripſeruntque: hi vero illis multa addiderunt: cycloidem, catenarias, velarias, aliaſque plurimas, quas calculis quibuſdam, & ſupputationibus comprehenderunt. Atque horum ego quidem operam, ſtudiumque mecum ipſe conſiderans, ſtatui poſtremum hoc genus hominum, qui nova inveniunt, maxime utile eſſe Reipublicæ; ſed illud tamen primum non minus: immo etiam fortaſſe magis; neque enim fieri poteſt, ut reconditiorem Geometriam penetret, niſi is, qui elementa prius, quaſi limen quoddam ingreſſus ſit; quare qui hæc explanant, viam aliis muniunt ad majora. Ego igitur, cum mihi non tantum ingenii eſſet, ut novi quidpiam molirer, eſſet vero tantum voluntatis, ut ne otioſus eſſe vellem; feci, ut Elementa Euclidis, quam potui diligentiſſimè exponerem, eaque Typis mandarem, ut eos, qui majori ingenio præditi ad altiora aſpirant, qua poſſem ope adiuvarem. Idem in Sphæricis, Coniciſque faciam aliquando, ſi hæc, quæ nunc edo, grata eſſe cognovero. Verum de his alias: nunc de Euclidis Elementis pauca dicam. Ac primum illud etiam atque etiam tibi perſuadeas velim, nihil iis in univerſa Geometria magis eſſe neceſſarium, quod etiam ipſum Elementi nomen nos docet; idcirco enim *Elementa* appellata ſunt, quod ſine iis in Mathematicis diſciplinis conſiſtere nihil poteſt. Neque tamen diſſimulo *Algebram* lineis parum uti, calculo, & literis conficere pene omnia. Verum ſi illam ad Geometriam traducamus, quid tandem efficere poterit ſine Elementis Euclideis? Nam eſt Algebra Scientia quædam cujuſdam univerſalis calculi, qui non ad hoc potius, quam ad illud quantitatis genus ſpectat, ſed omnia æque complectitur; nititur enim principijs abſtractis, quæ non extenſionis pro-

propria ſunt, non numeri, ſed quantitatis; hinc fit, ut ſi ad certum, & peculiare quoddam quantitatis genus traducenda ſit, oporteat etiam peculiarem ejuſdem generis naturam, atque intimas proprietates cognoſcere. Nam quid quæſo per Algebraicos calculos præſtare umquam in Arithmetica poteris? Niſi Arithmeticæ ipſius præcepta ante cognoveris? Niſi additionem, detractionem, multiplicationem, diviſionem, radicum quadrarum, cubarum &c. extractionem teneas? Numquid res aliter ſe habet in Geometria? Nihil minus. Quid enim aliud efficit Algebra, ſi ad Geometriam transferatur, niſi tantum præcipit, ad hoc, vel illud problema ſolvendum, has, vel illas lineas, aut ſuperficies, hæc vel illa ſolida addenda eſſe, demenda, multiplicanda, dividenda, illorum radices quadras, cubicas extrahendas? Hæc vero, quæ fieri præcipit, quomodo fiant, non docet; quod niſi Euclides, & lineam lineæ, & ſuperficiem ſuperficiei demere, & unam ſuperficiem pluribus æqualem facere, unum triangulum duobus, unum rectangulum duobus quadratis, pentagonis &c., quæ præcipit Algebra, exequi nullo modo poſſemus. Quid dicam de linearum, & ſuperficierum per lineas multiplicatione? Quid de diviſione? Quid tandem de radicum extractione? Quæ certe ſine abſtruſiſſima proportionum, quam Euclides tradidit, ſcientia fieri omnino non poſſunt; adeo ut nobiliſſima, atque magnificentiſſima ſcientia, *Algebra*, ſi Euclidis Elementa tollas, nulla ſit. Dixi particularitèr de Algebra, neque enim dubito, quin ſi illos, qui ad Algebram aſpirant, ad Euclidis Studium commoverim, reliquos quoque facile excitem. Verum erunt nonnulli, qui breviorem in hiſce Elementis exponendis methodum deſiderent. Ego vero, cum multæ, ac variæ ſint Elementorum tradendorum viæ, hanc potiùs Euclidis ſecutus ſum, quam aliorum, tum quia illi, qui

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breviorem tenent, non ita omnia ad unguem demonſtrant, ut Euclides facit; iuvat vero eos, qui primum in Geometriam ingrediuntur, ſtatim abſolutiſſimis, perfectiſſimiſque ex omni parte demonſtrationibus aſſuefieri: tum etiam quia ex multis viis, quas ad aliquid obtinendum tenere poſſumus, eam præſtare ſemper putavi, qua id, quod volumus, melius aſſequimur. Quod ſi in hiſce Elementis id unicè ageremus, ut veritates quaſdam demonſtraremus, non dubitarem eam viam omnibus anteferre, quæ eſſet, quam fieri poteſt, breviſſima. Verum quia non hoc tantum agimus, ſed illud etiam volumus, ut veritates, quas demonſtramus, in animis Adoleſcentulorum, qui ad hæc Studia ſe conferunt, penitus inſideant, ac firmiter adhæreant, ſimulque illorum ingenium illuſtrent, acuant, excitent, non breviorem Methodum tenendam eſſe exiſtimavi, qua ut facile res diſcuntur, ita facile excidunt; ſed eam potius, qua a facilioribus ad difficiliora paullatim deducti Adoleſcentes ita acuantur, ut vix quidquam eſſe amplius poſſit adeo difficile in Geometria, ut ipſos deterreat. Neque tamen hæc Elementa ita expoſui, ut mediocri ingenio Adoleſcentes aſſequi facile non poſſint: aſſequerentur fortaſſe brevius, & facilius, ſi alia methodo uſus eſſem; cum Euclidea uſus ſim, aſſequentur, ut ſpero, fructuoſius. Tu interim his utere, & ſi opinionem noſtram minus probas, voluntatem tamen ne improbes. Vale.

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DE MATHEMATICÆ OBIECTO.

DE Matheſeos principiis tantummodò in gratiam ſtudioſæ Iuventutis verba facturus, ut talis Doctrinæ ideam, quantum fieri poteſt, in aperto ponam, conſentaneum duco breviſſimè circa Matheſis obiectum præfari, antequam ad talium principiorum expoſitionem accedam, quia in hoc tractatu dumtaxat tyronum utilitati (ut Profeſſorij muneris ratio poſtulat) conſulere animus eſt.

Illa Doctrina, ſive Ars, quæ Mathematica appellatur, definiri ſolet *Scientia entis quatenus quantum eſt*: Illa enim entia, quæ a Mathematica conſiderantur, ſemper accipiuntur, *ut Quanta, ſeu ut æſtimabilia*. Quia verò entia per ſolam *Quantitatem Quanta* ſunt, ideo reſtat ut videamus quid *Quantitatis* nomine intelligendum ſit.

QUANTITAS univerſaliter accepta deſignatur illa Mentis conceptione, ſecundum quam de rebus quærere ſolemus per comparationis Aduerbium *Quam*? Uti evenit dum quærimus *Quam* ens ſit longum? *Quam* amplum? *Quam* grande? *Quam* grave? *Quam* diu? *Quam* magnum? *Quam* multum, ſeu *Quam* multa &c. Illa enim entia, de quibus licet has interogationes facere abſolutè *Quanta* ſunt, vel ſaltem, ut *Quanta* conſiderantur.

Quia verò circa entia quantitativa in ſe, vel ut talia conſiderata verſatur Mathematica, ut talium entium menſuram obtineat, ideo ens *ut Quantum* ponitur Mathematicæ obiectum, de quo a Matheſi demonſtrata ad ſolam *Entis Quanti* menſuram ordinantur.

Illa quantitas, per quam *Ens quantum* conſtituitur, univerſaliter accepta partitur in Quantitatem Continvam, atque Diſcretam. Continva quantitas dicitur illa, de qua ſolum quæritur Quanta, ſeu quam Multa ſit, ſola menſura, ſeu magnitudine inſpecta.

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Quantitas verò *diſcreta* appellatur illa, de qua quæri ſolet *Quot*, ſeu *quammulta*; & in hac quantitate ſola multitudo inſpicitur.

Continva quantitas ſubdividitur in *Permanentem*, atque *Succeſſivam*. Permanens quantitas dicitur illa, cuius omnes partes, quæ in quantitate deſignari poſſunt, permanent. Succeſſiva, ſive fluens quantitas appellatur illa, cujus partes ſunt in continvo fluxu poſitæ, uti evenit in tempore, atque Motu.

Inter quantitates continvas illæ, quæ ſunt localiter extenſæ, ut plurimum MAGNITUDINES vocantur, & hæ ſunt LINEA, SUPERFICIES, & CORPUS, ſiuè SOLIDUM, de quibus eſt principaliter intelligendum id, quod permulti dicunt, videlicet eſſentiam Quantitatis in partium extra partes poſitione conſiſtere. In hac quantitate illa, quæ ſignantur, indiviſibilia PUNCTA nominantur; indiviſibilia vero Temporis INSTANTIA vocantur.

Diſcreta, ſiue diſcontinvata quantitas illa nominatur, cujus partes communi termino copulatæ non ſunt, at diſtinctæ, atque ſeparatæ, & de his quæri ſolet quot ſunt, partes diſcretam quantitatem componentes. Hæc quantitas vocatur NUMERUS, ſeu MULTITUDO.

NUMERUS dicitur quantum diſcretum, quia componentes partes intelliguntur tamquam totidem res diverſæ ſeparatæ, atque diſtinctæ. Dum autem dicimus decem, vel centum homines, non unam rem nobis repræſentamus, ſed tot, quot indicat numerus denarius, vel centenarius. Exadverſo in quanto continvato partes componentes, ut unum intelliguntur, ſiquidem in acervo lapidum quamvis ſinguli lapides ſint ad invicem ſeparati, dum conſideratur ſola acervi moles, tunc quantum non eſt diſcretum, at continuum, quia ſolam quærimus magnitudinem, non autem NUMERUM partium componentium.

Quod in continva quantitate præſtant puncta indiviſibilia, in diſcreta faciunt unitates; nam quemadmodum puncta ponuntur continvatæ quantitatis principia, ita unitates conſiderandæ ſunt veluti principia omnium numerorum.

Reliquæ omnes quantitatis ſpecies juxtà Valeſium ad magnitudinem, vel Numerum reduci poſſunt, uti ſunt Locus, Pondus, vis Motrix, & ſimiles, quæ omnes conſiderantur a Ma-

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Matheſi non per ſe, ſed ut menſurabiles, vel numerabiles.

His poſitis concludendum videtur materiale Matheſeos obiectum eſſe Magnitudinem, & Numerum, formale vero (ut loquuntur Scholæ) exiſtere ſolam Menſurabilitatem, ſub qua unica ratione Matheſis quantitatem contemplatur.

Inquirenti poſtea cur quantitatis Doctrina Nomine Matheſis appelletur, reſpondeo, hanc Doctrinam, vel Artem a Veteribus tali nomine fuiſſe inſignitam, vel quia pænes antiquos ſolum erat Matheſeos ſtudium, vel quia Mathemata ante alias ſcientias ediſcebantur.

Quod attinet ad Mathematum ſpecies aliæ *Puræ*, aliæ *Mixtæ* dicuntur. *Puræ* Mathematicæ vocantur illæ, quæ quantitatem abſtractam conſiderant; *Mixtæ* verò appellantur, quæ Quantitatem in ſubiecto aliquo conſtitutam contemplantur. Inter puras Mathematicas connumerantur *Geometria*, atque Arithmetica, licet Arithmetica reputetur purior ipſa Geometria, quia Arithmeticæ obiectum eſt magis abſtractum, quam illud Geometriæ: qua de re Arithmetica principia, atque Theoremata habet magis univerſalia, quam ſint illa quæ ad Geometriam pertinent.

Mixtæ Mathematicæ iuxta diverſitatem illorum ſubiectorum, in quibus quantitas reperitur in varias ſpecies diſtribuuntur, ſiquidem illa Matheſis, quæ motum in Corporibus Cœleſtibus poſitum examinat, Aſtronomia vocatur, quæ Orbis noſtri, ſiuè Terræ quantitatem, & ſitum ſcrutatur, dicitur Geographia.

Statica appellatur illa, quæ agit de Ponderibus.

Mecanica motum, atque moventes vires inquirit: & ut paucis multa comprehendam, omnes illæ artes, quæ quantitatem in ſubiecto poſitam conſiderant, Mixtæ Mathematicæ nominantur, atque varijs nominibus appellantur, iuxta diverſitatem illorum ſubiectorum, in quibus quantitas reperitur.

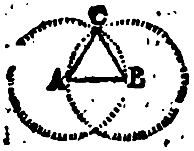
De Demonſtrationibus Mathematicis.

DOcente Ariſtotele *Demonſtratio* eſt ille Syllogiſmus, qui proprias affectiones de ſubiecto per proprias cauſas docet: verum hæc non recipitur tanquam univerſalis *Demonſtrationis* definitio, cum omni defi-

nito non conveniat: ſiquidem eſt & alia *Demonſtrationis* ſpecies, quam ſatis innuit idem Ariſtoteles, dum inquit ad Demonſtrationem ſufficere argumentum ab effectis deductum.

Quod ad Mathematicas demonſtrationes attinet, dico tales demonſtrationes, quæ ſcientificæ appellantur ad duas tantum claſſes eſſe reducendas, quarum prima continet omnes demonſtrationes, quas *Oſtenſiuas* vocant; circa quas eſt obſervandum, aliquas ſolum demonſtrare quod res ſint, alias vero quod ſint, & quare ſint.

Exemplum illarum demonſtrationum, quæ tantumodo demonſtrant quod res ſint, habemus in prima propoſitione Euclidis, in qua demonſtrat efformationem Trianguli æquilateri ABC, ex hoc ſolum quod duo latera AC, & BC ſint tertio lateri AB æqualia: ex quo poſtea infert dicta latera eſſe inter ſe æqualia: Hæc quidem demonſtratio libenter ab omnibus inter oſtenſivas recipitur, licet tantum demonſtret quod res ſit, non autem quare ſit: & ſanè æqualitas illa, quam habent duæ lineæ AC, & BC cum tertia AB non eſt cauſa talis æqualitatis, at ſolum indicium, ex quo colligitur illa æqualitas: duæ enim quantitates AC, BC non ſunt æquales per hoc ſolum quod æquationem habeant cum tertia quantitate AB, ut omnes concedunt.



Alterius demonſtrationis *Oſtenſiuæ* ſpecimen habemus, dum omnes eiuſdem circuli radij demonſtrantur æquales, poſita circuli definitione, videlicet *quod ſit figura plana unica curua linea contenta, quæ a medio comprehenſi ſpatij in omnibus ſuis partibus æqualiter diſtat*. Si enim talis eſt circuli eſſentia, tamquam a vera, & propria cauſa ſequitur, omnes eiuſdem circuli radios eſſe æquales: hæcque dicitur *Demonſtratio oſtenſiua* deſumpta a cauſa proxima, & immediata, quia radiorum circuli æqualitas tanquam veram cauſam agnoſcit definitionem circuli.

Altera demonſtrationis ſpecies eſt illa, quæ per deductionem ad abſurdum, ſiuè ad impoſſibile procedit. In hac demonſtratione oſtenditur, rem eſſe talem qualem dicimus, per hoc ſolum, quod aliter eſſe non poſſit.

Hæc pariter demonſtratio duplex eſt, quarum una ſimplici utitur impoſſibilitate, altera vero in geminata impoſſibilitate conſiſtit.

bamus

Exemplum huius demonſtrationis ſimplicis habetur, dum probamus rectam lineam AB tantummodo in unico puncto C poſſe bifariam dividi, & dicimus. Si recta AB in duobus punctis C, & D foret bifariam diviſa, ſequeretur, quod tam CB, quam DB æquales eſſent medietati ipſius AB: quare, & inter ſe æquales eſſent CD, BD. Cum autem hoc ſit impoſſibile, quia CB eſt totum, & DB pars; ex hoc inferri poteſt, datam rectam AB non eſſe bifariam diviſam in duobus punctis C, & D.



Fundamentum huius demonſtrationis pendet ex hoc, quod veritas non poſſit veritati contrariari, neque unam veritatem poſſe alteram deſtruere. At ſi verum foret quod AB poſſet dividi bifariam non ſolum in puncto C, verum etiam in puncto D, in hoc caſu una veritas foret contraria alteri veritati, nempe quod totum ſemper ſit maius ſua parte, quæ veritas deſtrueretur, quoties una recta linea in duobus punctis bifariam dividi poſſet.

Altera ſpecies demonſtrationis ab impoſſibilitate geminata utitur demonſtratione ad abſurdum, ut aliquam oſtendat veritatem, uti eveniret ſi ad probandam æqualitatem inter A, & B prius demonſtretur eſſe impoſſibile A maius eſſe quam B, poſtmodum verò eadem impoſſibilitate probetur quod A nequeat eſſe minus quam B: ex qua geminata impoſſibilitate demonſtrata infertur æqualitas inter A, & B.

Tali demonſtratione utitur Euclides in Libro 12. Elementorum, atque frequentiſſima eſt apud Archimedem. Tota vis huius demonſtrationis in hoc conſiſtere videtur, quod ſi A (ob illa quæ ſequerentur abſurda) nequit eſſe maius, nec minus quam B, neceſſaria illatione ſequitur quod A debeat adæquare B.

Huiuſmodi demonſtrationem tamquam nimis faſtidioſam rigorosè condemnant Sturmius in ſua Matheſi enucleata in Præfatione ad Lectores Art. 7., atque Renaldinus in Arte Analytica: minus rigorosè verò Franciſcus Vieta talem demonſtrationem culpat, dum dicit, hunc demonſtrandi uſum citra limitationem non eſſe admittendum: libenter enim concedit præfatus Author, quod ad inuicem comparando ea, quæ ex natura ſua æqualitatem non reſpuunt, ſi alterum altero neque maius, ne-

que minus eſſe poſſe demonſtretur, rectiſſime inferri poſſe talium rerum æqualitatem: non ſic vero quando ad inuicem forent comparata illa quanta, quorum natura æqualitatem recuſat: in quo caſu, licet fuerit demonſtratum unum neque maius, neque minus alio exiſtere, non inde licet inferre illa, quæ comparantur, eſſe æqualia, cum hanc æqualitatem habere nequeant, ut ſupponitur.



EU-

EUCLIDIS ELEMENTUM PRIMUM.

Deſ. I.

PUNCTUM illud dicitur, cujus nulla pars eſt.

AD integram hujus definitionis intelligentiam ſcire oportet, ab Euclide nomine PUNCTI intelligi id, quod in quantitate continva nullas habet partes: ſi enim in continva quantitate concipiamus aliquid, quod ſit quacumque parte ſpoliatum, illud erit PUNCTUM, ſiuè SIGNUM ab Euclide definitum. Quia verò punctum definitur in ordine ad partium negationem neceſſarium duxi animadvertere continvam quantitatem, quam Mathematici MAGNITUDINEM appellant, in triplici ordine diſpoſitas habere ſuas partes componentes, nempe ſecundum longitudinem, ſecundum latitudinem, atque craſſitiem; neque enim in magnitudine alia dari poteſt partium componentium ordinatio. Cum igitur inter magnitudines aliquæ ſuas partes habeant dumtaxat in longum ordinatas: aliæ vero in longum, & latum, & aliæ in longum, latum, atque profundum, (uti clarius ex infra dicendis conſtabit) dum in magnitudine aliquid deſignamus, quod nullam ex dictis habeat partium ordinationem, hoc eſt quod non ſit neque in longum, neque in latum, neque incraſſum extenſum, hoc erit PUNCTUM ab Euclide hoc in loco definitum, quod licet locum, & ſitum habeat in quantitate continva nullam tamen obtinet magnitudinem.

A B C
. . .

Mos eſt apud Mathematicos huiuſmodi Puncta ſola mente concepta acutiſſimi acus extremitate in Magnitudine ſignare, atque Alphabeticis Litteris, vel alio ſigno indicare dicendo Punctum A, Punctum B &c.

Def. II.

LINEA vocatur illa, quæ ſolam longitudinem obtinet.

SI magnitudinem aliquam mente concipiamus ſolumodo in longum extenſam, hæc erit LINEA ab Euclide definita. Mathematici ferè omnes ad indicandam veram lineæ intelligentiam aſſumunt motum puncti ſuperius deſcripti, dicendo ſcilicet, quod dum punctum e loco in locum mouetur,



cum ſit quacumque magnitudine ſpoliatum, veſtigium talis imaginarij erit ſola longitudine prædìtum. Si enim punctum A Intelligatur fluere ex loco A in B, hujus fluxus veſtigium AB erit LINEA; cum ſit quædam magnitudo ſola longitudine AB prædita; nam per ſuperiorem definitionem punctum fluens nullam obtinet dimenſionem.

SCHOLIUM.

PErmulti Mathematicarum diſciplinarum profeſſores lineam ex punctis numero infinitis tantumodo ſecundum longitudinem ordinatis compoſitam volunt, dum alij lineam ſolam exiſtere diſtantiam inter duo puncta repertum aſſeverant. Quia verò in data Lineæ definitione conſideratur ſola extenſio ſecundum longitudinem, non autem infinitorum punctorum ordinatio in longum facta, ideo ne infinito punctorum numero, quod vix adæquatè Præceptorum mentibus repræſentari poteſt, Tyrones confundamus, melius puto lineam concipere tanquam ſolam longitudinem, penitus negligendo componentia talis longitudinis, quam dictam lineam ex infinitis punctis compoſitam nobis repræſentare. Aliud enim eſt in linea prius concepta innumerabilia puncta ſignare, aliud lineam ex infinitis punctis componere. Et ut paucis multa comprehendam, dum magnitudinem in longum tantum exteuſam nobis repræſentamus integram LINEÆ *ideam efformamus definitioni ſuperius datæ conformem.*

Def. III.

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Def. III.

TERMINI lineæ dicuntur puncta.

HÆc definitio non eſt ita intelligenda, vt credatur quamlibet lineam eſſe à punctis terminatam; nam linea indefinita, circularis &c. non habent puncta terminantia, ſeu claudentia, at eſt dicendum, quod illæ lineæ, quæ extrema obtinent, in talibus extremitatibus habeant puncta: uti cernere licet in linea AB, quæ tanquam ſua extrema habet duo puncta A, & B.

Def. IV.

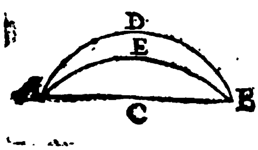
RECTA linea dicitur illa, quæ ex æquo ſua interiacet puncta.

APud Mathematicos omnes lineæ ſunt vel ſimplices, vel mixtæ. Simplices ſunt omnes lineæ rectæ, & circulares: Mixtæ dicuntur reliquæ lineæ curvæ, utpotæ ex rectis, & circularibus compoſitæ.

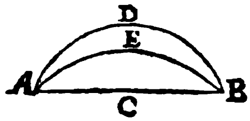
In hoc loco Euclides dat definitionem ſolius lineæ rectæ dicendo, eam eſſe, quæ æqualiter extenditur inter ſua puncta, hoc eſt nullum punctum inter extrema puncta ſignatum ab alijs, neque ſurſum, neque deorſum, neque ad dextram, neque ad ſiniſtram deflectit, ſed omnia puncta ſimul accepta totam occupant longitudinem ſitam inter lineæ extrema uti ſunt puncta A, & B: nam ſi omnia puncta deſignabilia in linea AB occupant diſtantiam cadentem inter terminos A, & B, ut inquit Proclus, talia puncta erunt ex æquo diſpoſita.



Archimedes lineam rectam definivit dicendo, eam eſſe minimam earum linearum, quæ eoſdem terminos habent: ut patet in linea ACB, quæ ſi erit minima omnium illarum linearum, quæ duci poſſunt ab A ad B, dicetur linea recta, non ſic verò lineæ ADB, AEB,



quæ



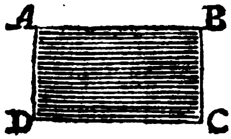
quæ cum non ſint minimæ, nec erunt dicendæ lineæ rectæ.

Inſuper quemadmodum Mathematici per imaginarium puncti fluxum quaſcumque lineas efformant, ita concipiendo punctum fluere a termino ad terminum per ſpatium omnium breviſſimum, talis fluxus ſignabit lineam rectam, qualis eſt linea ACB.

Def. V.

SUPERFICIES dicitur illa, quæ longitudinem atque latitudinem tantum habet.

QVemadmodum ſola extenſio in longum lineam efformat, ita duæ extenſiones in longum, & in latum ſuperficiem componunt. Si enim magnitudo ABCD fuerit tantum ſecundum longitudinem AB, atque ſecundum latitudinem AD extenſa, talis magnitudo erit ſuperficies ſuperius definita, a qua excluditur tertia extenſio hec eſt craſſities, ſiue profunditas, cum ſola longitudine, atque latitudine conſtituatur.



Nonnulli Authores ſuperficiem dicunt corporis terminum. Alij verò magnitudinem duobus tantum conſtantem intervallis.

Pariter ſicuti dependenter a fluxu puncti lineæ ideam efformamus, ita mediante fluxu lineæ ſuperficiem nobis poſſumus repræſentare. Si enim concipiamus lineam AB fluere in DC, hujus fluxus veſtigium erit ſuperficies ABCD.

Def. VI.

EXTREMA ſuperficiei ſunt lineæ.

IN tertia definitione puncta fuerunt poſita linearum extrema, ſiuè termini; in hac verò definitione lineæ ponuntur extrema, ſiuè termini ſuperficierum, non qui-

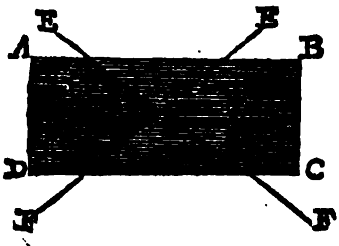
quidem omnium, ſed illarum tantum, quæ terminos habent, ſiquidem ſuperficies indefinita, atque Sphærica nullos habent terminos.

Def. VII.

PLANA ſuperficies vocatur illa, quæ ex æquo ſuas interiacet lineas.

HÆc planæ ſuperficiei definitio ſimilitudinem quam maximam habet cum definitione lineæ rectæ ſuperius datæ, ſolumque hoc intercedit diſcrimen, videlicet quod recta linea ſua puncta, ſuperficies verò ſuas lineas interiacet, hoc eſt harum linearum nulla debet ſupra, vel infra aliam reperiri.

Verum quia hoc haud ſatis eſt ad claram ideam ſuperficiei planæ efformandam, alij dicunt lineas in ſuperficie ex æquo interiacere, dum recta linea omnibus ſuperficiei partibus poteſt accomodari. Si enim quibuſcumque partibus ſuperficiei ABCD accommodetur recta linea EF, hæc dicenda erit ſuperficies plana: talis autem accommodatio, ſiuè adaptatio in hoc conſiſtit, quod omnia puncta lineæ rectæ tangant puncta ſuperficiei.



Demum attenta ſuperficiei generatione a motu lineæ facta, ſi recta AB in tranſverſum moveatur ſupra duas rectas AD, BC, feraturque in DC, hujus motus veſtigium dicendum erit ſuperficies plana ABCD, cui adæquatè conveniunt propoſitæ definitiones.

Hanc planam ſuperficiem Mathematici communiter appellant PLANUM.

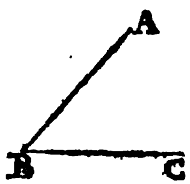
Def.

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Def. VIII.

ANGULUS PLANUS dicitur duarum linearum in plano ſe ſe mutuo tangentium, & non in directum jacentium alterius ad alteram inclinatio.

IN hac definitione Euclides ſolius Anguli plani eſſentiam exponit dicendo, inclinationem duarum linearum in eodem plano exiſtentium, atque ad unum punctum concurrentium exiſtere angulum planum. Si enim duæ lineæ AB, CB in eodem exiſtant plano, neque ſint in rectū conſtitutæ, at ad invicem inclinatæ concurrant in B, efformatus erit angulus planus ABC, cujus quantitas eſt inclinatio unius lineæ ad alteram: Magnitudo enim angulorum nō a linearum angulum efformantium longitudine, ſed a ſola inclinatione talium linearum dependet. Quantitas anguli ABC a ſola inclinatione linearum AB, CB conſtituitur. Duæ autem lineæ dicuntur non in directum jacere, quando earum altera poſt concurſum producta cum altera non coincidit, ſed ſecat; vel recedit; uti contingit in duobus circulis ſeſe tangentibus, atque in recta circulum tangente, quæ angulos efformat, licet circuli circumferentiam non ſecet.



Vltra angulum planum paulo ante deſcriptum, ſunt etiam alij anguli, qui ſolidi, atque ſphærici dicuntur. De ſolidis angulis agit Euclides in Stereometria: de Sphæricis tractat Theodoſius in ſuis Sphæricis Elementis.



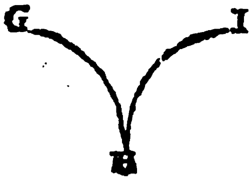
Def. IX.

RECTILINEUS Angulus dicitur ille, qui efformatur a lineis rectis.

QVilibet angulus planus efformatur a duabus lineis rectis, vel ab una recta, & alia curva, vel a duabus curvis. Angulus factus a rectis lineis, appellatur angulus

recti-

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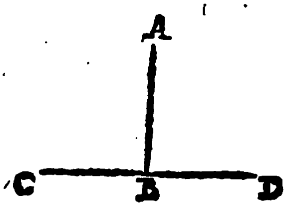


rectilineus, ut angulus ABC: ille vero qui efformatur ab una recta, & ab alia curva, vocatur angulus mixtilineus, ut angulus DEF: demum angulus comprehenſus a duabus lineis curvis, vocatur angulus curvilineus, ut angulus GHI.

Def. X.

Dum recta linea ſuper rectam conſiſtens lineam eos, qui deinceps ſunt angulos, æquales facit, uterque æqualium angulorum rectus dicitur, atque recta conſiſtens linea Perpendicularis illi, cui inſiſtit, appellatur.

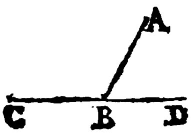
Cum apud Geometras frequentiſſimus ſit uſus anguli recti, atque lineæ perpendicularis, ideo in hac definitione Euclides exponit quiſnam ſit ille angulus, qui dicitur rectus, & quænam recta linea, quæ alteri lineæ, perpendicularis vocatur.



Si recta linea AB conſiſtat ſupra rectam CD, faciatque angulos deinceps ABC, ABD inter ſe æquales, quilibet horum æqualium angulorum rectus dicitur, atque recta AB, quæ rectos angulos efficit, vocatur perpendicularis ipſi CD.

Def. XI.

OBTUSUS Angulus ille dicitur, qui angulo recto major eſt.



Si angulus ABC major ſit angulo recto ſuperius definito, dicendus eſt angulus obtuſus.

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Def. XII.

ACUTUS Angulus ille vocatur, qui recto minor eſt.

VT Angulus A B D.

Def. XIII.

TERMINUS dicitur id, quod quantitatis extremum eſt.

QVia tres tantummodo ſunt Magnitudinum dimenſiones, nempe longitudo, latitudo, atque craſſities, ideo tres ſolum termini a Mathematicis aſſignantur, & ſunt Puncta, Lineæ, & Superficies. Puncta terminant lineas, lineæ terminant ſuperficies, atque ſuperficies terminant corpora; neque enim alius poteſt eſſe terminus, cum quantitas terminata ſemper ſuperet ſuum terminum una dimenſione: linea enim punctum ſuperat dimenſione longitudinis; ſuperficies lineam excedit latitudinis dimenſione: atque ſolidum, ſiue corpus ſuperficiem ſuperat dimenſione craſſitiei.

Def. XIV.

FIGURA dicitur illa, quæ ſub aliquo, vel aliquibus terminis comprehenditur.

IN hac definitione eſt obſervandum ad conſtituendam figuram haud ſat eſſe magnitudinem habere terminatam, nam linea quamvis terminata non dicitur figura, quia figura ab omni parte debet eſſe undequaque inter proprios terminos clauſa, qui termini nequeunt eſſe puncta, ſed lineæ in figuris ſuperficialibus, vel ſuperficies in figuris ſolidis, ſiue trinam dimenſionem habentibus.

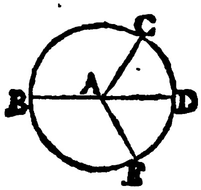
Def.

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Def. XV.

CIRCULUS vocatur illa figura plana ſub una linea comprehenſa, quæ peripheria, ſivè circumpherentia appellatur; ad quam ab uno puncto eorum, quæ intra figuram ſunt poſita, cadentes omnes rectæ lineæ ſunt æquales.

In hac definitione Euclides indicat eſſentiam illius figuræ planæ, quæ CIRCULUS dicitur, oſtendendo ad circuli conſtitutionem requiri, quod ſit figura plana unica linea comprehenſa, uti eſt figura BCE ab unica linea curua BCDE terminata, intra quam figuram datur punctum A, a quo omnes rectæ lineæ AB, AD, AC &c. ad comprehendentem lineam BDE ductæ ſunt æquales.



Linea curva comprehendens Circulum a Græcis Peripheria, & à Latinis Circumpherentia nuncupatur.

Def. XVI.

CENTRUM circuli dicitur illud punctum; ut punctum A, a quo omnes rectæ lineæ AB, AD &c. ad peripheriam BDE ductæ, ſunt æquales.

Def. XVII.

Diameter circuli appellatur quæcumque recta linea per circuli centrum ducta, & utrimque a circumpherentia terminata, quæ bifariam circulum dividit.

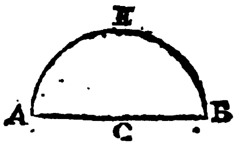
TAlis eſt recta BAC.

Def.

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Def. XVIII.

SEMICIRCULUS vocatur figura ſub diametro, & dimidia circumpherentia comprehenſa.



TAlis eſt figura AEB comprehenſa diametro ACB, atque dimidia circumpherentia AEB.

Def. XIX.

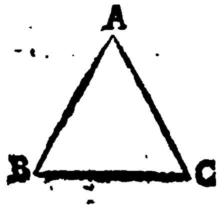
RECTILINEÆ figuræ appellantur illę, quæ a rectis lineis comprehenduntur.

OMnes illæ figuræ, quæ a rectis lineis comprehenduntur, rectilineæ figuræ appellantur. Si a curvis lineis terminentur, dicuntur figuræ curvilineæ: ſi vero inter alicujus figuræ terminos ſint & rectæ, curvæque lineæ, tunc figura nuncupatur mixtilinea.

Def. XX.

TRILATERÆ figuræ dicuntur, quæ a tribus lateribus, ſive lineis continentur.

CUm figura BAC comprehendatur a tribus lateribus BA, AC, CB, hæc dicenda eſt figura trilatera, ſiue, uti eſt communis loquendi uſus, Triangulum.



In Triangulo duo latera AB, AC dicuntur latera comprehendentia, ſive efformantia angulum BAC; latus vero BC dicitur ſubtenſum angulo BAC, atque angulus BAC dicitur comprehenſus a lateribus AB, AC, atque ſubtendens latus BC.

Def.

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Def. XXI.

QUADRILATERÆ appellantur figuræ, quæ ſub quatuor tantum lineis continentur.

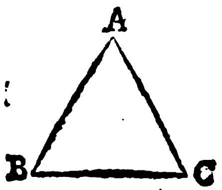
Def. XXII.

MULTILATERÆ figuræ nuncupantur, quæ ſub pluribus quàm quatuor lineis comprehenduntur.

MUltilateræ figuræ rectilineæ generali nomine POLIGONA nuncupantur.

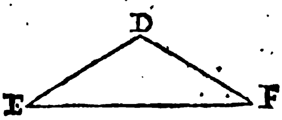
Def. XXIII.

Æquilaterum Triangulum illud dicitur, quod tria latera habet æqualia.



SI in triangulo BAC æqualia ſint tria latera BA, AC, CB, hoc dicendum erit triangulum æquilaterum, ſiue æquicrure.

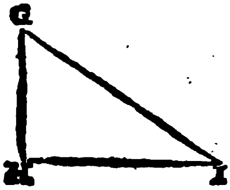
Def. XXIV.



ISOSCELES triangulum dicitur, quod duo tantum obtinet latera æqualia.

SI in triangulo EDF duo tantum reperiantur æqualia latera DE, DF, hoc triangulum vocatur Iſoſceles.

Def. XXV.



Scalenum triangulum illud appellatur, quod tria latera habet inæqualia.

SI triangulum HGI tria latera HG, GI, IH, inæqualia habeat, ſub nomine trianguli Scaleni indicatur.

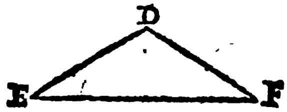
Def. XXVI.

ORTOGONIUM, ſive RECTANGULUM triangulum appellatur, quod habet unum angulum rectum.

SI triangulum GHI habet angulum H rectum, a Latinis dicitur triangulum rectangulum, a Græcis vocatur triangulum Ortogonium.

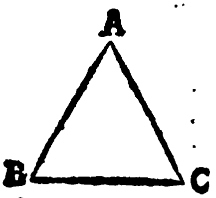
Def. XXVII.

AMBLYGONIUM, ſive obtuſangulum triangulum dicitur, quod unum obtuſum angulum habet.



TAle eſt triangulum EDF, quod habet angulum obtuſum D.

Def. XXVIII.



OXYGONIUM, ſive acutangulum triangulum appellatur, quod tres angulos habet acutos.

HUiuſmodi eſt triangulum BAC, cum habeat tres angulos A, B, C, acutos.

Def.



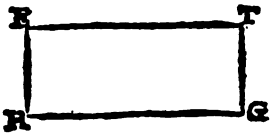
Def. XXXI.

QUADRATUM dicitur illa figura quadrilatera, cujus quatuor latera ſunt æqualia, & quatuor quoque anguli æquales.

SI quadrilatera figura ABCD quatuor habeat latera æqualia AB, BC, CD, DA, quatuorque angulos A, B, C, D, æquales, ſeu rectos, dicetur Quadratum.

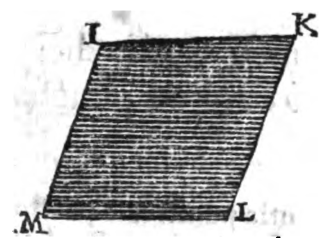
Def. XXX.

Altera parte longior, ſeu Quadrilongum vocatur illa figura quadrilatera, quę quatuor obtinet angulos æquales, non autem quatuor latera æqualia.



HOc cernere licet in figura ETGH, in qua omnes anguli E, T, G, H, ſunt æquales, atque recti, ut in quadrato, latera vero non omnia, at ſola oppoſita ET, HG, ſiue EH, TG æqualia.

Def. XXXI.



RHOMBUS dicitur figura quadrilatera, quæ æquilatera quidem eſt, ſed non rectangula.

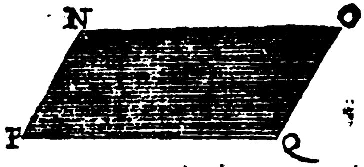
SI quadrilatera figura IKLM quatuor latera IK, KL, LM, MI habuerit æqualia, angulos vero I, K, L, M non rectos, dicetur, Rhombus.

 Def.

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Def. XXXII.

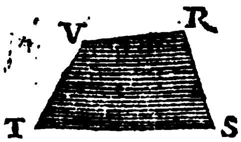
RHOMBOIDES nuncupatur illa figura quadrilatera, quæ oppoſita latera, & angulos habet æquales, cum neque æquilatera, neque æquiangula ſit.



SI in figura quadrilatera NOQP non tantum latera oppoſita NO, PQ: NP, OQ, verum etiam oppoſiti anguli O, & P; N, & Q æquales erunt, hæc Rhomboides nominabitur.

Def. XXXIII.

TRAPEZIA vocantur reliquæ omnes figuræ quadrilateræ.



SI figura quadrilatera RSTV non ſit Quadratum, nec Quadrilongum, nec Rhombus, nec Rhomboides, Trapezium appellabitur, in quo neque latera, neque anguli oppoſiti ſunt æquales.

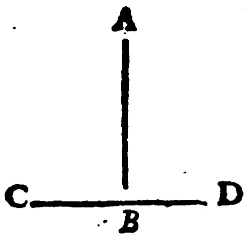
Def. XXXIV.

DISTANTIA unius puncti ab alio dicitur longitudo lineæ rectæ ab uno ad aliud punctum ductæ.



DIſtantia puncti A a puncto B eſt longitudo rectæ lineæ AB.

Def.



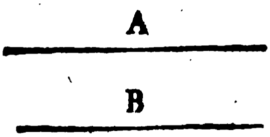
Def. XXXV.

DISTANTIA unius puncti ab aliqua recta linea vocatur perpendicularis a puncto ad rectam lineam ducta.

DIſtantia puncti A a recta CD eſt perpendicularis AB.

Def. XXXVI.

ÆQVALES dicuntur illæ Magnitudines, quarum differentia eſt nihil, vel quantitas inaſſignabilis.



SI differentia inter magnitudines A, & B ſit nihil, vel ſi ſit aliqua quantitas inaſſignabilis, magnitudines A, & B vocantur æquales.

Def. XXXVII.

INÆQVALES appellantur illæ Magnitudines, quarum differentia eſt aliqua quantitas aſſignabilis.

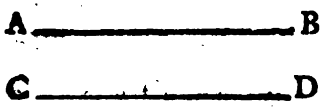
SI inter quantitates A, & B ſit aliqua aſſignabilis differentia; dictæ quantitates dicendæ ſunt inæquales.

Def. XXXVIII.

CONGRUENTES dicuntur magnitudines, quando facta talium magnitudinum ſuper-

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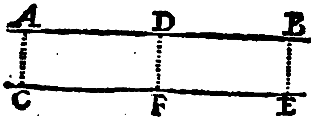
impoſitione, partes unius magnitudinis partibus alterius adaptantur.



SI recta AB ſuper impoſita rectæ CD habeat omnia ſua puncta adaptata punctis rectæ CD, hoc eſt quodlibet ipſius AB habeat aliud correſpondens punctum in recta CD, tunc dictæ magnitudines dicuntur congruentes.

Def. XXXIX.

PARALLELÆ rectæ lineæ appellantur, quæ in eodem exiſtentes plano, ſi ex utraque parte indefinitè producantur, ſemper ad invicem eamdem diſtantiam ſervant.



AD hoc ut duæ rectæ lineæ parallelæ, ſive non concurrentes dicantur, duæ requiruntur conditiones, quarum una eſt quod ſint in eodem plano conſtitutæ, altera quod quomodocumque productę eamdem diſtantiam ad invicem ſervent. Si enim duæ rectæ lineæ AB, CE fuerint in eodem plano ſignatæ, eamdemque in omnibus partibus habuerint diſtantiam, hoc eſt diſtantiæ AC, DF, BE &c. ſint æquales, tunc duæ rectæ lineæ AB, CD, dicendæ erunt Parallelæ, ſeu non concurrentes.

Def. XXXX.

PARALLELOGRAMMUM dicitur illa figura plana quadrilatera, in qua bina oppoſita latera ſunt parallela, ſeu æquidiſtantia.

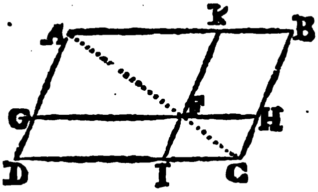
IN figura quadrilatera NOPQ, ſi latera oppoſita NO, PQ, nec non etiam NP, OQ, fuerint parallela, hæc vocabitur Parallelogrammum. In

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Parallelogramma illa, quæ habent angulos rectos, uti ſunt Quadratum, & Quadrilongum, dicuntur RECTANGULA; reliqua vero, quæ angulos habent obliquos, ut Rhombus, & Rhomboides, vocantur Parallelogramma non rectangula.

Def. XXXXI.



SI in Parallelogrammo DB ducta ſit diameter AC, atque per punctum F in diametro AC ſignatum, ductæ ſint rectæ GFH, KFI parallelæ lateribus Parallelogrammi, hoc eſt GFH parallela DC, & KFI parallella BC, totum Parallelogrammum erit diviſum in quatuor Parallelogramma, quorum duo, nempe DF, FB vocantur COMPLEMENTA, duo vero alia GK, IH, dicuntur Conſiſtentia circa diametrum AC.



PETITIONES, ſeu POSTULATA.

Petitio I.

A quovis dato puncto ad quodvis datum punctum rectam lineam ducere.



HÆc prima petitio poſtulat, ut ſemper concedatur a quovis puncto ad quodvis pnnctum lineam rectam ducere: ſi enim data ſint puncta A, & B, etiam permitti debet poſſe duci rectam AB.

Petitio II.

Rectam lineam terminatam ſemper ulterius in rectum producere.

Petitio III.

Quocumque dato centro, & intervallo Circulum deſcribere.

Petitio IV.

Quodcumque punctum in magnitudine ſignare.

Petitio V.

Unam Magnitudinem alteri ſuper imponere.

Petitio VI.

Quacumque magnitudine data aliam ſumere magnitudinem data magnitudine majorem, vel minorem.

AXIO-

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Axiomata, quæ communes Notiones, Pronunciata, atque Dignitates vocantur.

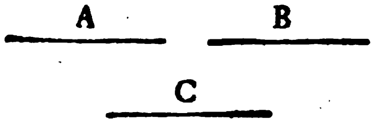
AXiomata ſunt quædam veritates adeo manifeſtæ, ut probatione non indigeant, quemadmodum ex infra ponendis Axiomatis conſtabit.

Axioma I.

Illæ quantitates, quæ eidem quantitati ſunt æquales, inter ſe ſunt æquales; & viciſſim illæ quantitates, quibus eadem quantitas eſt æqualis, ſunt æquales inter ſe.

Quod uno æqualium majus eſt, aut minus, majus quoque eſt, aut minus altero æqualium.

Si unum æqualium majus eſt, aut minus alio aliqua quantitate, alterum quoque æqualium illo alio majus, vel minus erit eadem quantitate.



SI A æquat C, atque B æquat eamdem C; A ęqualis erit ipſi B. Pariter ſi C adæquet A, & B; A, & B æquales erunt.

Denuo ſuppoſitis æqualibus A, & B, ſi C major eſt, aut minor A, major quoque, vel minor erit quam B.

Denum æqualibus exiſtentibus A, & B illum exceſſum, quem habet C ad A, habebit etiam ad B.

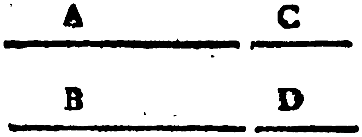
SCHOLIUM.

AD perfectam intelligentiam huius axiomatis neceſſarium putavi animadvertere Euclidem, alioſque Geometras æqualitatem

tatem identitati æquiparare, atque æqualitatem tamquam identitatem recipere; qua in re ſat erit unum proponere exemplum, nam dum Euclides ex tribus datis rectis lineis triangulum efformare vult, illud ex lineis, quæ ſint æquales lineis datis, atque hoc componit triangulum taliter efformatum recipit tamquam ex iiſdem lineis datis compoſitum, cum lineæ triangulum componentes ſint datis lineis æquales. Quo ſtante facile erit poſiti axiomatis veritatem dignoſcere; nam ſi A adæquat C: inde ſequitur A eſſe idem ac C: quod etiam valet de ipſo B: ſi igitur A, & B ſunt idem ac C; A erit idem ac B quo ad quantitatem, quæ ſola in hac doctrina conſideratur.

Axioma II.

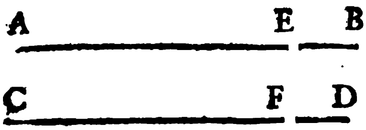
Si quantitatibus æqualibus adjiciantur æquales quantitates, illa tota, quæ reſultant, erunt æqualia.



SI æqualibus quantitatibus A, & B additæ ſint æquales quantitates C, D; totum compoſitum ex A, & C adæquabit totum factum ex B, & D.

Axioma III.

Si ab æqualibus quantitatibus æquales quantitates ablatæ ſint, quæ remanent quantitates erunt æquales.



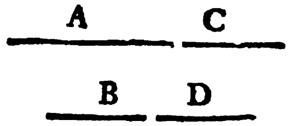
SI ab æqualibus quantitatibus AB, CD, detractæ ſint æquales quantitates EB, FD, reſidua AE, CF erunt æqualia.

Axioma IV.

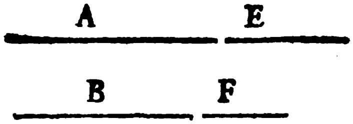
Si quantitatibus inæqualibus additæ ſint æquales quantitates, tota reſultantia erunt inæqualia.

Si

Si inæqualibus inæqualia adjecta ſint, majori majus, & minori minus, tota, quæ reſultant, erunt inæqualia, illud nimirum majus, & hoc minus.



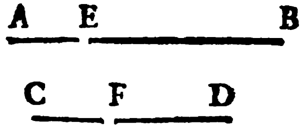
DUm inæqualibus quantitatibus A, & B adiunguntur æquales quantitates C, & D; tota reſultantia ex A, & C, & ex B, & D. erunt inæqualia.



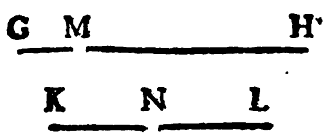
Pariter ſi inæqualibus quantitatibus A, & B additæ ſint inæquales quantitates E, & F, cum hac lege, quod majori A adiungatur major E, atque minori B addatur minor F; totum, quod reſultat ex A, & E majus erit toto reſultante ex B, & F.

Axioma V.

Si ab inæqualibus quantitatibus ablatæ ſint æquales quantitates, reſidua erunt inæqualia. Si ab inæqualibus quantitatibus inæquales quantitates ablatæ ſint, cum hac conditione quod ex majore auferatur minus, & ex minore majus, reſidua pariter erunt inæqualia.



Si a quantitatibus inæqualibus AB, CD detractæ ſint æquales quantitates AE, CF; reſiduum EB majus erit reſiduo FD.



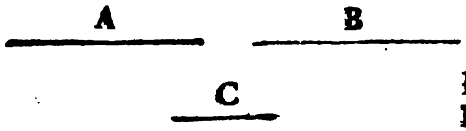
Pariter ſi ab inæqualibus quantitatibus GH, KL ablatæ ſint inæquales quantitates GM, KN, cum hac lege ut GM ablatum ex majore GH minus ſit

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ſit KN ablato ex minore KL: reſiduum MH majus erit reſi-duo NL.

Axioma VI.

Illæ quantitates, quæ ejuſdem quantitatis duplæ ſunt, inter ſe ſunt æquales. Id, quod unius æqualium duplum eſt, & alterius æqualium duplum erit. Quando unum æqualium alicujus duplum eſt, etiam cœtera æqualia ejuſdem dupla erunt.



SI tam A quam B ſint ipſius C dupla; A, & B erunt inter ſe æqualia. Poſitis æqualibus A, & B, ſi C ſit duplum ipſius A, idem C erit etiam duplum ipſius B. Pariter ſuppoſita æqualitate inter A, & B, ſi A duplum ſit ipſius C, etiam cætera alia ipſi A æqualia ut B, ejuſdem C dupla erunt.

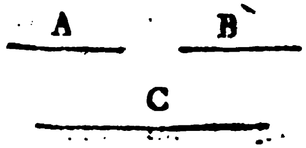
SCHOLIUM.

QVod de ſola duplicitate ab Euclide dictum fuit, valet etiam de triplicitate, quadruplicitate, ſubduplicitate, ſubtriplicitate &c. Nam ſi A, & B ſint tripla, vel ſubtripla ipſius C; æqualia erunt A, & B.

Axioma VII.

Illæ quantitates, quæ ejuſdem ſunt dimidiæ, inter ſe ſunt æquales; & viciſſim ſi una æqualium quantitatum ſit unius dupla, etiam aliæ ejuſdem duplæ erunt.

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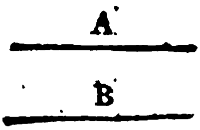
SI A, & B æqualia ſint, atque A dimidium ipſius C; etiam B dimidium erit ipſius C.

Quod de dimidia parte dictum eſt, valet etiam de quacumque alia parte.

Axioma VIII.

Quantitates, quæ ſibi mutuo congruunt, inter ſe ſunt æquales, & viciſſim quantitates congruentiæ capaces, ſi æquales ſint, erunt etiam congruentes.

PEr def. 38. Mathematici nomine congruentiæ intelligunt talem duarum quantitatum adaptationem, ſecundum quam neutra illarum quantitatum alteram excedit: quo ſtante facile erit comprehendere congruentes quantitates eſſe æquales, cum congruentium quantitatum differentia nulla ſit.



Pariter ſi quantitates congruentiæ capaces, uti ſunt omnes lineæ rectæ, ſuperficies planæ, & ſimiles, anguli plani &c. ſint æquales, erunt etiam congruentes, ſi enim recta linea A ſit rectæ lineæ B æqualis, etiam A congruet cum B.

Axioma IX.

Omne totum majus eſt ſua parte.

SI a toto pars auferatur, aliquid remanet, ſecus totum fuiſſet ablatum: quare totum majus erit ſua parte.

Axioma X.

Duæ rectæ lineæ nequeunt habere idem ſegmentum commune.

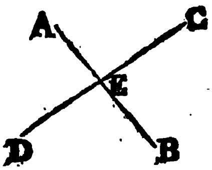
VEritas hujus axiomatis clare percipitur, ſi intelligamus naturam lineæ rectæ, quæ in hoc conſiſtit, videlicet quod pro-

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producta ſemper recto itinere procedat; unde ſequitur nec fieri, nec concipi poſſe, unam lineæ rectæ partem quantitativam, quæ nomine ſegmenti intelligitur, duabus rectis lineis communem exiſtere, quod eſt idem ac dicere, duas rectas lineas nullatenus commune ſegmentum habere poſſe.

Axioma XI.

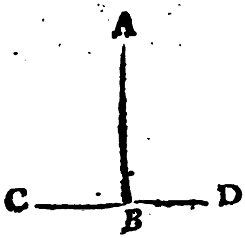
Duæ rectæ lineæ in uno puncto concurrentes, ſi ambæ producantur, neceſſario in eodem puncto ſe mutuo interſecabunt.



SI duæ rectæ lineæ AE, CE concurrant in E productæ verſus B, & D, ſe mutuo interſecabunt in puncto E, cum hoc ſolum punctum ſit commune utrique rectæ lineæ AB, CD.

Axioma XII.

Omnes anguli recti ſunt inter ſe æquales.

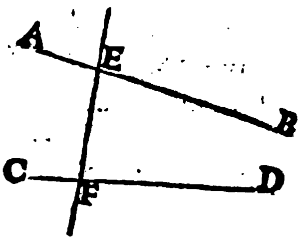


VEritas huius axiomatis apertè dignoſcitur ex definitione 10., in qua dicitur angulum rectum efformari a perpendiculari. Vt ex. g. ſi recta AB perpendiculariter inſiſtat rectæ CD, anguli ABC ABD erunt anguli recti: cum ergo immutabilis ſit conſtitutio lineæ perpendicularis, quia dependet ab æqualitate angulorum ABC, ABD; inde ſequitur etiam immutabilem eſſe illam inclinationem, per quam efformantur anguli recti: quare omnes anguli recti erunt inter ſe æquales.

Axioma

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Axioma XIII.

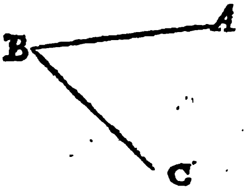


Si in duas rectas lineas AB, CD in eodem plano exiſtentes incidat recta EF, quæ faciat duos internos, & ad eaſdem partes angulos BEF, DFE duobus rectis angulis minores; duæ rectæ AB, CD indefinitè productæ debent concurrere ad partes B, & D, ad quas ſunt duo anguli duobus rectis minores.

PRæſens axioma, quod eſt *XI.* Euclidis, a quampluribus Scriptoribus, & merito quidem ab axiomatum numero excluditur, & inter propoſitiones demonſtrandas reponitur. Nos autem dicenda circa hanc veritatem ad prop. 29. remittimus, quo in loco dumtaxat hujus axiomatis veritas primitus locum habet.

Axioma XIV.

Duæ rectæ lineæ ſpatium non comprehendunt.

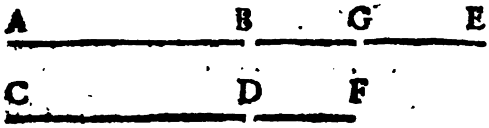


SI duæ rectæ lineæ AB, CB concurrant in B efformantes angulum ABC, propter eſſentiam lineæ rectæ ad partes A, & C ſemper debent ad invicem recedere; quare ad partes A, & C numquam poterunt concurrere; uti neceſſarium foret ad hoc ut ſpatium comprehenderent, ſive clauderent.

Axioma XV.

Si æqualibus quantitatibus AB, CD inæquales quantitates addantur BE, DF, erit totorum exceſſus adiunctorum exceſſui æqualis.

Po-



POſitis æqualibus quantitatibus AB, CD, ſi illis adiungantur inæquales quantitates BE, DF, atque BE excedat DF magnitudine GE; exceſſus totius AE ſupra totum CF erit GE, (*cum per Axio.* 1.) AG adæquet CF.

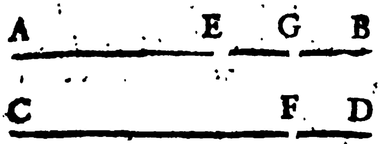
Axioma XVI.

Si quantitatibus inæqualibus, BE, DF æquales addantur quantitates AB, CD; exceſſus totius AE ſupra totam CF adæquabit exceſſum BE ſupra DF.

SIt GE exceſſus BE ſupra DF. Cum igitur AG adæquet CF, erit GE exceſſus AE ſupra CF, idem ac exceſſus BE ſupra DF.

Axioma XVII.

Si ab æqualibus quantitatibus AB, CD, auferantur inæquales quantitates EB, FD, ſitque EG exceſſus ablati EB ſupra ablatum FD, exceſſus reſidui CF ſupra reſiduum AE adæquabit EG exceſſum ablatorum EB, FD.



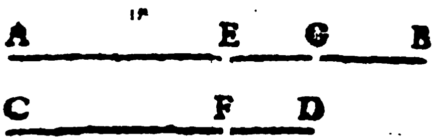
CUm ab æqualibus AB, CD, ablata ſint æqualia GB, FD (per Axio. 3.) erit AG æqualis ipſi CF: quare exceſſus reſidui CF ſupra reſiduum AE erit EG exceſſus ablatorum EB, FD.

Axioma

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Axioma XVIII.

Si ab inæqualibus quantitatibus AB, CD auferantur æquales quantitates AE, CF, exceſſus reſidui EB ſupra reſiduum FD, adæquabit exceſſum totius AB ſupra totum CD.



SIt GB exceſſus AB ſupra CD: hoc poſito erit AG æqualis ipſi CD, atque EG (*per axio.* 3.) æqualis ipſi FD; quare GB erit exceſſus reſidui EB ſupra reſiduum FD.

Axioma XIX.

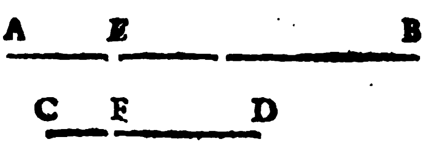
Totum adæquat omnes ſuas partes ſimul ſumptas.



QUantitas totius AB componitur ex quantitatibus omnium partium AC, CD, DB: quare evidens eſt totum AB adæquare omnes ſuas partes componentes AC, CD, DB.

Axioma XX.

Si totum AB ſit totius CD duplum, atque ablatum AE duplum ablati CF; erit reſiduum EB duplum reſidui FD.



QUia ablatum AE ſupponitur duplum ablati CF: pariter quia AB ponitur duplum CD, erit CF dimidium AE, & FD dimidium EB: quare reſiduum EB erit duplum reſidui FD.

Axioma XXI.

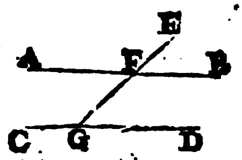
Si fuerint tres quantitates, quarum prima ſuperet ſecundam, atque ſecunda ſuperet tertiam, etiam prima tertiam ſuperabit.

Axioma XXII.

Illæ quantitates, quæ inæquales eſſe non poſſunt, ſunt æquales, ſi ſint ex eodem genere.

Axioma XXIII.

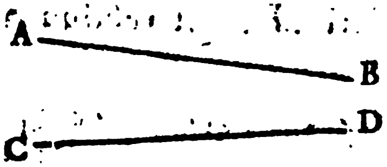
Quando recta linea unam parallelarum ſecat, producta ſecabit & alteram.



Si recta linea EF ſecet parallelam AB in F, producta ſecabit etiam alteram parallelam in G.

Axioma XXIV.

Duæ rectæ lineæ non parallelæ in eodem plano exiſtentes productæ ad unam partem concurrunt.



Si duæ rectæ AB, CD in eodem plano conſtitutæ non ſint parallelæ, productæ ad partes B,D, vel A,C debent concurrere.

Axioma XXV.

Recta linea in unico tantum puncto bifariam dividi poteſt.

Quid

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Quid ſit Definitio, quid Poſtulatum &c.

UT Tyrones facilius percipiant diſcrimen, quod eſt inter Mathematicorum Definitiones, Poſtulata, Axiomata, Propoſitiones, Problemata, Corollaria, Lemmata, & Scholia, neceſſarium duxi circa horum terminorum ſignificationem aliquid addere antequam ad propoſitionum demonſtrationes veniam.

Definitionis nomine nil aliud intelligitur quam illud determinatum rei eſſe, quod mens noſtra rebus tribuit; uti evenit, dum rectam lineam concipimus, cujus omnia puncta ſunt ex æquo diſpoſita; vel dum concipimus angulum planum rectilineum eſſe duarum rectarum linearum in eodem plano exiſtentium, atque ad punctum concurrentium inclinationem, vel Quadratum eſſe figuram quadrilateram, in qua quatuor latera, & quatuor anguli ſunt æquales. Hujuſmodi definitiones aliud non videntur eſſe, quam ſimplices noſtræ Mentis apprehenſiones, quibus res tali determinato modo nobis repræſentamus.

Poſtulatum dicitur illud, quod aliquid poſſibile expoſcit uti evenit dum a puncto ad punctum rectam lineam duci poſtulamus.

Axioma eſt illud, quod per ſe notam, atque indemonſtrabilem veritatem proponit, ut dum dicimus: quæ æqualia ſunt uni tertio, ſunt æqualia inter ſe.

Propoſitio generaliter accepta illa eſt, quæ aliquid proponit, uti eſt illa, quæ dicit in triangulo tres angulos eſſe æquales duobus rectis.

Theorema eſt illa propoſitio, in qua aliquid demonſtrandum proponitur; uti evenit dum dicimus in iſoſcele triangulo ad baſim angulos exiſtere inter ſe æquales.

Problema eſt illa propoſitio, in qua aliquid faciendum proponitur; uti contingit, dum angulum rectilineum bifariam diuidendum proponimus.

Poſtulatum convenit cum Problemate in hoc quod utrumque aliquid fieri expoſcat: differt verò, quia Poſtulati operatio, ut admittatur legitimè facta, non indiget demonſtratione, operatio verò a Problemate propoſita demonſtrationem requirit, ut legitimè facta concedatur.

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Pariter Axioma, atque Theorema hoc diſcrimen habent, quod ſcilicet Axioma aliquid proponit, quod libenter ſine demonſtratione verum conceditur, ex adverſo autem poſitum a Theoremate non eſt ulla ratione concedendum tamquam verum, niſi fuerit demonſtratum. Nullus enim demonſtrationem requirit, dum dicimus, æqualia uni tertio eſſe æqualia inter ſe; quam quidem demonſtrationem exoptamus, dum dicitur, in triangulo tres angulos eſſe æquales duobus rectis. Inſuper Poſtulatum eſt veluti principium operationis; axioma vero demonſtrationis.

Corollarium eſt veritas, quæ infertur ab aliqua demonſtratione: poſita namque demonſtratione quod in triangulo iſoſcele æquales ſint anguli ſupra baſim, ſi denuò inferatur quod in triangulo æquilatero tres anguli ſint æquales, quia triangulum æqualium laterum quaquauerſum eſt iſoſceles, hoc corollarium erit.

Lemma eſt propoſitio nullatenus ſpectans ad illam, quam pertractamus doctrinam, ſolumque accepta ad demonſtrandum id, quod opus eſt demonſtrare in aliqua doctrina.

Scholium eſt ſola adnotatio facta circa aliquam propoſitionem.

Signorum explicatio.

def.	Definitio.
Poſt., Pet.	Poſtulatum, Petitio.
Axio.	Axioma.
$=$	Æqualitas.
$+$	Plus.
$-$	Minus.
AB X CD	Rectangulum a rectis lineis AB, CD factum.
q. EF	Quadratum a recta linea EF factum.
q. e. d.	Quod erat demonſtrandum.
q. e. f.	Quod erat faciendum.

Propoſ.

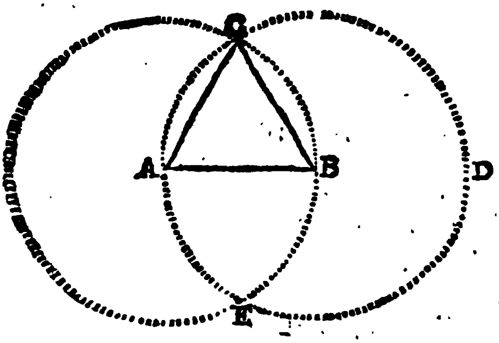
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Propoſ. I. Probl. I.

Super data recta linea AB terminata triangulum æquilaterum conſtituere.

Conſtructio.

CEntro A, & intervallo AB (*per. pet.* 3.) deſcribatur circulus CBE. Pariter centro B, & interuallo BA (*per pet.* 3.) notetur alius circulus CAE, qui priorem circulum neceſſariò interſecabit, quia punctum A ſpectans ad circulum CAE eſt intra circulum CBE, punctumque D, quod ſpectat ad eumdem circulum CAE, eſt extra circulum CBE, cum (*per Axio.* 9.) major ſit recta AD quam AB. Sit igitur talium circulorum interſecatio punctum C. A punctis A, & B ad punctum C (*per pet.* 1.) ducantur rectæ lineæ AC, BC: hoc facto ſupra datam rectam AB erit efformatum triangulum ACB, quod



Dico eſse æquilaterum.

Demonſtratio.

AB = AC (*per def.* 15.) BA = BC (*per eamdem def.*) ergo (*per Axio.* 1.) AC = BC: quare triangulum ABC erit æquilaterum. q.e.d.

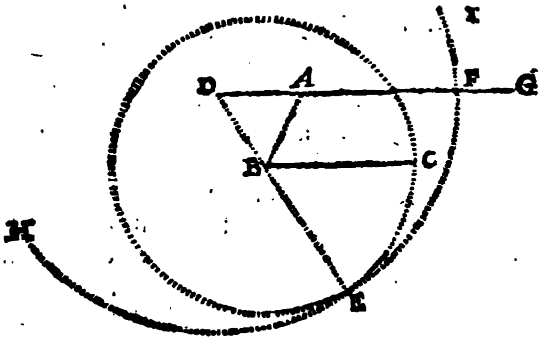
Propoſ. II. Prob. II.

Ad punctum datum A datæ rectæ lineæ BC æqualem rectam lineam ponere.

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HOc problema duas habet partes, unam ſcilicet, quando datum punctum A eſt extra lineam datam BC; alteram vero dum punctum A in recta BC reperitur.

Conſtructio primæ partis.



AB una extremitatum rectæ BC, ut a puncto B tanquam centro, & intervallo BC (*per pet.* 3.) deſcribatur circulus CE.

A centro B ad A (*per pet.* 1.) ducatur recta BA.

Super recta BA (*per prop.* 1.) ſignetur triangulum æquilaterum BDA.

Latus BD producatur uſque ad circumpherentiam in E, latuſque DA indefinitè producatur verſus G.

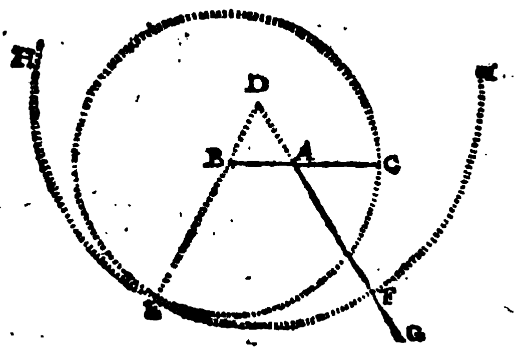
Demum centro D, & diſtantia DE (*poſt.* 3.) notetur circulus HEF, qui lineam AG ſecet in F.

Dico AF = BC.

Demonſtratio.

DE = DF (*per def.* 15.) DB = DA (per conſtructionem): ergo (*per axio.* 3.) AF = BE; ſed eidem BE = BC (*per def.* 15.) ergo (*per axio.* 1.) AF = BC. q. e. d.

Conſtructio ſecundæ partis.



CEntro B, uti prius, diſtantia BC (*per pet.* 3.) deſcribatur circulus CE.

Super BA (*per prop.* 1.) fiat triangulũ æquilaterum BDA, & DB producatur uſque ad circumferentiam CE. DA producatur indefinitè verſus G. De-

Demum centro D, & intervallo DE ſignetur circulus EF, qui rectam DG ſecet in F.

Dico AF = BC.

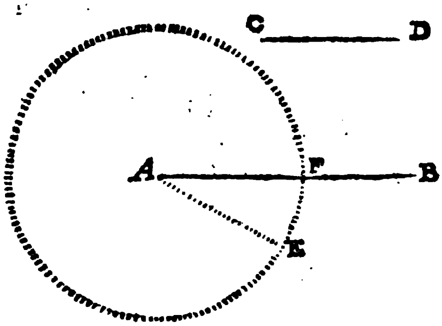
Demonſtratio hujus partis eadem eſt, quæ prioris, hoc eſt DE = DF (*per def. circuli*) DB = DA (*per conſtructionem*) ergo AF = BE (*per axio. 3.*) atqui etiam BC = BE (*per def. circuli.*) ergo (*per axio. 1.*) BC = AF. q. e. f.

Propoſ. III. Probl. III.

Datis duabus rectis lineis inæqualibus AB, CD, de maiore linea AB aufere partem æqualem minori lineę CD.



Conſtructio.



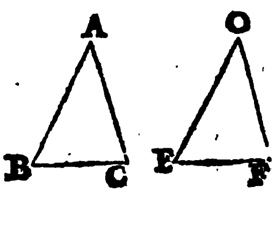
AD punctum A (*per prop. 2.*) ponatur recta AE = CD Centro A, & diſtantia AE (*per poſt. 3.*) ſignetur circulus EF, qui ſecet AB in F.

Dico AF = CD.

Demonſtratio.

CD = AE (*per conſtructionem*); AF = AE (*per def. 15.*): ergo (*per axio. 1.*) AF = CD. q. e. f.

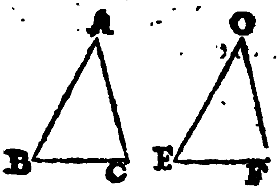
Propoſ. IV. Theor. I.



Si duo triangula BAC, OEF habeant duo latera ſingula ſingulis ęqualia, nempe latus BA = OE, & latus AC = OF,

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habeantque angulos BAC, EOF ab ęqualibus lateribus contentos ęquales.



Dico primo Baſim BC = baſi EF.

Dico ſecundo Triangulum BAC = triangulo EOF.

Dico tertio reliquos angulos, ſub quibus æqualia latera ſubtenduntur eſſe æquales: hoc eſt angulum ABC = angulo OEF; & angulum ACB = angulo OFE.

Demonſtratio primæ partis.

Per hypotheſim AB = OE; quare ſi AB ipſi OE ſuper imponatur taliter, ut duo puncta A, & O congruant (*per axio.*8.) etiam punctum B congruet cum puncto E, totaque AB congruet cum tota OE. Servata hac congruentia ſi angulus BAC fuerit ſuperpoſitus æquali angulo EOF, latus AC congruet cum latere OF, atque propter ſuppoſitam talium laterum æqualitatem punctum C cadet in F: Quo ſtante extrema baſium BC, EF erunt congruentia: quare & tota baſis BC congruet baſi EF, ſecus duæ rectæ lineæ ſpatium clauderent, quod eſt contra axio. 14. Cum igitur duæ triangulorum baſes BC, EF, ſint conguentes (*per axiom.* 8.) erunt æquales. q. e. d.

Demonſtratio ſecundæ partis.

Cum in prima parte fuerit demõſtratum, omnia latera trianguli ABC congruere lateribus trianguli OEF; inde ſequitur totum triangulum ABC congruere toti triangulo OEF: quare (*per axio.* 8.) dicta triangula erunt æqualia q. e. 2. d.

Demonſtratio tertiæ partis.

In prima parte hujus prop. demonſtratum eſt non ſolum latera AB, BC congruere lateribus OE, EF, verum etiam latus AC congruere lateri OF: inde ſequitur non tantum angulum ABC congruere angulo OEF, verum etiam angulum ACB congruere angu-

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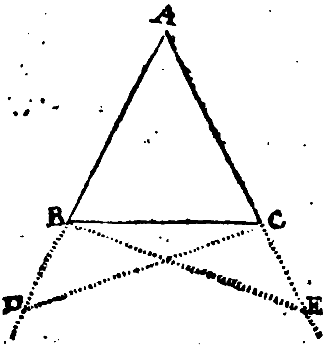
angulo OFE : ſed (*per axio.* 8.) illa, quæ congruunt, ſunt æqualia : ergo angulus ABC = angulo OEF, & angulus ACB = angulo OFE. q. e. 3. d.

SCHOLIUM.

PErmulti Authores hanc Euclidis demonſtrationem condemnant, utpote non Geometricam, cum utatur ſuperimpoſitione, quæ apud iſtos Authores potius Mecanicum, quam Geometricum ſapit. Verum ſi rem iſtam æquo animo perpendamus clarè apparebit, hanc non phyſicam, at mentalem rectarum linearum ſuperimpoſitionem exiſtere: qua de re talis ſuperpoſitio non erit verè mecanica, cum tantumodo oculis mentis, non autem corporis oſtendat talium triangulorum proprietates a primis principijs deductas. Ulterius liceat addere, quod illi Scriptores, qui datam Euclidis demonſtrationem nolunt admittere tamquam Geometricam, meliorem uſque adhuc non excogitarunt: quam ob rem ne Geometrica hæc Elementa hac propoſitione ſummopere neceſſaria deſtituantur, conſonum erit ſuperius propoſitam demonſtrationem tamquam geometricam recipere, quouſque melior ſit adinuenta.

Propoſ. V. Theor. II.

In triangulo Iſoſcele ABC, in quo duo latera æqualia ſint AB, AC.



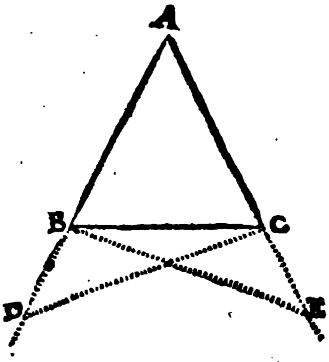
DIco primo angulos ſupra baſim ABC, ACB eſſe æquales. Dico ſecundo, productis æqualibus lateribus AB, AC in D, & E angulos pariter infra baſim CBD, BCE eſſe æquales.

Conſtructio.

In latere AB producto ſignetur quodcumque punctum, ut D; ex alio latere AC producto (*per. prop.* 3.) auferatur CE = BD. ducantur-

turque duæ rectæ BE, & CD.

Demonſtratio ſecundæ partis.

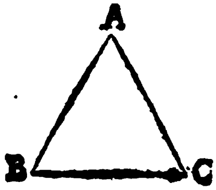


Cum per hypoteſim ſit AB = AC, atque (*per conſtructionem*) ſit BD = CE, (*per axio.* 2.) erit AD = AE, quare in triangulis ABE, ACD erunt duo latera duobus lateribus ſingula ſingulis æqualia, anguluſque A utrique triangulo communis: unde (*per prop.* 4.) baſis BE = baſi CD: angulus ABE = ang. ACD, atque angulus AEB = angulo ADC.

Pariter in triangulis CBD, BCE [*ex demonſtratis, & per conſtructionem*] ſunt duo latera BD, DC ſingula ſingulis æqualia duobus lateribus CE, EB, anguluſque BDC [*ex demonſtratis*] = angulo CEB: quamobrem (*per prop.* 4.) angulus CBE = angulo BCD, & angulus CBD = angulo BCE: cum autem dicti anguli CBD, BCE ſint infra baſim, demonſtrata erit ſecunda pars hujus propoſitionis, nempe quod in triangulo iſoſcele æqualibus productis lateribus, qui infra baſim ſunt anguli, ſint æquales.

Demonſtratio primæ partis.

In triangulis ABE, ACD ex demonſtratis angulus ABE = angulo ACD. Inſuper in triangulis CBD, BCE ex demonſtratis ang. BCD = ang. CBE: igitur ſi ab angulo ABE auferatur angulus CBE, & ab angulo ACD tollatur angulus BCD (*per axio.* 3.) remanebit ang. ABC = ACB. q. e. 2. d.



Corollarium.

Ex hac prop. colligitur in triangulo æquilatero ABC, omnes angulos eſſe æquales; nam triangulum æqualium laterum eſt quaqua verſum iſoſceles; quare & anguli ad baſim (*per hanc prop.*) erunt æquales;

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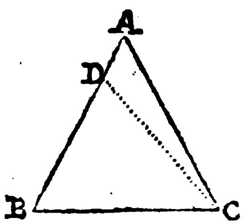
les: ſi enim accipiantur æqualia latera AB,AC [*per antecedentem prop.*] ang. B = ang. C: denuo ſi æqualia latera ſint BA, BC erit angulus C = ang. A, & ideo tres anguli A, B, C æquales.

Propoſ. VI. Theor. III.

In triangulo ABC ſi duo anguli B, & C ſint æquales:

Dico latera AB, AC æqualibus angulis ſubtenſa eſſe æqualia.

Conſtructio, & demonſtratio.



IN triangulo ABC poſita æqualitate duorum angulorum B,& C, ſi talibus angulis ſubtenſa latera nempe AB,AC nõ ſunt æqualia, unum erit altero majus. Ponamus AB majus AC: ex AB (*per prop.3.*) auferatur BD = AC, ducaturque CD.

Hoc facto in triangulis ACB, DBC erunt duo latera AC, CB, ſingula ſingulis æqualia duobus lateribus DB, BC, nec nõ etiam anguli ACB,DBC ab æqualibus lateribus comprehenſi (*per ipotheſim*) æquales: quare (*per prop.4.*) triang. ACB = triang. DBC: ergo pars æqualis toto contra axio.9. quapropter latus AB nequit eſſe majus latere AC: idem inconveniens ſequetur ſi AB dicatur minus quam AC: quare (*per axio. 22.*) AB = AC. q.e.d.

Corollarium.

EX hac prop. fit manifeſtum in illis triangulis, in quibus tres anguli ſunt æquales, tria pariter latera eſſe æqualia; quia in triangulo æqualium angulorum duo quomodocumque aſſumantur anguli, ſemper ſunt æquales; unde & dictis angulis ſubtenſa latera erunt æqualia, atque triangulum æquilaterum.

Propoſ.

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Propoſ. VII. Theor. 4.

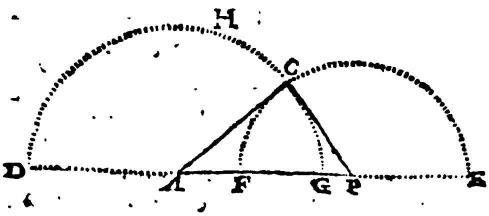
Super eadem recta linea AB ad punctum C ductis duabus rectis lineis AC, BC:

Dico ad eamdem partem, atque in eodem plano, in quo eſt punctum C ſuper eadem recta linea AB a terminis, A, & B ad aliud punctum non poſſe duci duas alias lineas ſeorſim prioribus AC, BC æquales, eoſdemque habentes terminos A, & B.

Conſtructio.

Centro A intervallo AC (*per pet.* 3.) deſcribatur ſemicirculus DCG: pariter centro *B* atque diſtantia BC notetur alius ſemicirculus ECF.

Demonſtratio.



Si ad partem puncti C, & in illo plano, in quo reperitur idem punctum C foret aſſignabile aliud punctum, ad quod duci poſſent duæ rectæ lineæ, quarum illa, quæ duceretur a termino *A* eſſet æqualis ipſi AC, altera vero ducta a termino *B* adæquaret *B*C, tale punctum ad partem puncti C ſignandum eſſet vel extra, vel intra ſemicirculos DCG, FCE, vel in circumferentijs talium ſemicirculorum. Si ponatur extra ſemicirculos DCG, FCE illa recta, quæ a termino A ad punctum extra ſemicirculos poſitum ducta erit, ſemper major erit ipſa AC; aliaque a puncto *B* ducta major quam *B*C: ſi verò punctum fuerit ſignatum intra ſemicirculos una tantũ linea fortaſſe æqualis erit ipſi AC, vel *B*C, ſed non ambæ æquales erunt: hoc idem dicendum ſi punctũ ſignatum ſit in una ſemicirculorum circum-

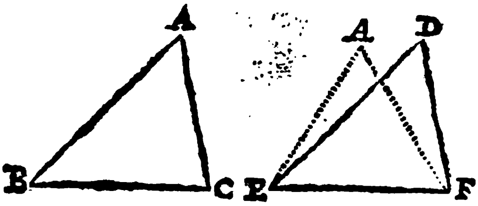
feren-

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ferentijs, ut in H, in quo caſu AH erit quidem æqualis ipſi AC, ſed *BH* non adæquabit *BC*, uti neceſſarium foret, ad hoc ut ad punctum H ductæ eſſent duæ rectæ AH, *BH* prioribus AC, *BC* ſingulæ ſingulis æquales. q. e. d.

Propoſ. VIII. Theor. V.

Si duo triangula ABC, DEF habeant duo latera AB, AC ſingula ſingulis DE, DF, æqualia, habeantque baſim BC = baſi EF.



Dico angulos BAC, EDF ab æqualibus lateribus contentos eſſe æquales.

Conſtructio, & demonſtratio.

Quia baſes BC, EF ſupponuntur æquales facta talium baſium ſuperimpoſitione (*per axio.* 8.) erunt congruentes, duoque extrema B, C congruent cum extremis E, F: ſervata hac baſium congruentia triangulum BAC ſuperponatur triangulo EDF. Hoc facto punctum A vel erit in D, vel extra; ſi ſit in D etiam latera AB, AC erunt lateribus DE, DF congruentia; quare angulus BAC congruet angulo EDF, & conſequenter erit æqualis (*per axio.* 8.) Si poſtea punctum A cadat extra punctum D, contra prop. præcedentem ab extremitatibus E, & F ad aliud punctum extra D duci poſſunt duæ rectæ EA, FA prioribus ED, FD æquales. q. e. d.

Corollarium.

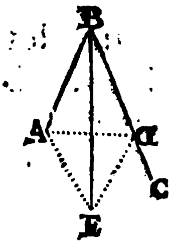
Ex hac propoſ. ſequitur in illis triangulis, in quibus omnia latera unius trianguli ſunt ſingula ſingulis lateribus alterius trianguli æqualia, non ſolum omnes talium triangulorum angulos ab æqualibus lateribus comprehenſos eſſe æquales, verum etiam totum triangulum eſſe toti triangulo æquale, quia, ut demon-

monſtratum eſt , anguli congruunt angulis, & triangulum triangulo .

Propoſ. IX. Probl. IV.

Datum angulum rectilineum ABC, bifariam dividere.

Conſtructio.



IN uno ex lateribus BA , vel BC comprehendentibus datum angulum ABC utcumque ſignetur punctum A : ex latere BC (*per prop.* 3.) abſcindatur BD æqualis ipſi BA : ducatur AD : ſupra AD (*per prop.* 1.) conſtituatur triangulum AED æquilaterum : ducatur BE.

Dico rectam BE datum angulum ABC bifariam dividere.

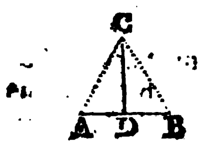
Demonſtratio.

Per conſtructionem *BA* $=$ BD , atque AE $=$ DE: quare in triangulis ABE , DBE erunt duo latera AB , BE ſingula ſingulis æqualia duobus lateribus DB, BE : (nam BE eſt latus commune utrique triangulo) baſiſque AE $=$ baſi DE : quare (*per prop.* 8.) angulus ABE $=$ ang. DBE.: unde datus angulus ABC erit bifariam diviſus. q. e. f.

Propoſ. X. Probl. V.

Datam rectam lineam AB bifariam dividere.

Conſtructio.



SUper data *AB* (*per prop.* 1.) conſtituatur triangulum æquilaterum AC*B*. Angulus AC*B* (*per prop.* 9.) bifariam dividatur a recta CD.

Dico datam rectam lineam *AB* bifariam eſſe diviſam in D.

Demon-

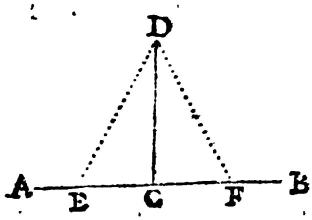
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Demonſtratio.

Per conſtructionem AC = BC; quamobrem in duobus triangulis ACD, *B*CD duo latera AC, CD ſingula ſingulis æqualia erunt duobus Lateribus *B*C, CD: pariter cum duo anguli ACD, *B*CD ab æqualibus lateribus contenti ſint æquales per conſtructionem: ſequitur (*per prop.* 4.) baſim AD = baſi D*B*: quare data A*B* erit biſariam diviſa in D. q. e. f.

Propoſ. XI. Probl. VI.

Datæ rectæ AB a puncto C in ea dato perpendicularem ducere.



Conſtructio.

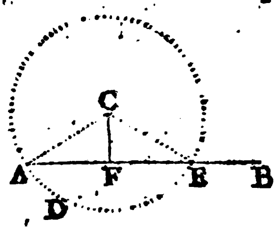
IN data recta A*B* ſignetur punctum E quod non ſit C: (*per prop.* 3.) fiat EC = CF. Super EF (*per prop.* 1.) conſtituatur triangulum æquilaterum EDF. A puncto C ad D ducatur recta CD.

Dico CD perpendicularem eſſe ipſi A*B*.

Demonſtratio.

Per conſtructionem EC = CF, atque ED = FD: ergo in triangulis DCE, DCF duo latera EC, CD ſingula ſingulis erunt æqualia lateribus FC, CD: baſiſque ED = baſi FD: quare (*per prop.* 8.) angulus DCE = ang. DCF: igitur (*per def.* 10.) recta DC erit perpendicularis ipſi A*B*. q.e.f.

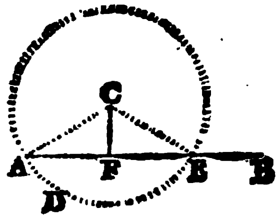
Propoſ. XII. Probl. VII.



Super data recta AB a puncto C extra datam rectam conſtituto, ipſi AB perpendicularem ducere.

Con-

Conſtructio.



Centro C. tanto intervallo deſcribatur circulus AE ut ſecet rectam *AB* in A, & E. Recta AE (*per prop.* 10.) biſariam dividatur in F.

A puncto C ducantur rectæ CA, CF, CE.

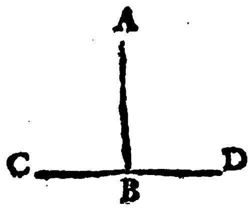
Dico CF ipſi *AB* eſſe perpendicularem.

Demonſtratio.

Recta CA = CE (*per def. Circuli*) AF = FE (*per conſtructionem*) ergo in triangulis CFA, CFE duo latera CF, FA ſingula ſingulis æqualia erunt duobus lateribus CF, FE: baſiſque CA = baſi CE, quare (*per prop.* 8.) ang. CFA = ang. CFE: igitur (*per def.* 10.) dicti anguli erunt recti; & recta CF ipſi *AB* perpendicularis. q. e. f.

Propoſ. XIII. Theor. VI.

Fig. 1.

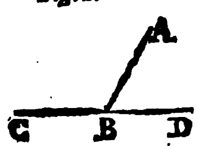


Si Recta AB cadat ſupra rectam CD.

Dico duos, quos deinceps facit angulos A*B*C, A*B*D, eſſe rectos, vel duobus rectis æquales.

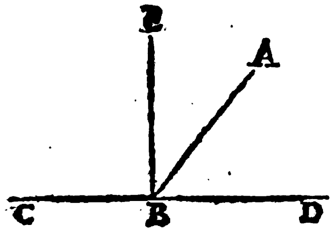
Conſtructio, & Demonſtratio.

Fig. 2.



Dum recta *AB* ſuper recta CD conſiſtit hoc dupliciter poteſt contingere, primo ſcilicet, quando *AB* eſt ipſi CD perpendicularis, ut (*Fig.* 1.); ſecundo dum *AB* eſt ipſi CD inclinata, ut (*Fig.* 2.). Quando *AB* eſt ipſi CD perpendicularis (*Fig.* 1.) tunc (*per def.* 10.) duo deinceps anguli *ABC*, *ABD* ſunt recti: quare (*per axio.* 12.) duobus rectis angulis æquales.

Si



Si vero AB(*Fig.2.*)non ſit ipſi CD per pendicularis, neque duo anguli ABC, ABD erunt recti, at unus obtuſus, alter vero acutus; quos pariter

Dico duobus rectis æquales.

A puncto B (*per prop.* 11.) ducatur BE ipſi CD perpendicularis. (*Fig.3.*)

Demonſtratio.

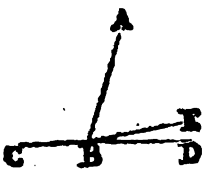
Duo anguli DBA, ABE, cum ſint partes anguli recti DBE, (*per axio.* 19.) erunt æquales uni recto; addito angulo recto CBE (*per axio.* 2.) erunt tres anguli DBA, ABE, EBC duobus rectis angulis æquales; ſed duo anguli ABE, EBC (*per axio.* 19.) æquant angulum obtuſum ABC: ergo obtuſus angulus ABC cum acuto ABD erunt duobus rectis angulis æquales q. e. d.

Propoſ. XIV. Theor. VII.

Si ad B punctum rectæ AB in eodem plano e diverſis partibus ductæ ſint duæ rectæ lineæ CB, DB, quæ duos deinceps angulos ABC, ABD efficiant duobus rectis angulis æquales.

Dico rectas CB, BD in directum jacere.

Conſtructio, & Demonſtratio.



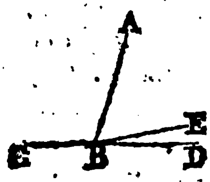
SI linea CB non eſt in directum conſtituta cum BD, linea CB in rectum producta neceſſario cadere debet ſupra, vel infra BD. Ponamus igitur quod cadat ſupra, quodque CBE ſit una recta linea.

Hoc ſuppoſito anguli $\left.\begin{matrix}ABC\\ABE\end{matrix}\right\}$ (*per prop.* 13.) ſunt duobus rectis æquales.

Anguli $\left.\begin{matrix}ABC\\ABD\end{matrix}\right\}$ (*per hypoteſim*) ſunt æquales duobus rectis.

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Ergo (*per axio.* 1.) ang. $\left.\begin{matrix}ABC\\ABE\end{matrix}\right\} =$ ang. $\left.\begin{matrix}ABC\\ABD\end{matrix}\right\}$ & dempto communi angulo ABC (*per axio.* 3.) erit ang. ABD = ang. ABE, quod fieri nequit, quia pars adæquaret totum contra veritatem axio. 9. quapropter CBE non erit una linea recta: quod demonſtratum eſt de linea BE, valet etiam de quacumque alia linea; excepta BD, igitur BD cum BC efformabunt lineam rectam. q. e. d.



SCHOLIUM.

Etiam huic propoſitioni (quemadmodum, & in prop. 7.) factum ſuit) placuit adiungere hæc verba in eodem plano, *cum euidens ſit quod ſi duo puncta C, & D non fuerint in eodem plano, in quo reperitur & ipſa recta AB, amplius non verificatur hæc propoſitio; nam ſi punctum D. acceptum fuerit extra illud planum, in quo ſunt recta AB, & punctum C, manifeſtum eſt duos angulos ABC, ABD poſſe exiſtere duobus rectis æquales, quamvis recta BD non ſit in rectum cum CB (ut conſtat ex lib. 11. elem.) Ut ergo verificetur hæc propoſitio neceſſarium eſt quod duo puncta C, & D reperiantur in illo plano, in quo eſt recta AB.*

Propoſ. XV. Theor. VIII.

Si duæ rectæ lineæ AB, CD ſe mutuo ſecent in E Dico angulos ad verticem AEC, BED, vel ang. AED, BEC eſſe æquales.

Demonſtratio:

Anguli $\left.\begin{matrix}AEC\\AED\end{matrix}\right\}$ (*per prop.* 13.) = ſunt duobus rectis.

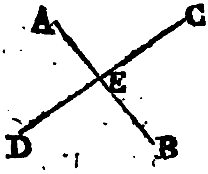
Anguli $\left.\begin{matrix}AED\\DEB\end{matrix}\right\}$ (*per prop.* 13.) = ſunt duobus rectis: ergo

Anguli $\left.\begin{matrix}AEC\\AED\end{matrix}\right\}$ (*per axio.* 1. = *Ang.* $\left.\begin{matrix}AED\\DEB\end{matrix}\right\}$

Dem-

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Dempto communi angulo AED remanebit angulus AEC $=$ angulo BED. q. e. d.



Corollarium primum.

EX hac prop. manifeſtè colligitur quatuor angulos ad punctū E cōſtitutos adæquare ſummam quatuor rectorum; quia tam duo ang. $\frac{AEC}{AED}$ quam duo ang. $\frac{DEB}{BEC}$ (*per prop.* 13.) ſunt duobus rectis æquales.

Corollarium Secundum.

OMnes anguli in uno plano circa idem punctum conſtituti, ſunt quatuor rectis angulis æquales, cum omnes dicti anguli ſint partes quatuor rectorum, juxta generalem regulam, videlicet quodcumque ſpatium in aliquo plano circa unum punctum quatuor rectis angulis æquivalere.

Corollarium Tertium.

SI duo anguli æquales AED, CEB ad verticem uniantur in puncto E, taliter ut unius anguli latus, nempe DE, ſit in rectum cum EC latere alterius anguli

Dico etiam alia latera AE, EB eſſe in directum poſita.

Cum enim per ſuppoſitionem ang. AED $=$ ſit ang. CEB, addito communi ang. AEC; erunt

ang. $\left.\begin{matrix}AED\\AEC\end{matrix}\right\} =$ ang. $\left.\begin{matrix}CEB\\AEC\end{matrix}\right\}$: ſed (*per prop.* 13.)

ang. $\left.\begin{matrix}AED\\AEC\end{matrix}\right\} =$ ſunt duobus rectis: ergo etiam

ang. $\left.\begin{matrix}CEB\\AEC\end{matrix}\right\} =$ erunt duobus rectis: quare (*per prop.* 14.) rectæ lineæ AE, EB erunt in rectum conſtitutæ.

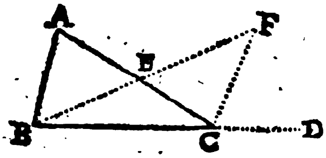
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Propoſ. XVI. Theor. IX.

Si trianguli ABC unum latus, ut BC, producatur in D.

Dico angulum externum ACD majorem eſſe utrolibet angulo interno, & oppoſito ABC, BAC.

Conſtructio primæ partis.



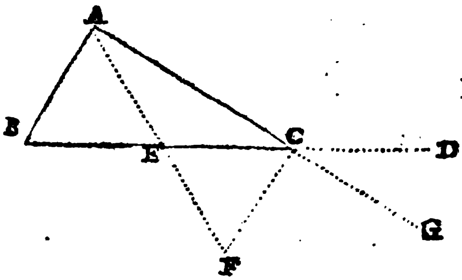
Recta AC (*per prop.* 10.) bifariam dividatur in E.

Ducatur recta *BE*, ex qua indefinitè producta (*per prop.* 3.) auferatur EF = *BE*: jungatur FC.

Demonſtratio.

In triangulis AEB; CEF (*per conſtructionem*) ſunt duo latera AE, E*B* ſingula ſingulis = lateribus CE, EF: pariter in dictis triangulis (*per prop.* 15.) angulus *AEB* = ang. CEF: ergo (*per prop.* 4.) ang. *EAB* = ang. *ECF*: ſed externus ang. *A*CD (*per axio.* 9.) major eſt angulo *ACF*: ergo (*per axio.* 1.) idem angulus externus *A*CD major erit angulo interno, & oppoſito C*AB*. q. e. 1. d.

Conſtructio ſecundæ partis.



Latus *B*C (*per prop.* 10.) bifariam dividatur in E; ducatur AE, ex qua producta (*per prop.* 3.] auferatur EF = AE. Signetur FC: atque AC producatur in G.

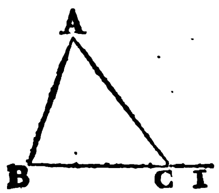
De-

Demonſtratio.

PEr conſtructionem BE = EC atque AE = EF; quare in triang. AEB, FCE erunt duo latera AE, EB ſingula ſingulis æqualia lateribus FE, EC, angulique AEB, FEC ab æqualibus lateribus comprehenſi æquales (*per prop.* 15.): ergo (*per prop.* 4.) angulus ABC = ang. FCB: ſed ang. GCB [*per axio.* 9.] major eſt angulo FCB: ergo idem angulus GCB erit etiam major angulo ABC: quia vero [*per prop.* 15.] angulus GCB = eſt ang. ACD, etiam angulus ACD [*per axio.* 1.] major erit angulo interno, & oppoſito ABC. q. e. d.

Propoſ. XVII. Theor. X.

In triangulo ABC.
Dico duos angulos omnifariam ſumptos exiſtere duobus rectis minores.



Conſtructio.

Producatur latus BC in I.

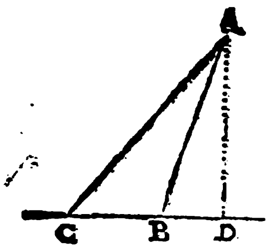
Demonſtratio.

Angulus externus ACI [*per prop.* 16.] major eſt interno, & oppoſito angulo BAC; addito communi angulo ACB [*per axio.* 4.] erunt ang. $\left.\begin{matrix}ACI\\ACB\end{matrix}\right\}$ majores angulis $\left.\begin{matrix}BAC\\ACB\end{matrix}\right\}$ ſed anguli $\left.\begin{matrix}ACI\\ACB\end{matrix}\right\}$ [*per prop.* 13.] ſunt æquales duobus rectis: ergo duo anguli $\left.\begin{matrix}BAC\\ACB\end{matrix}\right\}$ duobus rectis minores erunt. q. e. d.

Corollarium primum.

EX hac propoſitione conſtat in rectangulis, atque obtuſangulis triangulis duos angulos eſſe acutos.

Corollarium ſecundum.



PAriter ex hac propoſ. ſequitur quod dum recta *AB* cadens ſupra rectam CD angulos facit inæquales, unum ſcilicet obtuſum ut *ABC*, alterum vero acutum, ut *ABD*, perpendicularis ducta a puncto A ſupra CD, uti eſt AD, cadit ad partes anguli acuti: nam ſi caderet ad partes anguli obtuſi ut *AC*: in hoc caſu in triangulo *ABC* duo angli *ABC*, *ACB* duobus rectis majores forent, quia ang. *ABC* eſt obtuſus, *ACB* eſſet rectus.

Corollarium tertium.

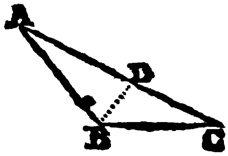
IN triangulo æquilatero omnes anguli ſunt acuti: ſunt enim omnes æquales [*per corollar. prop.* 5.] quare ſi unus eſſet rectus, vel obtuſus, etiam alii forent recti, vel obtuſi.

Corollarium quartum.

IN triangulo iſoſcele anguli ad baſim ſunt ſemper acuti; nam ſi eſſent recti, vel obtuſi, in triangulo darentur duo anguli non minores duobus rectis, quod eſt contra ſupra poſitam propoſitianem.

Propoſ. XVIII. Theor. XI.

In dato triangulo ABC
Dico quod ſub majore latere AC ſtat major angulus ABC, & ſub minore latere AB ſtat minor angulus ACB.



Conſtructio.

EX latere *AC*, quod ſupponitur majus latere *AB* (*per prop.* 3.] auferatur *AD* = *AB*. notetur BD. De-

Demonſtratio.

Per conſtructionem AD = AB: quare BAD erit triangulum iſoſceles; unde [*per propſ.* 5.] ang. ABD = ADB; ſed angulus ADB [*per prop.* 16.] eſt major angulo C: ergo (*per axio.* 1.) & angulus ABD major erit angulo C: atqui angulus ABC (*per axio.* 9.) major angulo ABD: ergo (*per axio.* 21.) ang. ABC multo major ang. C: igitur ſub majore latere AC ſtat major ang. ABC, & ſub minore latere AB ſtat minor angulus ACB, q. e. d.

Corollarium.

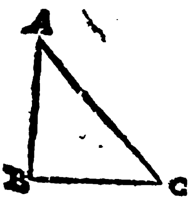
EX hac prop. colligitur in triangulo Scaleno, omnes angulos eſſe inæquales, quia trianguli ſcaleni latera ſunt inæqualia.

Propoſ. XIX. Theor. XII.

In triangulo ABC

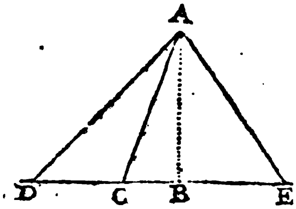
Dico quod ſub majore angulo B reperitur majus latus AC, & ſub minore ang. C ſtat minus latus AB.

Demonſtratio.



SI dicamus latus AC = AB [*per prop.* 5.] ſequitur ang. B = ang. C contra ſuppoſitum.

Si dicamus latus AC minus AB [*per prop.* 18.] aſſerendum eſt angulum C majorem ang. B. contra ſuppoſitum, cum ſupponatur ang. B major ang. C: cum igitur latus AC nequeat eſſe minus, nec æquale lateri AB [*per axio.* 23.] erit majus, q. e. d.



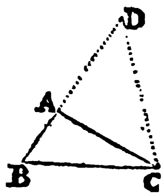
Corollarium.

SI a puncto A ad rectam DE ductæ ſint, & perpendicularis AB, & inclinatæ AD, AC, AE. Dico perpendicularem AB exiſtere omnium minimam. quia ſola perpendicularis *AB* [*per def.* 10.] angulos rectos efformat cum DE. Si ergo in triangulo *ABC* (idem intelligendum de reliquis) angulus *AEC* eſt rectus, & angulus *ACB* acutus, ſeu recto minor: ex hac propoſitione ſequitur latus *AB* minus eſſe latere *AC*: quare AB omnium minima.

Propoſ. XX. Theor. XIII.

In triangulo BAC.

Dico duo latera quomodocumque accepta, uti ſunt BA, AC, majora eſſe tertio latere BC.



VEritas hujus propoſitionis in triangulo æqualium laterum eſt ſatis manifeſta, cum ex tribus lateribus æqualibus duo tertium excedant.

Pariter in triangulo iſoſcele apertè conſtat unum æqualium cum baſe alterum æqualium ſuperare.

Quamobrem tota dubietas reducitur ad duo latera æqualia in triangulo iſoſcele, an ſint baſe majora, & an in triangulo ſcaleno duo minora latera *BA*, *AC* ſuperent maximum latus BC.

Conſtructio.

PRoducatur BA verſus D, fiatque (*per prop.* 3.) *AD* == *AC*: ducaturque DC.

Demonſtratio.

Triangulum DAC (*per conſtructionem*) eſt iſoſceles: quare (*per prop.* 5.) ang. ADC == ang. ACD: ſed ang. BCD (*per axio.*

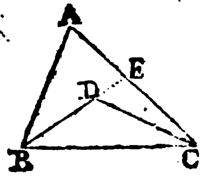
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axio. 9.) major ang. ACD: ergo. BCD (*per axio. 1.*) major ang. ADC. Cum igitur in triangulo BDC ang. BCD ſit major ang. CDB (*per prop. 19.*) latus BD erit majus latere BC: ſed latus BD (*per axio. 2.*) adæquat duo latera BA, AC: ergo latera BA, AC majora erunt latere BC. q. e. d.

Propoſ. XXI. Theor. XIV.

In triangulo BAC ſi a terminis lateris BC intra triangulum ad punctum D ductæ ſuerint duæ rectæ lineæ BD, CD.

Dico has lineas minores eſſe reliquis duobus trianguli lateribus BA, AC, angulum vero BDC majorem eſſe ang. BAC.



Conſtructio.

Producatur BD, uſque ad AC in E.

Demonſtratio.

In Triangulo BAE (*per prop. 20.*) latera $\left.\begin{matrix}BA\\AE\end{matrix}\right\}$ majora ſunt BE, quare addito communi latere EC

erunt latera $\left.\begin{matrix}BA\\AE\\EC\end{matrix}\right)$ hoc eſt $\left.\begin{matrix}BA\\AC\end{matrix}\right\}$ majora $\left.\begin{matrix}BE\\EC\end{matrix}\right\}$

Pariter in triang. CED (*per prop. 20.*) latera $\left.\begin{matrix}CE\\ED\end{matrix}\right\}$ majora ſunt latere CD quare addito communi latere DB;

erunt latera $\left.\begin{matrix}CE\\ED\\DB\end{matrix}\right)$ hoc eſt $\left.\begin{matrix}BE\\EC\end{matrix}\right\}$ majora $\left.\begin{matrix}CD\\DB\end{matrix}\right\}$ ergo (*per axio. 11.*)

latera $\left.\begin{matrix}BA\\AC\end{matrix}\right\}$ majora lateribus $\left.\begin{matrix}CD\\DB\end{matrix}\right\}$ q. e. p. d.

Quod

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Quod poſtea angulus BDC, ſit major ang. BAC ita demonſtratur.

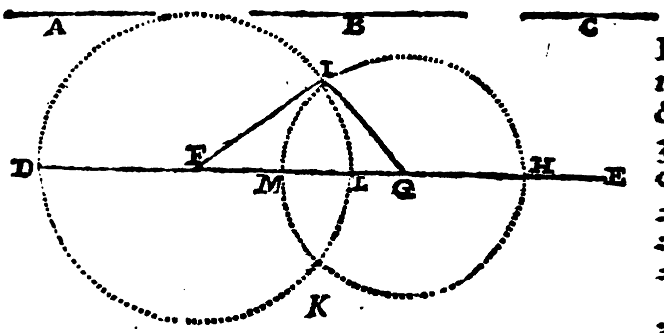
Angulus BDC (*per prop.* 16.) major ang. DEC.

Angulus DEC (*per eamdem* 16.) major ang. EAB: ergo (*per axio.* 21.) angulus BDC major ang. BAC. q. e. 2. d.

Propoſ. XXII. Probl. VIII.

Ex tribus datis rectis lineis A, B, C, quarum duæ omnifariam ſumptæ reliqua ſint majores, triangulum componere.

Conſtructio.



A recta DE indefinitè producta (*per prop.* 3.) abſcindantur DF = A: FG = B: GH = C. Centro F intervallo FD (*per poſt.* 3.) notetur circulus DIK: pariter centro G intervallo GH deſcribatur alius circulus HIK, qui priorem circulum DIK ſecabit in punctis I, & K, ut patet ex ſequenti demonſtratione.

Per conſtructionem facta eſt DF = A: FG = B: GH = C: duæ autem rectæ A, & C ſupponuntur majores quam B: quare DF, & GH majores erunt quam FG: quo ſtante ſi ex FG ablata fuerit FL = FD, reſiduum GH majus erit reſiduo LG: quia verò (*per def. circuli*) eſt GH = GM; inde ſequitur GM majorem eſſe quam GL: quare punctum M ſpectans ad circulum IHK erit intra circulum DIK; punctum vero H ſpectans ad circulum IHK, quia à centro F magis diſtat, quam FL; erit extra cir-

circulum DIK: quare dicti circuli ſe ſe interſecabunt in punctis I, & K.

Poſita demonſtratione quod duo circuli, ut ſupra deſcripti ſe interſecent in punctis I, & K a duobus centris F, & G ad punctum interſecationis I rectæ ducantur FI, GI.

Dico triangulum FIG eſſe efformatum a tribus datis lineis A, B, C.

Demonſtratio.

Per conſtructionem A == DF: (*per def. circuli*) DF == FI: ergo (*per axio. 1.*) A == FI.

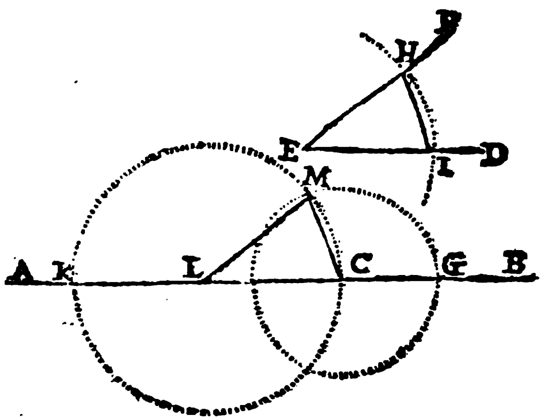
Pariter per conſtructiouem C == GH: (*per def. circuli*) GH == GI: ergo (*per axio. 1.*) C == GI.

Cum autem & FG facta ſit æqualis ipſi *B*; triangulum FIG erit ex tribus datis rectis lineis A, *B*, C efformatum. q. e. f.

Propoſ. XXIII. Probl. IX.

Ad rectam AB, atque ad punctum in ea datum L efformare angulum rectilineum æqualem dato angulo rectilineo DEF.

Conſtructio.



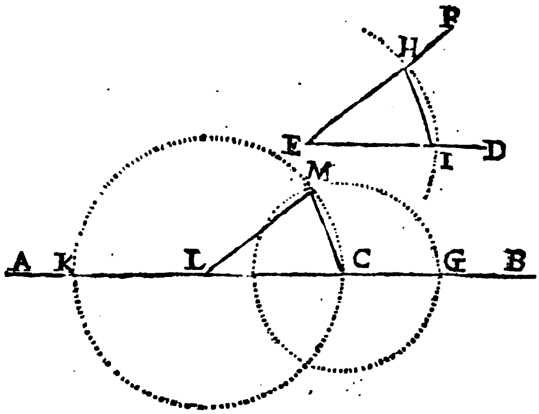
Centro E quocumque intervallo notetur circulus HI, atque recta HI. Ex LA (*per pro. 3.*) abſcindatur LK == EH; pariterque ex LB auferatur LC == EI: demumque ponatur CG == IH.

Centro L intervallo LK deſcribatur

tur circulus CMK, item centro C intervallo CG ſignetur circulus GM, qui priorem circulum (*per prop.* 22.) ſecabit in M. ductis rectis LM, CM.

Dico angulum CLM ad datam rectam AB, & datum punctum L conſtitutum, æqualem eſſe dato angulo DEF.

Demonſtratio.



Per conſtructionem EH = LK: ſed (*per def. circuli.*) LK = LM: unde (*per axio.* 1.) LM = EH.

Pariter (*per conſtructionem*) LC = EI, atque CG = IH: ſed (*per def. circuli.*) CG = CM: igitur CM = IH.

Quibus poſitis in triangulo IEH erunt duo latera IE, EH ſingula ſingulis æqualia lateribus CL, LM; baſiſque IH = baſi CM: quare (*per prop.* 8. *hujus*) ang. IEH = ang. CLM. q. e f.

Propoſ. XXIV. Theor. XV.

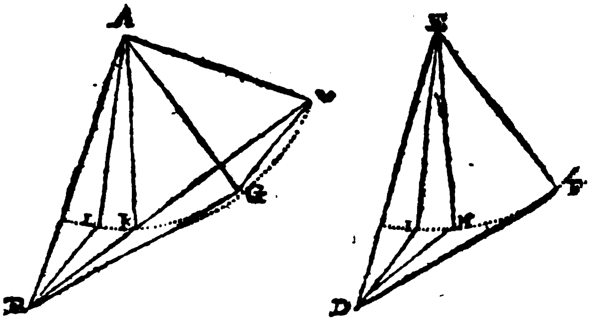
Si duo triangula BAC; DEF habeant duo latera BA, AC ſingula ſingulis æqualia lateribus DE, EF; angulus vero BAC major ſit angulo DEF.

Dico baſim BC majorem eſſe baſi DF.

Con-

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Conſtructio.



Cum angulus *BAC* ſupponatur major angulo DEF, ſi ad punctum A & ad rectam AB (*per prop. 23.*) conſtituatur angulus *BAG* == ang. DEF; linea AG debet cadere intra lineas *AB*, *AC*. Hac linea A G producta donec ſit æqualis rectæ EF, vel AC, terminus G poteſt eſſe infra baſim *BC*, in ipſa baſi *BC*, atque ſupra eamdem baſim *BC*, & hoc propter datorum triangulorum diverſitatem.

Primo. Data triangula cum ſupra notatis conditionibus ſint ABC, DEF in hoc caſu punctum G cadit infra baſim BC.

Secundo. Si data triangula fuerint BAC, DEH, punctum K quod æquivalet ipſi G erit in baſi BC.

Tertio. Si propoſita triangula ſint BAC, DEI, in hoc caſu punctum L, quod æquivalet puncto G, erit ſupra baſim BC.

Quando punctum G, ut in primo caſu, cadit infra baſim BC ducantur duæ rectæ GB, GC.

Quando verò cadit ſupra baſim in L, ut in tertio caſu, notetur recta BL.

Demonſtratio primæ partis.

In triangulis BAG, DEF [*per conſtructionem, & ſuppoſitionem*] latera BA, AG ſingula ſingulis æqualia ſunt lateribus DE, EF, & angulus BAG == DEF: quare (*per prop. 4.*) baſis BG == baſi DF. Triangulum GAC (*per conſtructionem*) habet duo latera æqualia AC, AG: ergo [*per prop. 5.*) angulus AGC == ACG: ſed angulus ACG [*per axio. 9.*] major angulo GCB: quare (*per axio. 1.* & angulus AGC major angulo GCB: Pariter cum angulus BGC (*per axio. 9.*] major ſit angulo AGC,

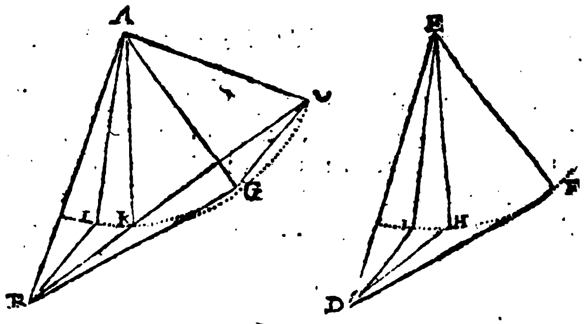
AGC; inde ſequitur (*per axio.* 21.) angulum BGC, majorem eſſe angulo GCB: Quo ſtante in triangulo BGC (*per prop.* 19.) latus BC majus erit latere *B*G; ſed *B*G ſuperius demonſtrata eſt æqualis ipſi DF: ergo *B*C major DF. q. e. p. d.

Demonſtratio ſecundæ partis.

IN trianguli *B*AK, DEH (*per ſuppoſitionem, & per conſtructionem*) ſunt duo latera *B*A, AK ſingula ſingulis lateribus DE, EH æqualia, atque angulus *B*AK = ang. DEH: quare (*per prop.* 4. *B*K = DH; at *B*C (*per axio.* 9.) major *B*K: ergo & *B*C major DH. q. e. 2. d.

Demonſtratio tertiæ partis.

IN triangulis BAL, DEI (*per ſuppoſitum, & per conſtructionem*] latera *B*A, AL ſunt ſingula ſingulis æqualia lateribus DE, EI, nec non etiam angulus *B*AL = ang. DEI: ergo (*per prop.*4.] baſis *B*L = baſi DI. Quia vero (*per prop.* 21.) *B*C major eſt quam *B*L: erit, & baſis BC major baſe DI: quo ſtante in omni caſu erit verum aſſerere in illis Triangulis, in quibus ſunt duo latera ſingula ſingulis æqualia, & angulus ab æqualibus lateribus comprehenſus angulo major; ſub majore angulo exiſtere majorem baſim, & ſub minore angulo eſſe minorem baſim. q. e. d.



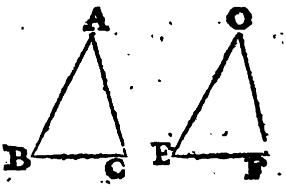
Propoſ.

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Propoſ. XXV. Theor. XVI.

Si duo triangula ABC, OEF habeant duo latera BA, AC ſingula ſingulis æqualia duobus lateribus EO, OF, baſiſque BC major ſit baſe EF.

Dico angulum A oppoſitum majori baſi BC, majorem eſſe angulo O ſub minore baſe EF conſtituto.



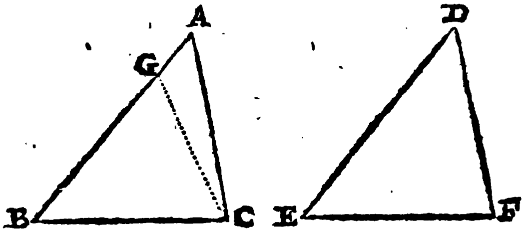
Angulus A nequit adæquare angulum O, quia *per prop.* 4.] baſis BC adæquaret EF contra ſuppoſitum. Pariter idem angulus A non poteſteſſe minor angulo O: quia (*per prop.* 24.] baſis BC minor foret baſe EF; quod pariter eſt contra ſuppoſitum: ergo angulus A major erit angulo O. q. e. d.

Propoſ. XXVI. Theor. XVII.

Si duo triangula ABC, DEF duos angulos ABC, ACB duobus angulis DEF, DFE ſingulos ſingulis æquales habuerint, unumque latus uni lateri æquale; ſiuè quod æqualibus angulis adjacet, ſiuè quod uni æqualium angulorum ſubtenditur.

Dico reliqua latera quæ æqualibus angulis ſubtenduntur, reliquis lateribus ſingula ſingulis eſſe æqualia, reliquumque angulum A reliquo angulo D æqualem.

Conſtru-



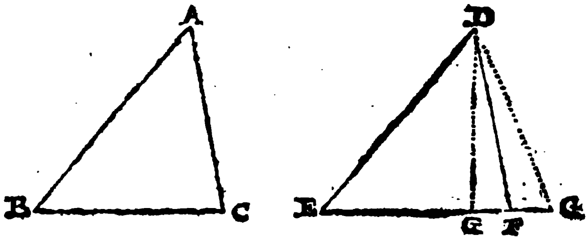
Conſtructio, & demonſtratio illius partis, in qua latera æqualia ſupponuntur illa, quæ æqualibus adiacent angulis, uti ſunt BC, & EF.

SI latus BA non eſt æquale lateri ED, ſit majus, ſi hoc fieri poteſt. Poſito igitur quod BA ſit majus ED (*per prop.* 3.) ex BA auferatur BG æquale ipſi ED: hoc ſtate duo triangula BGC, EDF habebunt duo latera GB, BC ſingula ſingulis æqualia duobus lateribus DE, EF, atque angulum B (*per ſuppoſitionem*) angulo E æqualem: quare (*per prop.* 4.) angulus BCG erit æqualis angulo EFD: ſed eidem ang. EFD (*ex hypoteſi*) eſt æqualis angulus BCA: ergo (*per axio.* 1.) angulus BCG = ang. BCA: quod eſt impoſſibile cum unus ſit pars alterius. Idem demonſtrabitur ſi dicatur BA minus, quam ED, quare BA = ED, & CA = FD.

Demonſtrata laterum æqualitate, facile erit oſtendere æqualitatem inter reliquos angulos A, & D, quia duo triangula BAC, EDF habebunt latera BA, AC ſingula ſingulis æqualia lateribus ED, DF, & baſim BC (*per ſuppoſitum*) æqualem baſi EF: quare (*per prop.* 8.) angulus A = ang. D. q. e. d.

Con-

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Conſtructio, & demonſtratio alterius partis, in qua latera, quæ ſupponuntur æqualia, ſubtenduntur angulis æqualibus, uti ſunt BA, & ED.

IN triangulis BAC, EDF, eiſdem manentibus quoad angulorum æqualitatem, ſi latera BA, ED æqualibus angulis C, & F ſubtenſa ſupponantur æqualia, etiam in hoc caſu.

Dico latera BC, CA fore æqualia ſingula ſingulis lateribus EF, FD.

Demonſtratio.

SI latus EF poteſt eſſe majus latere BC, ex majore latere EF (*per prop.* 3.) auferatur EG ipſi BC æquale: hoc facto ducatur DG. In duobus triangulis ABC, DEG (*per ſuppoſitionem, & per conſtructionem*) duo latera AB, BC erunt ſingula ſingulis æqualia lateribus DE, EG, & angulus B = ang. E, quare (*per prop.* 4.) angulus BCA = ang. EGD: ſed angulo BCA ſupponitur æqualis angulus EFD: ergo (*per axio.* 1.] angulus EGD = ang. EFD, quod cum ſit contra prop. 16. non erit EF majus BC. Eodem modo demonſtrabitur quod BC nequeat eſſe majus EF: ergo erit æquale. q. e. d.

Corollarium.

SEquitur ex hac prop. poſita triangula BAC, EDF eſſe æqualia, cum habeant latera BA, AC ſingula ſingulis æqualia lateribus ED, DF, angulumque A = ang. D, uti demonſtratum fuit in ſecunda parte propoſitionis quartæ.

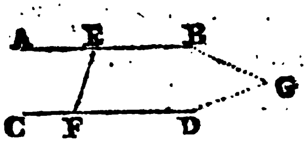
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Propoſ. XXVII. Theor. XVIII.

Si recta EF incidat in rectas AB, CD in eodem plano exiſtentes, faciatque angulos alternos AEF, DFE æquales.

Dico rectas AB, CD eſſe parallelas.

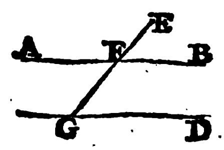
Conſtructio, & demonſtratio.



SI rectæ AB, CD, non ſunt parallelæ (*per axio.* 24.) debent concurrere: concurrant igitur in G, conſtituantque triangulum EGF, in quo (*per ſuppoſitum*) angulus externus FEA æqualis erit interno, & oppoſito EFG: quod eſſe non poteſt (*per prop.* 16.): ergo duæ rectæ AB, CD erunt parallelæ. q. e. d.

Propoſ. XXVIII. Theor. XIX.

Si recta EG incidat in rectas AB, GD in eodem plano exiſtentes, faciatque angulum externum EFB æqualem interno, & oppoſito ad eaſdem partes FGD, vel duos internos, & ad eaſdem partes BFG, FGD duobus rectis æquales.



Dico AB, GD eſſe parallelas.

Demonſtratio primæ partis.

ANgulus externus EFB (*per hypotheſim*) = ang. FGD. Angulus EFB (*per prop.* 15.) = ang. GFA: ergo (*per axio.* 1.) angu-

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angulus AFG = ang. FGD; qui cum ſint alterni (*per prop.* 27.) AB, & GD erunt parallelæ. q. eſt 1. d.

Demonſtratio ſecundæ partis.

PEr ſuppoſitionem duo anguli BFG, FGD = duobus rectis. Sed (*per prop.* 13.) etiam duo anguli BFG, AFG = ſunt duobus rectis.

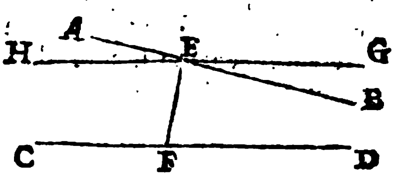
Ablato communi angulo BFG (*per axio.* 3.) erit angulus FGD = ang. AFG: cum autem iſti anguli ſint alterni (*per prop.* 27.) rectæ AB, GD erunt parallelæ. q. e. 2. d.

SCHOLIUM.

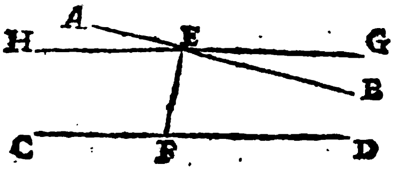
AXioma ſuperius ſub numero 13. propoſitum, & ab Euclide numero 11. indicatum primitus aſſumitur ad ſequentem propoſ. demonſtrandam. Quia vero id Mathematici, & jure merito, a numero axiomatum excludunt, cum ejus veritas non ſit ſatis per ſe nota: hac unica de cauſa neceſſarium duxi talis axiomatis demonſtrationem hic apponere, ut ea, quæ a tali principio dependent, ſcientificè colligantur.

Axio. XIII.

Si in duas rectas lineas AB, CD in eodem plano exiſtentes incidat recta EF, quæ faciat duos internos, & ad eaſdem partes angulos BEF, EFD duobus rectis angulis minores.



Dico rectas AB, CD productas tandem debere concurrere ad partes B, & D, ubi ſunt anguli duobus rectis minores.



Conſtructio.

AD punctum E, & ad lineam EF (*per prop.* 23.) conſtituatur angulus FEG = ang. EFC.

Demonſtratio.

ANgulus FEG (*per conſtructionem*) eſt = angulo EFC quare addito communi angulo EFD.

erunt (*per axio.* 2.) anguli $\left.\begin{matrix}EFC\\EFD\end{matrix}\right\}$ = ang. $\left.\begin{matrix}FEG\\EFD\end{matrix}\right\}$

ſed (*per prop.* 13.) anguli $\left.\begin{matrix}EFC\\EFD\end{matrix}\right\}$ = duobus rectis: ergo

& anguli $\left.\begin{matrix}FEG\\EFD\end{matrix}\right\}$ = duobus rectis:

quare (*per prop.* 28.) duæ rectæ FD, EG erunt parallelæ. Rurſus quia (*per hypotheſim*) anguli $\left.\begin{matrix}EFD\\FEB\end{matrix}\right\}$ minores ſunt duobus rectis, ſequitur angulos $\begin{matrix}FEG\\EFD\end{matrix}$ majores eſſe ang. $\begin{matrix}EFD\\FEB\end{matrix}$ ablato communi angulo *EFD* (*per axio.* 5.) erit angulus *FEG* major angulo *FEB*: quare recta *EB* intra *EG*, & *FD* cadet, ſecabitque rectam EG: quia vero *EG* demonſtrata eſt parallela ipſi CD: ſequitur (*per axio.* 13.) quod EB debeat etiam ſecare alteram parallelarum CD: ſed recta nequit rectam ſecare niſi cum illa concurrat: ergo EB producta concurret cum CD. q. e. d.

Pro-

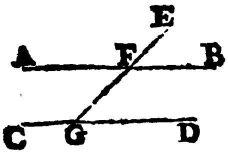
Propoſ. XXIX. Theor. 20.

Si recta EFG incidat in parallelas AB, CD.

Dico primo. Angulos alternos AFG, FGD æquales efficere.

Dico ſecundo. Angulum externum EFB interno, & oppoſito, & ad eaſdem partes FGD æqualem eſſe.

Dico tertio. Internos, & ad eaſdem partes angulos BFG, FGD exiſtere duobus rectis æquales.



Demonſtratio primæ partis.

SI dicatur ex alternis angulis AFG, FGD minorem eſſe angulum FGD, addito communi angulo GFB

erunt duo ang. $\left.\begin{matrix}FGD\\GFB\end{matrix}\right\}$ minores duobus ang. $\left\}\begin{matrix}AFG\\GFB\end{matrix}\right.$

ſed (*per prop.* 13.) ang. $\left.\begin{matrix}AFG\\GFB\end{matrix}\right\}$ = ſunt duobus rectis: ergo

anguli $\left.\begin{matrix}FGD\\GFB\end{matrix}\right\}$ erunt duobus rectis minores:

quare (*per axio* 13.) rectæ AB, CD, neceſſario debent concurrere ad partes BD, contra ſuppoſitum, nam AB, CD ſuppoſitæ ſunt parrallelæ. q. e. i. d.

Demonſtratio ſecundæ partis.

EXternus angulus EFB (*per prop.* 15.) = ang. AFG; ſed angulus AFG (*per primam partem*) = ang. al terno FGD: ergo

 angu-

Angulus externus EFB [*per axio.* 1.] = interno, & oppoſito FGD. q.e.2.d.

Demonſtratio tertiæ partis.

ANgulus AFG (*per primam partem*) = alterno FGD; quare addito communi angulo GFB,

erunt anguli AFG, GFB = ang. FGD, GFB ſed (*per prop.* 13.)

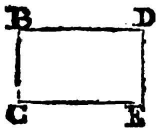
anguli AFG, GFB = duobus rectis: quare, &

anguli FGD, GFB duobus rectis æquales erunt. q. e. 3. d.

Corollarium primum.

Si in Parallelogrammo BE unus angulus ſit rectus, ut ang. B,

Dico omnes angulos eſſe rectos.



CUm recta BC (*per hypoteſim*) cadat in parallelas BD, CE, duo ang. B, & C [*per hanc prop.*) erunt æquales duobus rectis: ſed ang. B ſupponitur rectus: ergo & ang. C rectus erit: eodem modo demonſtrabitur cæteros angulos D, & E eſſe rectos. q. e. d.

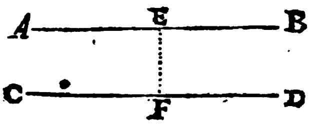
Corollarium ſecundum.

Si recta EF uni parallelarum, nempe AB fuerit perpendicularis,

Dico etiam alteri parallelarum CD perpendicularem eſſe.

CUm [*per hyp.*] EF ſit ipſi AB perpendicularis, (*per def.* 10.) Angulus FEB rectus erit: ſed (*per* 3. *partem hujus prop.*

duo



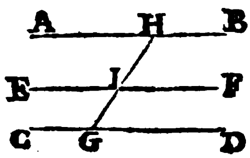
duo ang. FEB) EFD) = ſunt duobus rectis: ergo ang. EFD erit rectus; quare (*per def.* 10.) recta EF ipſi CD perpendicularis erit. q. e. d.

Propoſ. XXX. Theor. XXI.

Si duæ rectæ lineæ AB, CD, in eodem plano exiſtentes cum recta EF, ſint eidem EF parallelæ.

Dico AB, & CD eſſe inter ſe parallelas.

Conſtructio.



Ducatur recta HG, quæ incidat in rectas AB, CD: cum autem ſupponatur, etiam EF eſſe in eodem plano cum rectis AB, CD; eadem recta HG incidet in EF in I.

Demonſtratio.

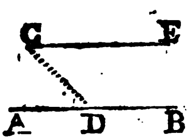
Duæ rectæ AB, EF (*per hyp.*) ſunt parallelæ: ergo (*per prop.* 29.) angulus AHI = ang. alterno HIF. Pariter (*per ſuppoſ.*) parallelæ ſunt EF, CD quare (*per eandem* 29.) externus ang. HIF = interno, & oppoſito IGD: quare (*per axio.* 1.) ang. AHI = ang. IGD: cum autem iſti anguli ſint alterni (*per prop.* 27.) rectæ AB, CD erunt parallelæ. q. e. d.

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Propoſ. XXXI. Probl. X.

Datæ rectæ AB, a dato puncto C parallelam lineam ducere.

Conſtructio.



A Dato puncto C utcumque ducatur recta CD, quæ cum data AB angulos faciat CDB, CDA. Ad punctum datum C (*per prop.* 23.) conſtituatur angulus DCE = ang. CDA. Dico rectam CE parallelam AB.

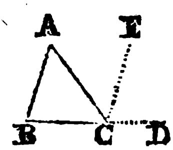
Demonſtratio.

ANguli CDA, DCE [*per conſtructionem*] facti ſunt æquales: ſed ſunt anguli alterni: ergo (*per prop.* 27.) rectæ CE, AB erunt parallelæ. q. e. f.

Propoſ. XXXII. Theor. XXII.

In triangulo ABC producto latere BC in D, Dico primo. Angulum externum ACD æqualem eſſe duobus angulis internis, & oppoſitis, A, & B.

Dico Secundo. Trianguli ABC, tres internos angulos A, B, & C æquales eſſe duobus rectis.



Conſtructio.

A puncto C ipſi BA (*per prop.* 31.) parallela ducatur CE.

De-

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Demonſtratio primæ partis.

ANguli $\left.\begin{matrix}ACE\\ECD\end{matrix}\right)$ (*per axio.19.*) = augulo externo ACD.

Angulus ACE (*per prop.* 29.) = alterno *B*AC.
Angulus ECD (*per prop.* 29.) = interno, & oppoſito A B C: quare
Angulus externus ACD = duobus internis, & oppoſitis A, & B, q. e. 1. d.

Demonſtratio ſecundæ partis.

ANgulus externus ACD (*ex demonſt. in prima parte.*) = angulis A, & B,

addito communi ang. ACB, erunt ang. $\left.\begin{matrix}ACD\\ACB\end{matrix}\right)$ = ang. $\left.\begin{matrix}A\\B\\ACB\end{matrix}\right\}$ ſed

Anguli $\left.\begin{matrix}ACD\\ACB\end{matrix}\right\}$ (*per prop.* 13.) = duobus rectis: ergo etiam

Anguli $\left.\begin{matrix}A\\B\\ACB\end{matrix}\right\}$ = erunt duobus rectis. q. e. 2. d.

Corollarium primum.

EX hac propoſ. fit manifeſtum, tres angulos cuiuſcumque trianguli, eſſe æquales tribus angulis alterius cujuſuis trianguli, cum in omni triang. tres anguli ſint duobus rectis æquales.

Corollarium ſecundum.

SI unius trianguli duo anguli æquales fuerint duobus angulis alterius trianguli, etiam tertius angulus in uno triangulo, erit æqualis tertio angulo in alio triangulo.

Corollarium tertium.

IN triangulo iſoſcele rectangulo anguli ad baſim ſunt ſemirecti, quia ſunt æquales, inter ſe, & ambo æquivalent uni recto

recto. In triangulo iſoſcele obtuſangulo, ad baſim anguli, ſemirecto ſunt minores, & in iſoſcele acutangulo ſunt majores ſemirecto.

Corollarium quartum.

IN triangulo æquilatero quilibet angulus adæquat duas tertias partes unius recti, vel tertiam partem duorum rectorum, quia duobus angulis rectis in tres partes æquales diviſis, quælibet tertia pars duas tertias partes unius anguli recti continet.

Corollarium quintum.

IN triangulo, quando unus angulus duos alios adæquat, ille angulus eſt rectus.

Corollarium ſextum.

CUm duo triangula unum angulum æqualem habent, etiam reliquorum ſummæ ſunt æquales.

SCHOLIUM.

SUperius propoſitum Theorema, quod per totam ferè Matheſim uſus propè immenſos habet, teſtante Eudemo a Pytagora propoſitum fuit. Ex veritate huius Theorematis non ſolum dignoſcimus, quot rectis æquivaleant triangulorum anguli, verum etiam cujuſlibet figuræ rectilineæ, quot ſcilicet rectis angulis ſint æquales rectilinearum figurarum anguli, tam interni, quàm externi; uti videre licet apud Autores hanc rem pertractantes.

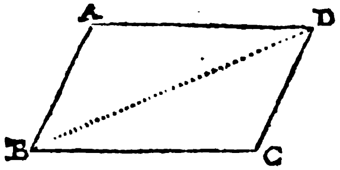
Propoſ. XXXIII. Theor. XXIII.

Si duæ rectæ AB, DC ad eaſdem partes coniungant rectas lineas AD, BC æquales, & parallelas.

Dico

Dico AB, DC, quæ coniungentes dicuntur, quoque eſſe æquales, & parallelas.

Conſtructio.



A Puncto B ad punctum D, ducatur recta BD.

Demonſtratio primæ partis.

REctæ AD, BC (*per hyp.*) ſunt parallelæ: quare [*per prop.* 29.] Anguli alterni ADB, DBC ſunt æquales: Pariter cum *AD* ſit æqualis BC, in duobus triangulis ADB, DBC erunt duo latera AD, DB, ſingula ſingulis æqualia lateribus CB, DB, atque ang. ADB = angulo DBC: quare [*per prop.* 4.] AB = DC. q. e. 1. d.

Demonſtratio ſecundæ partis.

TRiangula ADB, DBC, (*per primam partem*) habent conditiones propoſ. 4: ergo (*per eamdem* 4.) ang. ABD = ang. BDC: quia vero iſti anguli ſunt alterni (*per prop.* 27.]: parallelæ erunt coniungentes AB, DC. q. e. 2. d.

Propoſ. XXXIV. Theor. XXIV.

In parallelogrammo AC

Dico primo. Latera oppoſita AD, BC, atque AB, DC eſſe æqualia.

Dico ſecundo. Angulos oppoſitos A, C, atque B, D eſſe æquales.

Dico tertio. Diametrum DB bifariam diuidere parallelogrammum AC.

Con-

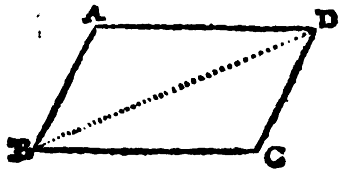
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Conſtructio.

A Puncto B ad D, ducatur recta BD.

Demonſtratio primæ partis.

QVia rectæ AB, DC (*per def.* 40.) ſunt parallelæ: [*per prop.* 29.] erit ang. ABD = alterno BDC. Pariter quia [*per eandem def.* 40.] parallelæ ſunt AD, BC (*per prop.* 29.) angulus ADB = angulo alterno DBC: quo ſtante duo triangula ADB, CDB habebunt duos angul. $\left.\begin{matrix}ADB\\ABD\end{matrix}\right\}$ ſingulos ſingulis æquales duobus ang. $\left.\begin{matrix}DBC\\BDC\end{matrix}\right\}$, latuſque BD commune utrique triangulo: quare (*per prop.* 25.) latera oppoſita AD = BC, atque AB = DC. q. e. 1. d.



Demonſtratio ſecundæ partis.

IN duobus triangulis BAD, BCD [*per prop.* 26.] ang. A = ang. C.

Pariter [*uti demonſtratum eſt in* 1. *parte.*] cum ang. $\left.\begin{matrix}ABD\\DBC\end{matrix}\right\}$ = ſint ſinguli ſingulis angulis $\left.\begin{matrix}ADB\\BDC\end{matrix}\right\}$: erit totus angulus ADC = toti angulo ABC: quare anguli oppoſiti ſunt æquales. q. e. 2. d.

Demonſtratio tertiæ partis.

TRiangula DAB, BCD (*per demonſtrata in prima, & ſecunda parte.*] habent omnia latera, & angulos ſingulos ſingulis æquales: quare (*per prop.* 4.] triangulum BAD = triang. BCD; ergo Parallelogrammum AC a diametro BD erit bifariam diviſum. q. e. 3. d.

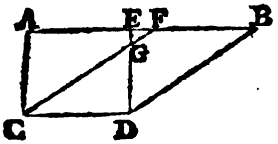
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Propoſ. XXXV. Theor. XXV.

Parallelogramma AD, CB ſuper eadem baſi CD, atque inter parallelas AB, CD conſtituta. Dico eſſe æqualia.

Demonſtratio.

PEr prop. 34. AE = CD, FB = CD } unde (*per axio.* 1.) AE = FB; addita communi quantitate EF; erit AF = EB. Pariter



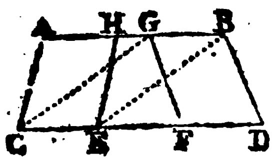
Per prop. 34. AC = ED. Rurſus
Per propoſ. 29. ang. BED = ang. FAC; ergo
Per prop. 4. triang. FAC = trian. BED; ablata communi parte, hoc eſt triangulo EGF
Per axio 3. Trapezium AEGC = trapezio FGDB. addito communi triangulo GCD, erit
Per axio. 2. Parallelogrammum AD = Parallelog. CB. q. e. d.

Propoſ. XXXVI. Theor XXVI.

Si duo Parallelogramma EA, FB ſuper æqualibus baſibus, CE, FD, & inter eaſdem parallelas AB, CD fuerint conſtituta.

Dico parallelogrammum EA = parallelogrammo FB.

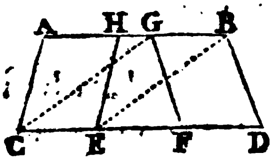
Conſtructio.



Ducantur duæ rectæ CG, EB.

Demonſtratio.

In parallelog. DG (*per prop.* 34.) GB = FD. baſis



baſis F D (per hyp.) = baſi CE : ergo GB = CE (per axio. 1.)

Cum autem [per hyp.] GB ſit parallela CE :

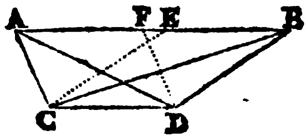
Figura quadrilatera GBEC [per def. 40.] erit parallelogrammum, ſed [per prop. 34.] tam parallelog. EA, quam parallelog. DG = ſunt parallelog. CB : ergo (per axio. 1.) parallelog. EA = parallelog. DG. q. e. d.

Propoſ. XXXVII. Theor. XXVII.

Si triangula CAD, CBD ſuper eadem baſi CD, & inter parallelas AB, CD fuerint conſtituta.

Dico triang. CAD = triang. CBD.

Conſtructio.



A Puncto C [per. prop. 31.] ducatur CE parallela DB : pariter a puncto D notetur DF parallela ipſi AC.

Demonſtratio.

Quia CD [per hyp.] eſt parallela AB, atque DF [per conſtructionem] eſt parallela AC ; quadrilaterum ACDF [per def. 40.] erit parallelog. eadem ratione parallelog. erit quadrilaterum CEBD.

Quare [per prop. 35.] parallelog. DA = parallelog. DE.

Quia vero triangula CAD, CBD [per prop. 34.] ſunt medietates parallelogrammorum æqualium DA, DE (per axio. 7.) dicta triangula CAD, CBD erunt æqualia q. e. d.

Prop.

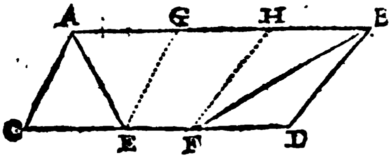
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Propoſ. XXXVIII. Theor. XXVIII.

Si triangula C A E, D B F ſuper æqualibus baſibus C E, F D, & inter eaſdem parallelas AB, CD conſtituta fuerint.

Dico triangulum CAE = triang. DBF.

Conſtructio.



A Punctis, E, & F [*per prop.* 31.] ducantur *E* G, F H, quarum *EG* ſit parallela AC, & FH parallela DB.

Demonſtratio.

Cum AB, *CD* ſupponantur parallelæ, erit *AG* parallela CE, & HB, parallela F*D*. Pariter *EG* [*per conſtructionem*] facta eſt parallela CA, & FH parallela *DB*: quare [*per def.* 40.) CG, *D*H erunt parallelogramma, quæ [*per prop.* 36.] ſunt æqualia.

Sed triangula CA*E*, *D*BF [*per prop.* 34.] ſunt medietates æqualium parallelogrammorum C G, *D*H: ergo dicta triangula [*per axio.* 7.] erunt æqualia. q. e. d.

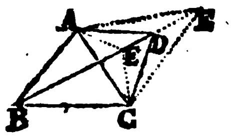
Propoſ. XIL. Theor. XXIX.

Si æqualia triangula BAC, BDC ſuper eadem baſi BC, atque ad eaſdem partes conſtituta fuerint.

Dico rectam per A, & D ductam, ipſi BC eſſe parallelam.

Con-

Conſtructio, & Demonſtratio.



SI AD, & BC non ſunt parallelæ, ex puncto A [*per prop.*31.] ducatur recta parallela BC, quæ ſupra, vel infra AD cadat. Primo cadat ſupra in E.

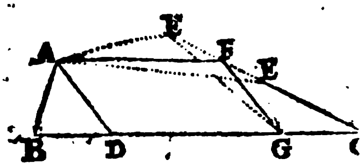
Quo facto producatur BD, donec concurrat cum AE in E; debent enim concurrere, quia duo ang. BAE, ABE ſunt duobus rectis minores; pariter ducatur CE.
His poſitis [*per. prop.* 37.] triang. BAC = triang. BEC; ſed
Triang. BAC (*per hyp.*] = triang. BDC: ergo
Triang. BEC [*per axio.* 1.] = triang. BDC: quod fieri nequit, cum totum ſit ſemper ſua parte majus: quare AE non eſt EC parallela: idem demonſtrabitur ſi dicatur rectam per A ipſi BC ductam parallelam cadere infra AD in E, in quo caſu pars adæquaret ſuum totum. Cum igitur ex A neque ſupra, neque infra AD poſſit duci parallela ipſi BC, ſola AD erit ipſi BC, parallela. q. e. d.

Propoſ. XL. Theor. XXX.

Si duo triangula BAD, GFC ſuper æqualibus baſibus BD, GC ad eamdem partem ſint æqualia.

Dico rectam AF ipſi BC eſſe parallelam.

Conſtructio, & Demonſtratio.



SI AF non eſt ipſi BC parallela (*per prop.* 31.) ducatur AE parallela FC, quæ, vt in præcedente demonſtratione, ſupra, vel infra AF cadere debet. Si cadat ſupra in E producatur CF uſque quo concurrat cum AE in E; debet enim concurrere, quia ſi concipiamus lineam rectam ductam ab A ad C, clarè apparebit, duas rectas AE, CE diſcedere a duobus angulis minoribus duobus rectis:

quare: (*per axio.* 13.) CF debet concurrere cum AE: ducatur recta EG.

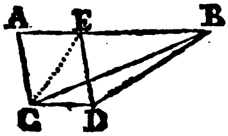
His poſitis [*per prop.* 38.] triang. BAD = triang. GEC: ſed
Triang. BAD = triang. GFC [*per hyp.*]: ergo
Triang. GEC = triang. GFC (*per axio.* 1.): quod eſt impoſſibile [*per axio.* 9.] quare AE nequit eſſe parallela BC: hujuſmodi inconveniens demonſtrabitur de quacumque alia linea, excepta ſola AF; ergo AF erit ipſi BC parallela: q. e. d.

Propoſ. XLI. Theor. XXXI.

Si parallelogrammum AD, atque triangulum CBD ſuper eadem baſi CD, & inter eaſdem parallelas AB, CD fuerint conſtituta.

Dico parallelogrammum AD eſſe duplum trianguli CBD.

Conſtructio.



A Puncto C ad E ducatur recta CE.

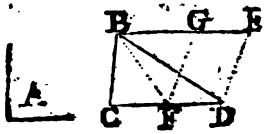
Demonſtratio.

Triang. CAE (*per prop.* 34.) = triang. CED: ergo
Parallelog. DA duplum trianguli CED: ſed
Triang. CED (*per prop.* 37.) = triang. CBD: quare
Parallelog. DA (*per axio.* 6.) duplum trianguli CBD: q. e. d.

Propoſ. XLII. Probl. XI.

Dato triangulo CBD, & angulo rectilineo A. In dato angulo A conſtituere parallelog. DG æquale dato triangulo *CBD*.

Conſtructio.



QVodvis latus dati trianguli, ut CD (*per prop.* 10.) bifariam dividatur in F.
Ad punctum F, & ad rectam FD (*per prop.* 23.) conſtituatur angulus DFG = ang. dato A.
A puncto B [*per prop.* 31.] ducatur BE parallela CD:
Per punctum D notetur DE parallela FG.
Dico Parallelog. DG = eſſe triang. CBD, & habere ang. DFG = dato angulo A.
Ducatur recta BF.

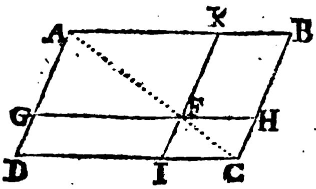
Demonſtratio.

TRiangulum CBF (*per prop.* 38.) = triang. *F*BD: unde Triang. CBD duplum triang. *F*BD: ſed Parallelog. DG (*per prop.* 41.) eſt duplum trianguli *F*BD: ergo Parallelog. DG [*per axio.* 1.] = triang. CBD.
Pariter cum angulus *D*FG [*per conſtructionem*] ſit = dato ang. A factum eſt q. e. f.

Propoſ. XLIII. Theor. XXXII.

In Parallelogrammo DB
Dico complementum DF = complemento FB.

Conſtructio.



PRo efformandis complementis, atque conſiſtentibus circa diametrum parallelogrammi, conſtructio habetur in definit. 41. quæ conſtructio hoc in loco ex integro ſupponitur.

De-

Demonſtratio.

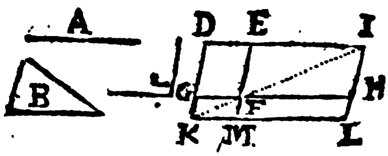
TRiang. ADC = triang. ABC
Triang. AGF = triang. AKF } [*per prop.* 34.] ergo
Triang. FIC = triang. FHC

Si a triang. ADC auferantur triang. AGF, FIC (*per axio.* 3) & pariter ſi a triang. ABC auferãtur triang. AKF, FHC erit complementum *DF* = complemento *FB*. q. e. d.

Propoſ. XLIV. Probl. XII.

Triangulo B, angulo rectilineo C, & recta A datis; efformare parallelogrammum æquale triang. B in angulo rectilineo C, atque in recta A.

Conſtructio.

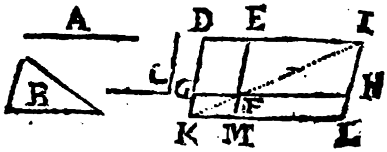


TRiangulo dato B (*per prop.* 42.) in angulo C conſtituatur æquale parallelog. GE, ſitque ang. GFE = ang. dato C.

Producatur GF, fiatque [*per prop.* 3.) *FH* = A.
per H ducatur HI parallela E*F*, producaturque DE quouſque concurrat cum HI in I.
Notetur I*F*, quæ producatur donec occurrat ipſi DG in K.
Per K notetur KL parallela GH.
Producantur IH, E*F* quouſque occurrant ipſi KL in punctis M, & L.

Dico parallelogrammum MH æquale eſſe triang. B, habere angulum M*F*H æqualem ang. dato C, atque latus *F*H æquale datæ rectæ A.

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Demonſtratio.

PArallelogrammum MH [*per prop.43.*] = parallelog. GE; ſed parallelogrammum GE (*per conſtuctionem*) = triangulo dato B : quare

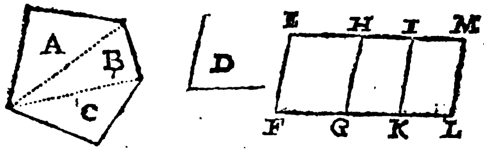
Parallelogrammum MH [*per axio.1.*] = triangulo dato B.

Ulterius in parallelogrammo MH angulus MFH = angulo dato C, quia [*per prop.15.*] eſt æqualis ang. GFE, qui (*per conſtructionem*) factus eſt æqualis angulo C, demumque latus FH [*per conſtructionem*] æquale dato lateri A : unde parallelog. MH erit æquale triangulo B, habebit angulum MFH æqualem angulo C, atque latus FH æquale lateri A. q. e. f.

Propoſ. XLV. Probl. XIII.

Dato rectilineo ABC, angulo rectilineo D, atque recta EF Parallelogrammum conſtituere æquale rectilineo dato ABC, quod habeat angulum æqualem angulo dato D, habeatque latus æquale rectæ lineæ EF.

Conſtructio.



DAtum rectilineum reſoluatur in triangula numero minora quantum fieri poteſt, ducendo ab uno angulo ad reliquos

toti-

totidem lineas rectas. Hæc triangula in hoc caſu tria ſunt, videlicet A, B, C. Hoc facto triang. A [*per propoſ.* 44.] in angulo *D*, & in latere *EF* efformetur æquale parallelog. *FH*, ſitque angulus EFG = angulo D. Pariter [*per prop.* 44.] triangulo B in angulo *D*, & in latere EF, ſeu HG efformetur aliud æquale parallelogrammum GI, in quo angulus HGK ſit dato angulo *D* æqualis. Eadem regula ad rectam IK in angulo *D* efformetur parallelog. KM æquale triangulo C, & angulus IKL æqualis ſit dato angulo *D*.

Dico figuram FM eſſe parallelog. æquale rectilineo *ABC*, in quo latus EF eſt lateri dato æquale, atque angulus EFL æqualis dato angulo D.

Demonſtratio.

Anguli EFG, HGK ambo (*per conſtruct.*) facti ſunt æquales ang. dato D: quare (*per axio.* 1.) dicti anguli erunt inter ſe æquales. Addito communi angulo FGH, erunt angul.

$$\left.\begin{matrix}EFG\\FGH\end{matrix}\right\} = \text{ang.} \left.\begin{matrix}FGH\\HGK\end{matrix}\right\}$$

At (*per prop.* 29.) duo ang. priores ſunt duobus rectis æquales: ergo etiam duo poſteriores duobus rectis æquales erunt:

ſi ergo duo ang. $\left.\begin{matrix}FGH\\HGK\end{matrix}\right\}$ = ſunt duobus rectis, (*per prop.* 14.)

FG, & GK erunt in rectum conſtitutæ: eodem modo demonſtrabitur KL, in rectum eſſe cum FK, ideoque FL exiſtere lineam rectam; quod pariter de linea EM intelligendum venit.

Cum autem (*per conſtructionem*) EH ſit ipſi FG parallela, etiam EM, & FL erunt parallelæ:

Pariter cum (*per prop.* 30.) EF, ML ſint parallelæ, figura FM erit parallelogrammum, in quo angulus EFG (*per conſtructionem*) eſt = ang D, latuſque EF data recta.

Quia vero omnes partes parallelogrammi FM numero, & magnitudine æquant partes dati rectilinei A, B, C.

Parralielog. FM æquale erit rectilineo dato ABC. q. e. f.

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Propoſ. XLVI. Prob. XIV.

Datæ rectæ lineæ AB quadratum deſcribere.



Conſtructio.

A Duobus punctis A, & B ipſi AB, (*per prop.* 11.) ducantur duæ perpendiculares AD, BC, & [*per prop.* 3.) fiant ipſi AB æquales: ducatur DC. Dico AC eſſe quadratum ſuper data AB deſcriptum.

Demonſtratio.

DVo ang. DAB, CBA (*per conſtructionem*) ſunt duobus rectis æquales, quare [*per prop.* 28.) AD, BC erunt parallelæ.

Cum autem AD, BC ambæ factæ ſint æquales ipſi AB erit (*per axio.*1.) AD = BC: quare (*per prop.*33.) coniungentes AB, DC erunt, & ipſæ æquales, & parallelæ: ideoque AC (*per def.*40.) erit parallelogrammum, in quo recta AD (*per conſtructionem*) æqualis eſt AB: quare (*per prop.* 34.) etiam æqualia erunt reliqua latera BC, CD. Rurſus cum duo anguli DAB, ABC (*per conſtruct.*) ſint recti, etiam oppoſiti C, & D (*per prop.* 34.) recti erunt: quamobrem figura quadrilatera AC (*per def.*29.) erit quadratum datæ rectæ AB. q. e. f.

SCHOLIUM.

AB æqualibus rectis lineis deſcripta quadrata ſunt æqualia, & viciſſim ſi quadrata fuerint æqualia, etiam lineæ, à quibus deſcribuntur, æquales erunt. Hæc veritas ſatis patet, quia ſi quadrata ab æqualibus lineis deſcripta, ſibi ſuperponantur, congruent: cum habeant latera, & angulos æquales, utpotè rectos; e contra, ſi quadrata ſint æqualia debent congruere: ergo, & latera erunt congruentia; ideoque æqualia.

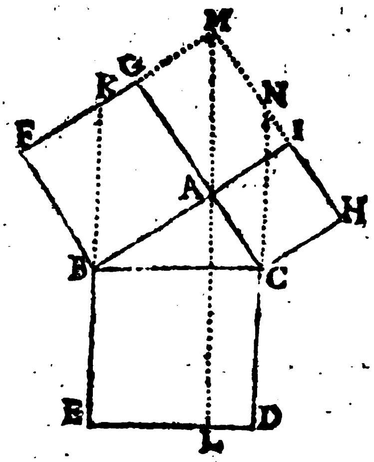
Propoſ.

Propoſ. XLVII. Theor. XXXIII.

In triangulo rectangulo BAC cum angulo recto A,

Dico quadratum lateris BC ſubtenſi angulo recto A adæquare duo quadrata laterum AB, AC comprehendentium angulum rectum A.

Conſtructio.

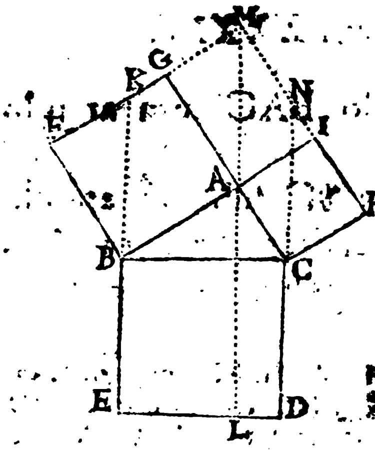


Lateris BC (*per prop. 46.*) deſcribatur quadratum BD. Pariter laterum AB; AC notentur quadrata AF, AH.
A puncto A (*per prop. 31.*) ducatur AL parallela CD, vel BE, quæ verſus M indefinitè producatur.
EB producatur uſque ad K, & DC uſque ad N.
Pariter FG producatur uſqueque concurrat cum AM in M: ſic etiam continvetur HI donec, & ipſa concurrat cum AM in M.

Demonſtratio.

In triangulo BAC duo anguli $\left.\begin{matrix}BAC\\ABC\end{matrix}\right\}$ ſinguli ſingulis ſunt equales duobus angulis $\left.\begin{matrix}BFK\\FBK\end{matrix}\right\}$ in triangulo BFK, quia anguli $\left.\begin{matrix}BAC\\BFK\end{matrix}\right\}$ (*per ſuppoſitum, & per conſtructionem*) ſunt recti, pariter cum recti ſint anguli $\left.\begin{matrix}CBK\\ABF\end{matrix}\right\}$ dempto communi ang. ABK qui reſtant ang. $\left.\begin{matrix}ABC\\FBK\end{matrix}\right\}$ (*per axio. 3.*) erunt æquales,

 Ul-



Ulterius in triang. BAC eſt latus BA = lateri BF (*per def. quadrati*) in triangulo BFK; quare (*per prop. 26.*) BC = BK: ſed (*per def. quad.*) BC = BE: ergo (*per axio. 1.*) BK = BE.

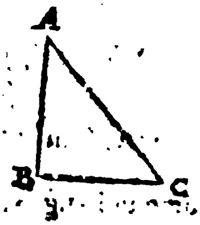
His ſtantibus, quia KE (*per conſtructionem*) eſt parallela ML (*per prop. 36.*) erit Parallelogrammum BL = parallelog. BM: ſed parallelog. BM (*per prop. 35.*) eſt = parallelog. ſeu quadrato AF: ergo parallelog. BL (*per axio. 1.*) erit = quadrato AF.

Eodē prorſus modo demōſtrabitur parallelog. LC. = quadr. AH. Cum autem duo parallelogramma BE LC adæquēt quadratū *BD* quia ſunt omnes dicti quad. partes, etiam quad. *BD* æquale erit duobus quad. AF AH q. e. d.

SCHOLIUM.

HUjus Theor[illegible] ad Pytagoricum dicitur ab inventore Pytagor[illegible] [illegible]uens eſt uſus in tota Matheſi cum non tantum loc[illegible] ſe[illegible] in [illegible]atis, verùm etiam in cæteris figuris ſimilibus, [illegible]monſtra[illegible]ur in prop. 30. lib. 6. Ab hoc Theoremate velut fonte [illegible]er[illegible] problemata, ac corollaria deducuntur a Geometris, quorum unum tantum placet hic addere.

Corollarium.



IN triangulo iſoſcele rectangulo ABC quadratum lateris AC duplum eſt quadrati lateris AB, vel BC: quia (*per demonſtrata in hac prop.*) quadr[illegible] AC = quad. AB BC, & propter æqualitatem late-

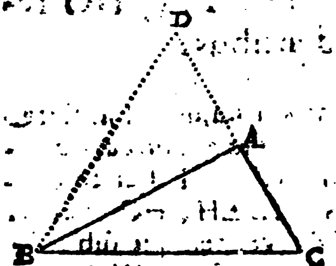
laterum $\left.\begin{matrix}AB\\BC\end{matrix}\right\}$ etiam talium laterum quad. ſunt æqualia ; unde ſequitur quad. AC ſolius quad. AB eſſe duplum. q. e. d.

Propoſ. III. Theor. XXXIV.

In triangulo BAC ſi quad. lateris BC ſit = quadratis laterum BA, AC,

Dico angulum BAC eſſe rectum, & conſequenter triang. BAC rectangulum.

Conſtructio.



A Puncto A ſupra BA (*per prop. 11.*) erigatur perpendicularis AD quæ ſit ipſi AC æqualis, ducaturque BD.

Demonſtratio.

IN triangulo rectangulo BAD (*per prop. 47.*) = quad. BD quad. $\left.\begin{matrix}BA\\AD\end{matrix}\right.$ ſed

Quad. AD (*per conſtructionem.*) = quad. AC: ergo

Quad. BD = q. $\left.\begin{matrix}BA\\AC\end{matrix}\right\}$ ſed (*per hypoteſim*)

Quad. BC = quad. $\left.\begin{matrix}BA\\AC\end{matrix}\right\}$ quare (*per axio. 1.*)

Quad. BD = quad. BC.

Quo ſtante in triangulo BAC erunt latera $\left.\begin{matrix}BA\\AC\end{matrix}\right\}$ ſingula ſingulis æqualia lateribus $\left.\begin{matrix}BA\\AD\end{matrix}\right\}$ in triang. BAD: pariterque baſis BC = baſi BD: unde (*per prop. 8.*) angul. BAC = angul. BAD: ſed ang. BAD (*per conſtructionem*) rectus eſt: ergo etiam ang. BAC rectus erit. q. e. d.

Elementi primi finis.

LIBER II.

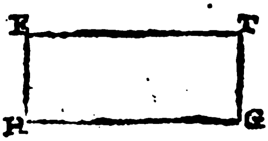
EUCLIDIS

ELEMENTUM SECUNDUM.

Definitio I.

Parallelogrammum rectangulum EG, dicitur contineri ſub duabus rectis lineis EH, HG rectum angulum comprehendentibus.

AD clariorem intelligentiam hujus defin. ſcire oportet, capacitatem, ſiuè aream cujuſcumque parallelogrammi rectanguli dici contentam ſub duabus rectis lineis EH, HG: vel ſub HG, GT vel GT, TE: vel denique ſub TE, EH, quæ unum angulum rectum EHG comprehendant: quælibet enim duæ lineæ ſic acceptæ oſtendunt totam magnitudinem parallelogrammi rectanguli EG: nam EH exprimit latitudinem, HG vero longitudinem. Quo ſtante duabus datis lineis æqualibus, vel inæqualibus, uti ſunt EH, HG rectum angulum EHG efformantibus, ex ſola talium linearum magnitudine intelligitur quantitas parallelogrammi rectanguli EG; cum una ex dictis lineis EH oſtendat latitudinem, altera vero HG longitudinem demonſtret. Et ſanè ſi concipiamus rectam EH perpendiculariter moueri per totam HG, tali motu producetur area parallelogrammi rectanguli EG: idem prorſus eveniet, ſi HG perpendiculariter moueatur ſuper HE: unde ſequitur hujuſmodi figuræ quantitatem haberi per ductum, ſiue multiplicationem duarum rectarum angulum rectum comprehendentium, uti ſunt EH, HG.



SCHO-

SCHOLIUM.

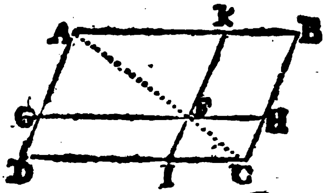
Quando recta lineæ parallelogrammum rectangulum comprehendentes ſunt æquales, reſultat quadratum, quando inæquales oblongum parallelogrammum continent.

Quodcumque parallelogrammum rectangulum ſiuè ſit quadratum, ſiue oblongum nomine RECTANGULI appellatur.

Signum, quod rectangulum indicat, eſt X, ut in principio primi elementi dictum fuit.

Def. II.

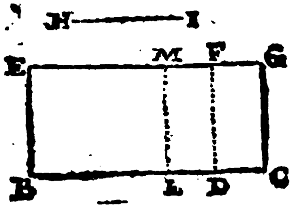
In parallelogrammo AC, ſpatium illud, quod continetur ab uno eorum, quæ circa diametrum ſunt parallelogrammorum, videlicet GK, vel IH, & duobus complementis DF, & FB, GNOMON appellatur.



Propoſ. I. Theor. I.

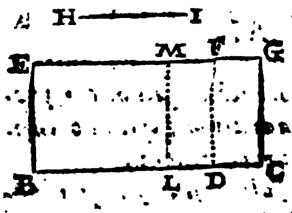
Si fuerint duæ rectæ lineæ BC, & HI, quarum una, nempe BC ſit in quaſcumque partes diviſa, ut in D, & L;

Dico rectangula facta ab integra HI, & a ſegmentis BL, LD, DC eſſe æqualia rectangulo comprehenſo a duabus datis BC, HI.



Conſtructio.

Supra BC ad punctum B. (*per 11. pri.*) conſtituatur perpendicularis BE æqualis HI. Per E notetur EG (*per 31. pri.*) parallela BC. Ulterius a punctis C, D, & L ſignentur rectæ



rectæ CG, DF, LM, parallelæ BE, quæ producantur donec occurrant ipſi EG in punctis G, F, M.

Demonſtratio.

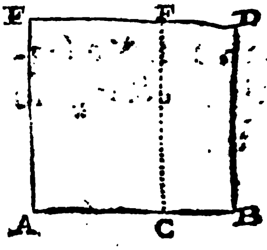
IN figura quadrilatera BG (*per conſtructionem*) latera oppoſita ſunt parallela, & anguli recti: quare BG erit rectangulum a datis rectis BC, & BE, ſeu HI efformatum, cujus partes ſunt rectangula BM, LF, DG (*per axio.19.*) æque ſunt æquales rectangulo BG. Cum autem (*per 34. pri.*) æquales ſint rectæ BE, LM, DF, atque BE ſit facta æqualis HI: ſequitur BE, LM, DF adæquare HI: quare

EB × BL, ML × LD, FD × DC } facta ſunt ab integra HI, & a ſegmentis *BL*, LD, DC: ſed hæc rectangula (*per axio. 19.*) æqualia ſunt HI × BC: ergo rectangula comprehenſa a data HI, & a ſegmentis alterius datæ BC æqualia ſunt rectangulo ſub datis HI, BC contento. q. e. d.

Propoſ. II. Theor. II.

Si fuerit recta AB ſecta in quaſcunque partes AC, CB,

Dico rectangula a tota AB, & a quolibet ſegmento AC, & CB deſcripta, æqualia eſſe quadrato totius AB.



Conſtructio.

SUper data AB (*per 46. pri.*) conſtituatur quadratum AD: ex puncto C notetur CF parallela AE, vel BD.

Demonſtratio.

AF, & CD ſunt duo rectangula (*per axio. 19.*) = q. AD: ſed

AF eſt BA × AC, CD eſt BA × *CB* } ergo

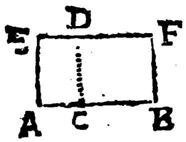
BA × AC, BA × *CB* } = q. AD. q. e. d.

Pro-

Propoſ. III. Theor. III.

Si recta AB in duas partes AC, CB utcumque ſecta ſit.

Dico rectangulum, ſub tota AB, & partium alterutra BC comprehenſum, adæquare rectangulum partium AC, CB una cum quadrato prædictæ partis BC.



Conſtructio.

Super CB (*per 46. pri.*) conſtituatur quadratum CF. Ex A ducatur AE parallela CD, quæ occurrat ipſi FD in E.

Demonſtratio.

Per conſtructionem CF eſt quad. quare *BC* = *BF*: atqui [*per pri. hujus.*]

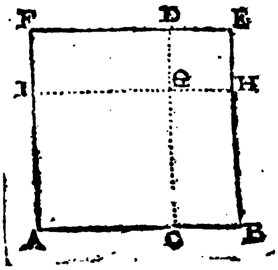
$BC \times BA = \left\{ \begin{array}{l} BC \times CA \\ BC \times BC \end{array} \right\}$: & *BC* × *BC* eſt quad. ergo

$BC \times BA + q.\ BC = BC \times BA$. q.e.d.

Propoſ. IV. Theor. IV.

Si recta AB utcumque in C ſecta fuerit,

Dico quad. totius AB æquale quad. ſegmentorum AC, CB, atque rectangulo bis ſub ſegmentis AC, CB contento.

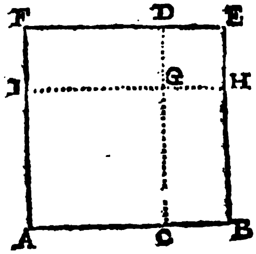


Conſtructio.

Super tota *AB* [*per 46. pri.*] conſtituatur quad. AE; ex puncto C ducatur CD parallela AF, vel *BE*: fiat CG = CA: per G, ducatur IH parallela *AB*, vel FE.

De-

Demonſtratio.



QVia (*per conſtructionem*) CG = CA fig. AG erit quad. ſegmenti AC. Pariter quia *B*A = CD, ablatis æqualibus CG, CA; remanebit DG = C*B*: ſed C*B* (*per 34. pri.*) æquat GH: ergo DG = GH: quare fig. GE erit quad. ſegmenti C*B*. Demum figuræ ID, CH ſunt rectangula ſegmentorum AC, C*B*; his ſtantibus

Quad. AE (*per 2. hujus*) = { *B*A × AC, *B*A × C*B* }: ſed [*per 3. hujus.*]

*B*A × AC = AC × C*B* + q. AC)
*B*A × C*B* = AC × C*B* + q. C*B*) : quare
AC × C*B* bis)
quad. AE = q. AC) q. e. d.
q. C*B*)

Corollarium.

EX hac prop. conſtat quod ſi in Quadrato AE efformatum ſit quad. unius ſegmenti AC, nempe AG: productis lateribus CG, IG; rectangulum ex GD, GH efformatum eſt quad. alterius ſegmenti C*B*: quia latera GD, GH ſunt æqualia.

Propoſ. V. Theor. V.

Si recta AB fuerit ſecta in partes æquales in C, & in partes inæquales in D.

Dico rectangulum ſub partibus inæqualibus AD, DB contentum, una cum quadrato intermediæ partis CD, æquale eſſe quadrato dimidiæ CB.

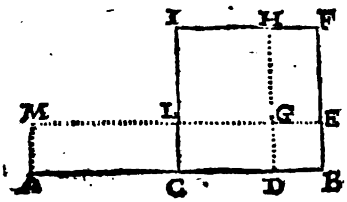
Conſtructio.

SVper dimidia C*B* [*per 46. pri.*] conſtituatur q. CF: ſupra *B*D. fiat. q. DE. ex A ad A*B* [*per 11. pri.*] erigatur perpen-

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pendicularis AM; producaturque EG donec concurrat cum AM in M: ulterius ex puncto D ſignetur DH parallela CI, vel BF.

Demonſtratio.



QVia [*per conſtructionem*] CF, & DE ſunt quadrata, (*per corol. 4. hujus*) LH erit quadratum intermediæ CD: Pariter quia (*per eſſentiam quadrati*) GD = DB, rectangulum AG erit comprehenſum ab inæqualibus partibus AD, DB.

His ſtantibus, cum CF, & DE ſint quadrata erit CB = BF, atque DB = BE: quare CB × BE = FB × BD: ſed (*per 36. pri.*) CB × BE = AC × CL: ergo (*per axio. 1.*)
FB × BD = AC × CL
addito communi CD × DG erit
FB × BD + CD × DG = AC × CL + CD × DG
demuo addito communi q. LH, quod eſt q. rectæ CD erit
FB × BD + CD × DG + q. CD = AC × CL + CD × DG + q. CD, ſed
FB × BD + CD × DG + q: CD (*per axio. 19.*) = q. CB: ergo
AC × CL + CD × DG: ſiuè AD × DG + q. CD = q. CB. q.e.d.

Propoſ. VI. Theor. VI.

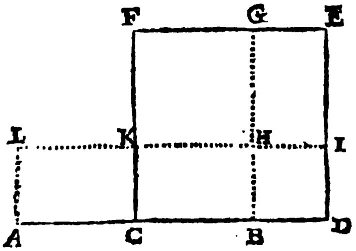
Si recta AB ſit ſecta bifariam in C, & illi in rectum adiecta ſit BD.

Dico rectangulum contentum ſub compoſita ex AB, BD, hoc eſt AD, & ſub adiecta BD, una cum quadrato dimidiæ CB: æquale eſse quadrato compoſitæ ex dimidia CB, & adiecta BD, ſeu quadrato CD.

Con-

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Conſtructio.



SUper CD [*per 46. pri.*] conſtituatur q. CE: pariter ſupra *BD* efformetur q. BI. Ex A ad AD ducatur perpendicularis AL. Producatur IH quouſque concurrat cum AL in L. Pariter producatur BH uſque ad G.

Demonſtratio.

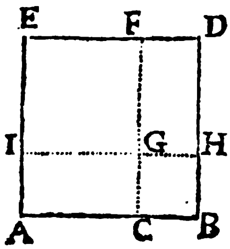
CUm [*per conſtructionem*) BI ſit quadratum ipſius BD [*per corol. 4. bujus.*)
*K*G erit quad. ipſius *CB*: his ſtantibus
CD X DI = ED X D*B*.
Dempto communi, hoc eſt q. BI; erit
CB X BH = EI X IH: ſed (*per 36. pri.*)
CB X BH = A*C* X CK: quare
EI X IH = A*C* X CK.
Addito communi *CD* X DI; erit
AC X C*K* + *CD* X *DI*, hoc eſt *AD* X *DI* = EI X IH + *CD* X DI
Denuo addito communi q. rectæ C*B* hoc eſt q. KG: erit
AD X *DI* + q. *CB* = EI X IH + *CD* X *DI* + q. *CB*: ſed
EI X IH + C*D* X *DI* + q. *CB*, hoc eſt q. KG = q. C*D*, ideſt q. CE.
quare etiam
AD X *DI* + q. CB = q. *CD*: hoc eſt CE: q. e. d.

Propoſ. VII. Theor. VII.

Si recta AB utcumque ſecta fuerit in C.

Dico quadrata totius AB, atque unius ſegmenti CB, æqualia eſſe rectangulo bis ſub A*B*, & C*B* contento, una cum quadrato alterius ſegmenti AC.

Con-



Conſtructio.

Super AB fiat quad. AD: pariter ſupra CB conſtituatur quadratum CH, producanturque CG, HG, in F, & I.

Demonſtratio.

Ex demonſtratis in 4. hujus.

$$\text{q. AB, hoc eſt AD} = \left(\begin{array}{l} \text{AC} \times \text{CB bis} \\ \text{q. AC} \\ \text{q. CB} \end{array} \right.$$

Addito communi q. CB, hoc eſt CH: erunt

$$\left. \begin{array}{l} \text{q. AB} \\ \text{q. CB} \end{array} \right\} = \left. \begin{array}{l} \text{AC} \times \text{CB bis} \\ \text{q. AC} \\ \text{q. CB bis} \end{array} \right)$$

Sed per tertiam hujus.

$$\left. \begin{array}{l} \text{AC} \times \text{CB bis} \\ \text{q. CB bis} \end{array} \right\} = \text{AB} \times \text{BC bis: ergo}$$

$$\left. \begin{array}{l} \text{q. AB} \\ \text{q. CB} \end{array} \right\} = \left. \begin{array}{l} \text{AB} \times \text{BC bis} \\ \text{q. AC} \end{array} \right\} \text{q. e. d.}$$

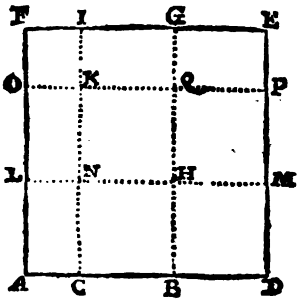
Propoſ. VIII. Theor. VIII.

Si recta AB utcumque ſecta ſit in C.

Dico rectangulum quater ſub AB, & uno ſegmentorum CB una cum quadrato alterius ſegmenti AC, æquale eſſe quadrato ex compoſita AB, & BC deſcripto.

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Conſtructio.



DAtæ rectæ AB in rectum adda-tur BD = BC. Super AD conſtituatur quad. AE. Ex punctis C, & B notentur rectæ CI, BG parallelæ AF, vel DE. Super BD fiat quad. BM, producatur MH uſq; ad L ſupra FI fiat quad. IO, pariter producatur OK uſque ad P.

Demonſtratio.

QVia (*per conſtructionem*] BM, & OI ſunt quadrata (*per collor.*4. *hujus*) quad. etiam erunt LG, CP, & NQ: quia vero BD facta eſt ipſi BC æqualis; q. BD = q. BC, atque AB × BC = AB × BD,

Pariter quia AC = FI; quad. IO erit quad. ipſius AC.
his manentibus [*per* 7. *hujus*.]

AB × BC bis) = quad. AB)
q. AC) = quad. BC) ſed [*per conſtructionem.*]
BC = BD: ergo

AB × BC = AB × BD; atque q. BC = q. BD: quo ſtãte erit
AB × BD bis) = q. AB
q. AC) = q. BD

Additis duobus AB × BD; erunt.
AB × BD quater } = q. AB
q. AC } = q. BD
AB × BD bis:

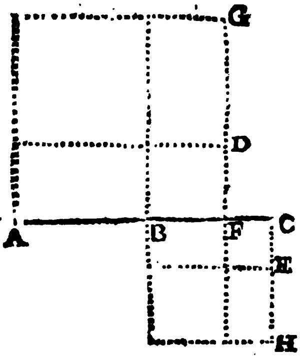
Sed [*per* 4. *hujus.*]
q. AB }
q. BD } = q. AD: ergo etiam
AB × BD bis }
AB × BC quater } = q. AD. q. e. d.
q. AC }

Pro-

Propoſ. IX. Theor. IX.

Si recta AC ſit bifariam diviſa in B, & non bifariam in F.

Dico quadrata inæqualium partium AF, FC ſimul accepta eſſe dupla quadratorum dimidiæ AB, & intermediæ ſectionis BF.



Conſtructio.

Super AF [*per 46. pri.*] efformetur quad. AG. Supra *B*C conſtituatur quad. *B*H: pariter ſupra *B*F quad. *B*D; & ſupra FC quad. FE.

Demonſtratio.

Per 4. *bujus*) q. AF hoc eſt AG $= \begin{cases} AB \times BF \text{ bis} \\ q.\ AB \\ q.\ BF \end{cases}$

Addito communi quad. FC, hoc eſt FE erunt.

$$\begin{cases} q.\ AF \\ q.\ FC \end{cases} = \begin{cases} AB \times BF \text{ bis} \\ q.\ AB \\ q.\ BF \\ q.\ FC \end{cases}$$

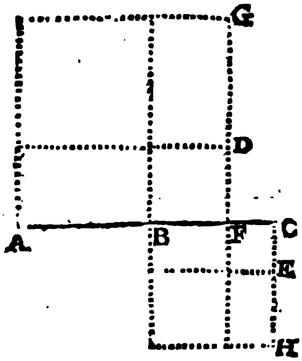
Sed (*per hyp.*)
$AB \times BF = CB \times BF$: ergo

$$\left.\begin{matrix} q.\ AF \\ q.\ FC \end{matrix}\right\} = \begin{cases} CB \times BF \text{ bis} \\ q.\ AB \\ q.\ BF \\ q.\ FC \end{cases}$$; ſed (*per 7. bujus.*)

$$\left.\begin{matrix} CB \times BF \text{ bis} \\ q.\ FC \end{matrix}\right\} = \left.\begin{matrix} q.\ BC \\ q.\ BF \end{matrix}\right\} \text{ſeu} \left.\begin{matrix} q.\ AB \\ q.\ BF \end{matrix}\right\} \text{ quare}$$

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Si hæc illis ſubſtituantur; erunt

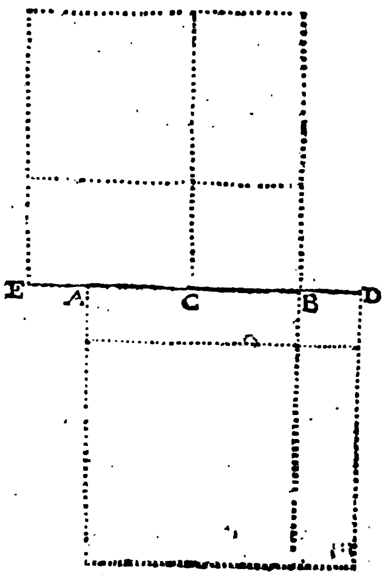
$$\left.\begin{matrix} q.\ AF \\ q.\ FC \end{matrix}\right) = \left.\begin{matrix} q.\ AB \text{ bis} \\ q.\ BF \text{ bis} \end{matrix}\right) :$$ quare

$$\left.\begin{matrix} q.\ AF \\ q.\ FC \end{matrix}\right\} \text{dupla} \left.\begin{matrix} q.\ AB \\ q.\ BF \end{matrix}\right\}$$ q.e.d.

Propoſ. X. Theor. X.

Si recta AB ſit bifariam ſecta in C, eique in rectum adiecta ſit BD.

Dico duo quadrata rectarum AD, & BD dupla eſſe quadratorum, rectarum AC, & CD.



Conſtructio.

PRoducatur *BA* verſus E, ſitque AE = *BD*.

Demonſtratio.

QVia [*per hyp.*] A*B* eſt bifariam diviſa in C, atque EA = *BD* [*per axio.2.*] EC = CD.

[*per*

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[*per 9. hujus.*] $\left.\begin{matrix}q.EB\\q.BD\end{matrix}\right\}$ dupla $\begin{matrix}q.EC\\q.CB\end{matrix}$:ſed

$$\text{[per conſtruc.]}\ \begin{matrix}q.EB = q.AD)\\ q.AC = q.CB)\\ q.EC = q.CD)\end{matrix}\ \text{:quare}$$

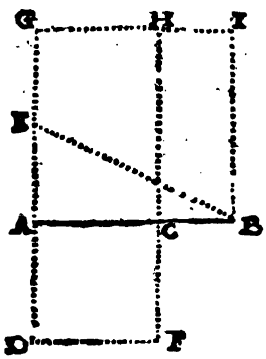
Facta mutatione horum æqualium quadratorum, nempe q. *EB* in q. AD : q. EC in q. *CD*: & q. C*B* in q. AC erunt

$$\left.\begin{matrix}q.AD\\q.BD\end{matrix}\ \text{dupla}\right\}\left.\begin{matrix}q.CD\\q.AC\end{matrix}\right\}\ \text{q.e.d.}$$

Propoſ. XI. Probl. I.

Datam rectam AB ita ſecare in C, ut rectangulum ſub tota AB, & parte BC contentum, ſit æquale quadrato alterius partis AC.

Conſtructio.



SUper data *AB* [*per 46. pri.*] deſcribatur q. AI, cujus latus AG, bifariam dividatur in E. Notetur E*B*. Et producta EA verſus D fiat ED = E*B*. Ex *AB* majore quàm AD abſcindatur AC = AD,

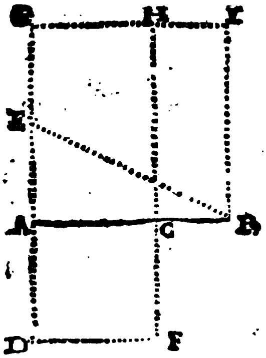
Dico AB × *BC* = q. AC.

Per C notetur CH parallela AG, vel *B*I, & ex D ducatur DF parallela *AB*, quæ occurrat ipſi HC in F.

Demonſtratio.

QVia [*per def. quad.*] IB = *BA* rectangulum CI erit contentum ſub data *BA*, & parte BC. Pariter quia [*per conſtruct.*] AC = AD rectangulum AF erit quad. alterius partis AC:

Modo eſt demonſtrandum rectangulum CI = q. AF
Recta AG [*per hyp.*] eſt bifariam diviſa in E, & illi in rectum addita AD: quare [*per 6. hujus.*



GD × DA + q. AE = q. ED: hoc eſt q. EB; ſed [*per 47. pri.*

q. EB = q. EA } q. AB } : ergo

GD × DA + q. EA = q. EA / q. AB

Dempto communi q. EA: erit
GD × DA = q. AB: ſed
GD × DA eſt Parallelogramum GF, & q. AB; eſt AI: ergo
Communi parte ablata, nempe AH: erit rectangulo. CI hoc eſt *AB* × *BC* = rectangulum AF, quod ex dictis eſt q. *A*C: quare data recta *AB* eſt in C diviſa, quemadmodum erat dividenda.

SCHOLIUM.

Hoc problema numeris accomodari nequit, quia [uti demonſtratum eſt a P. Clauio ad prop. 14. lib. 9.) numerus non poteſt taliter in duos dividi, ut productus ex toto, & una parte adæquet quadratum alterius partis.

Propoſ. XII. Theor. XI.

In triangulo obtuſangulo BCA, cum angulo obtuſo C.

Dico quadratum lateris BA ſubtenſi angulo obtuſo C excedere quadrata laterum BC, CA comprehendentium angulum obtuſum C, rectangulo bis contento ſub BC, CD. Eſt vero CD pars lineæ BC productæ uſque ad perpendicularem AD.

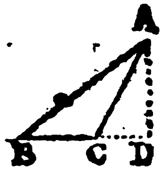
De-

Demonſtratio.

QVia *BD* eſt vtcumque diviſa in C [*per 4. hujus*] erunt

$$\left.\begin{array}{l} q.\ BC \\ q.\ CD \\ BC \times CD\ bis \end{array}\right\} = q.\ BD.$$ addito communi q. *AD*, erunt

$$\left.\begin{array}{l} q.\ BC \\ q.\ CD \\ BC \times CD\ bis \\ q.\ AD \end{array}\right\} = \left.\begin{array}{l} q.\ PD \\ q.\ AD \end{array}\right\}$$ [ſed *per 47. pri.*]



$$\left.\begin{array}{l} q.\ CD \\ q.\ AD \end{array}\right\} = q.\ CA:$$ pariter $$\left.\begin{array}{l} q.\ BD \\ q.\ AD \end{array}\right\} = q.\ BA:$$ ergo

$$\left.\begin{array}{l} q.\ EC \\ q.\ CA \\ EC \times CD\ bis \end{array}\right\} = q.\ BA:$$ quare

q. *BA* ſuperabit $\left.\begin{array}{l} q.\ BC \\ q.\ CA \end{array}\right\}$ *BC* × *CD* bis. q. e. d.

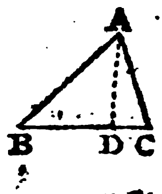
Propoſ. XIII. Theor. XII.

In quocumque triangulo BAC.

Dico quadratum lateris B A ſubtenſi angulo acuto C defficere a quadratis laterum BC, C A comprehendentium angulum acutum C, re-ctangulo bis contento ſub alterutro laterum comprehẽdentium angulum acutum C, in quod cadit perpendicularis AD ex oppoſito angulo A demiſſa, nempe BC; atque portione ejuſdem lateris intercepta inter perpendicularem AD, & angulum acutum C, nempe DC.

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Demonſtratio.



QVia recta BC eſt utcumque diviſa in D, [*per 7. huius.*]

q. *BC*) = *BC* X CD bis
q. *CD*) = q. *BD*

Addito communi q. *AD*, erunt

q. *BC* ⎫ *BC* X *CD* bis ⎫
q. CD ⎬ = q. *BD* ⎬ ſed [*per 47. pri.*
q. *AD* ⎭ q. *AD* ⎭

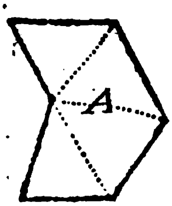
q. *CD*)
q. *AD*) = q. *AC*; & q. *BD*)
q. *AD*) = q. *BA*: ergo

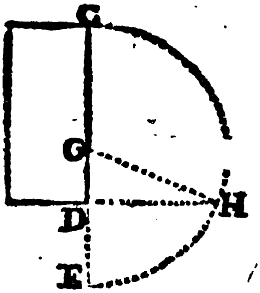
q. *BC*) = q. *BA*):
q. *AC*) = *BC* X *CD* bis) quare

q. *BA* minus erit q. BC)
q. *AC*) BC X CD bis. q. e. d.

Propoſ. XIV. Probl. II.

Dato rectilineo A æquale quadratum conſtituere.





Conſtructio.

REctilineo *A* (*per 45. pri,* in angulo recto cõſtituatur æquale parallellogrammum FC. Producatur CD, fiatque DE = DF. Recta CE [*per 10. pri.*] bifariam diuidatur in G. Centro G intervallo GE, vel GC notetur ſemicirculus CHE: producatur FD uſque ad circumpherentiam in H: notetur GH.

Dico quad. DH = dato rectilineo A.

De-

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Demonſtratio.

CUm recta CE [*per conſtructionem*] ſit bifariam diviſa in G, & non bifariam in D. [*per* 5. *hujus*.]
CD X DE + q. GD = q. GE, ſeu GH: ſed [*per* 47. *pri.*]
q. GH = q. DH + q. DG: ergo
CD X DE + q. GD = q. DH + q. GD,
Dempto communi quad. GD; erit
CD X DE, hoc eſt parallelogrammum FC = q. DH:
Sed (*per conſtructionem.*)
Parallelogrammum FC = dato rectilineo *A*: ergo
rectilineum A = q. DH. q. e. f.

Elemendi Secundi Finis.



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LIBER III.

EUCLIDIS

ELEMENTUM TERTIUM.

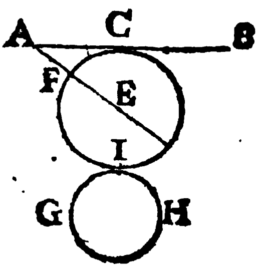
DEFINITIONES.

Def. I.

ÆQUALES CIRCULI dicuntur illi, quorum diametri, vel rectæ lineæ à centris ad circumferentias ductæ, ſunt æquales.

Def. II.

TANGERE circulum dicitur recta linea, quando circulum tangit, & ſi producatur, non ſecat.



SI recta AC tangat circulum in C, atque producta in B circulum non ſecet, appellatur TANGENS, ad differentiam rectæ AE, quæ cum circulum ſecet, dicitur SECANS. Punctum C dicitur punctum CONTACTUS, atque angulus a recta CB, & periferia comprenenſus vocatur ANGULUS CONTACTUS.

Def.

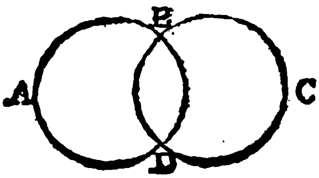
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Def. III.

TANGENTES circuli dicuntur illi, qui ſe invicem tangunt, non autem ſecant.

Tales ſunt Circuli CFI, GIH, qui ſe tangunt in puncto I, quod dicitur CONTACTUS circulorum.

Def. IV.

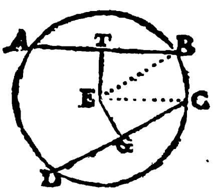


SECANTES circuli dicuntur illi, qui ſe mutuo ſecant.

Tales ſunt circuli ABD, BCD, qui ſe mutuo ſecant in punctis *B*, & D.

Def. V.

Rectæ Lineæ in circulo ductæ dicuntur ÆQUALITER a centro diſtare, cum perpendiculares a centro ad illas rectas ductæ, ſunt æquales. LONGIUS autem a centro diſtare dicitur illa recta, in quam major perpendicularis cadit.



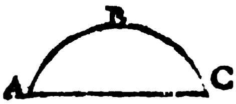
Si in circulo ABCD, cujus centrum E, ductæ fuerint duæ rectæ AB, C*D*, atque perpendiculares ET, EG in illas a centro E cadentes, ſint æquales; rectæ *AB*, CD dicuntur æqualiter diſtare a centro E: nam dictæ perpendiculares ſunt menſura diſtantiarum a centro circuli.

Ex adverſo ſi perpendicularis ET major ſit quam EG; recta *BA* magis a centro E diſtare dicitur quam recta CD.

Def.

Def. VI.

SEGMENTUM circuli dicitur illa ſigura, quæ a recta linea, & circuli circumferentia comprehenditur.



QVia ſigura ABC comprehenditur a recta AC, atque a circumferentia ABC, SEGMENTUM circuli vocatur; rectaque AC CORDA, ſive SUBTENSA nominatur.

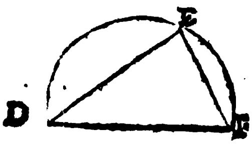
Def. VII.

SEGMENTI angulus dicitur, qui a corda, & circuli circumferentia comprehenditur.

ANGULUS enim mixtilineus comprehenſus a recta, ſiue corda AC, & circumferentia ABC, appellatur angulus SEGMENTI.

Def. VIII.

In SEGMENTO autem vocatur angulus, qui a rectis lineis a puncto in periferia ſignato ad cordæ extremitates ductis comprehenditur.



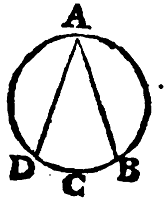
SI in circumferentia ſegmenti DEF, acceptum fuerit punctum E, a quo ad terminos cordæ, ſiuè rectæ DF ductæ ſint rectæ ED, EF: ex his rectis lineis comprehenſus angulus rectilineus DEF, dicitur angulus IN SEGMENTO.

Def.

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Def. IX.

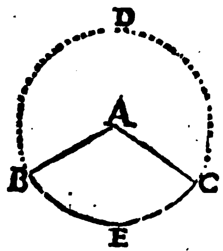
Angulus a duabus rectis lineis ab uno circumferentiæ puncto in circulo ductis comprehenſus INSISTERE dicitur illi periferiæ parti, quam aſſumunt rectæ lineæ angulum comprehendentes.



SI in circumferentia circuli DAB accipiatur punctum A, a quo ductæ ſint rectæ AD, AB comprehendentes angulum *DAB*, & aſſumentes circumferentiam DC*B*: angulus DAB dicitur inſiſtere circumferentiæ DCB tamquam baſi ipſius anguli *DAB*.

Def. X.

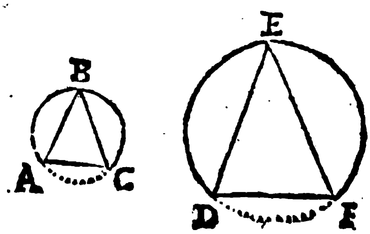
SECTOR circuli vocatur figura terminata a duabus rectis lineis in circuli centro angulum efformantibus, & a periferia ab illis lineis aſſumpta.



TAlis eſt figura *BA*C terminata a rectis AB, AC, & ab arcu *BEC*. Sector etiam dicitur figura BDCA ab ijſdem rectis *A*B, *A*C, & a periferia BDC terminata, cum hoc ſolum diſcrimine, quod ille dicitur minor ſector, hic verò major.

Def. XI.

SIMILIA circulorum SEGMENTA vocantur, illa, in quibus anguli ſunt æquales,



SI circulorum ſegmenta A*B*C, DEF angulos capiant æquales B, E; talia ſegmenta SIMILIA appellantur.

Pro-

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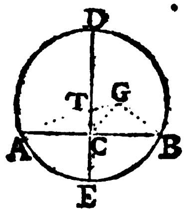
Propoſ. I. Probl. I.

Dati circuli ADBE centrum invenire.

Conſtructio, & Demonſtratio.

IN dato circulo utcumque ducatur recta *AB*, quæ (*per* 10. *pri.*) bifariam dividatur in C. *A* puncto C (*per* 11. *pri.*) erigatur perpendicularis *CD* a circuli periferia terminata in D, & E.

Dico in recta DE eſſe dati circuli centrum.



Si hoc centrum extra rectam DE eſſe poteſt, ponatur in G, notenturque rectæ *GA*, GC, *GB*.

Hoc ſuppoſito, quia in triangulis GC*A*, GCB (*per conſtructionem*) latera GC, C*A* ſunt ſingula ſingulis æqualia lateribus *GC*, C*B*, atque baſis *GA* (*per hyp.*) æqualis baſi *GB*: erit (*per* 8. *pri.*) ang. *GCA* = ang. *GCB*: quare (*per def.* 10. *pri.*) dicti anguli erunt recti. Cum autem (*per conſtructionem*) & angulus *ACD* ſit rectus; ſequitur (*per axio.* 1.) ang. *ACD* = ang. *ACG*; partem toti: quod repugnat: ergo punctum *G* nequit eſſe dati circuli centrum. Idem de quocumque alio puncto extra rectam DE accepto dici poteſt; quare in recta DE erit centrum quæſitum.

Hoc demonſtrato, ſi recta DE bifariam fuerit diviſa in T, hoc punctum erit centrum quæſitum, quia (*per deſ. circuli.*) omnes rectæ a centro ad periferiam ductæ, ſunt æquales. q. e. f.

Corollarium.

SI ex duabus rectis lineis in circulo ſignatis vna alteram ſecet bifariam, & ad angulos rectos; in ſecante erit circuli centrum.

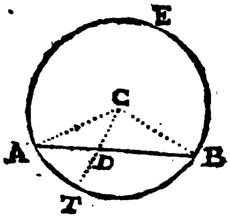
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Propoſ. II. Theor. I.

SI in periferia circuli ABE duo quęlibet puncta A, & B fuerint accepta,

Dico, rectam AB talia puncta coniungentem, intra circulum cadere.



Conſtructio.

DAti circuli AEB (*per* 1. *hujus.*) inveniatur centrum C. In recta AB accipiatur quodvis punctum D, ducanturque rectæ CA, CB, CDT.

Demonſtratio.

REctæ CA, CB (*per def. circuli.*) ſunt æquales: ergo (*per* 5.*pri.*) ang. CAB ≐ ang. CBA: ſed ang. CDB [*per* 16.*pri.*] major ang. CAB: quare (*per axio.* 1.) idem ang. CDB maior ang. CBA: quare [*per* 19.*pri.*] latus CB majus latere CD: quia vero hæc duo latera diſcedunt a centro C, majuſque latus CB habet ſuum terminum B in circuli circumferentia; punctum D, quod eſt terminus minoris lateris CD, erit intra circumferentiam: hæc demonſtratio quibuſcumque punctis intermediis rectæ AB applicari poteſt: unde recta coniungens puncta A, B, intra circulum cadit. q. e. d.

Propoſ. III. Theor II.

In circulo ACBD, ſi recta AB tranſiens per centrum T, bifariam ſecet rectam CD non per centrum tranſeuntem in E.

Dico primo. AB ſecare ad angulos rectos CD.
Dico ſecundo. Si AB ſecat ad angulos rectos

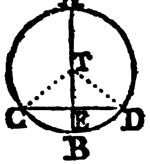
CD

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CD, ſecare etiam bifariam in E.

Conſtructio.

Ex centro T ducantur duæ rectæ TC, TD.



Demonſtratio primæ partis.

Triangula CET, DET, [*per hyp.*] habent latera CE, ET ſingula ſingulis æqualia lateribus DE, ET, baſiſque CT [*per def. circuli.*] eſt æqualis baſi DT: quare [*per 8. pri.*] ang. CET = ang. DET: unde dicti ang. æquales [*per def. 10. pri.*] erunt recti. q: e. 1. d.

Demonſtratio ſecundæ partis.

(P*er def. circuli.*) TC = TD: quare (*per 5. pri.*) ang. TCE = TDE: ergo in triangulis TEC, TED erunt anguli TEC, TCE ſinguli ſingulis æquales angulis TED, TDE, latuſque TC = lateri TD: igitur (*per 26. pri.*) CE = ED: q. e. 2. d.

Corollarium.

Ex hac propoſitione colligi poteſt, quod ſi in triangulo iſoſcele ab angulo baſi opoſito ad ipſam baſim perpendicularis ducta fuerit, hæc baſim bifariam ſecabit; & ſi bifariam ſecet erit perpendicularis.

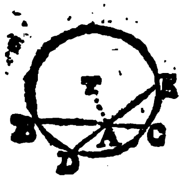
Propoſ. IV. Theor. III.

Si in circulo BDCE duæ rectæ BC, DE non per centrum tranſeuntes ſe mutuo ſecent in A. Dico has rectas ambas non ſe mutuo bifariam ſecare.

Con-

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Conſtructio,



DAti circuli BDCE [*per* 1. *hujus.*] inveniatur centrum T, a quo ad A recta ducatur TA.

Demonſtratio.

SI BC eſt bifariam ſecta in A (*per* 3. *hujus.*) ang. TAC rectus erit. Pariter ſi DE & ipſa eſt in duas æquales partes diviſa in A (*per eamdem* 3.) ang. TAE erit rectus: quare [*per axio.* 12. ang. TAC = ang. TAE; quod fieri nequit, cum ſemper totum ſit ſua parte majus: ergo rectæ BC, DE non ſe mutuo bifariam ſecant in A. q. e. d.

Propoſ. V. Theor. IV.

Si duo circuli ADE, ABC ſe mutuo ſecent.

Dico eos non habere idem centrum.

Conſtructio.



VNius circuli, nempe ADE [*per* 1. *hujus.*) inveniatur centrum T.

In circumferentia ADE accipiatur punctum E, quod ſit intra circulum ABC: notentur à centro T duæ rectæ TA, TEC.

Demonſtratio.

CUm punctum T [*per conſtructionem*] ſit centrum circuli ADE (*per def. circuli*.) erit TA = TE.

Pariter, ſi idem punctum foret centrum alterius circuli ABC [*per eamdem def.*] eſſet TA = TC: quare [*per axio.* 1.] TE = TC, quod eſſe nequit, quia pars ſemper eſt minor ſuo toto: ergo punctum T nequit eſſe centrum commune circulis ſe ſe ſecantibus. q. e. d.

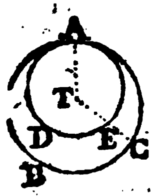
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Propoſ. VI. Theor. V.

Si duo circuli ABC, ADE ſe interius tangant in A,

Dico eos non habere idem centrum.

Constructio,



Circuli ADE, vel ABC (*per 1. hujus.*) inveniatur centrum T, a quo ad contactum A notetur recta TA: pariter ex T notetur recta TEC, quæ utramque circumferentiam ſecet in E, & C.

Demonſtratio.

Quia (*per conſtruc.*) punctum T eſt centrum circuli ADE; erit TA = TE. Pariter ſi punctum T eſt centrum circuli ABC; erit TA = TC: quare (*per axio. 1.*) TE = TC, quod repugnat, cum TE ſit pars, & TC totum: ergo T nequit eſſe centrum commune circulis ſeſe intus tangentibus. q. e. d.

Propoſ. VII. Theor. VI.

Si in circulo BCDT, cujus centrum G accipiatur punctum A, quod non ſit centrum, & ab illo puncto ad circumferentiam ducantur quæcumque rectæ lineæ AB, AC, AD, AE, AT. &c.

Dico primo. Harum linearum maximam eſſe AB tranſeuntem per centrum G.

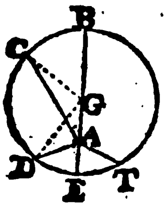
Dico ſecundo. AE reſiduum diametri BA, eſſe minimam.

Dico tertio. Ex aliis quibuſcumque rectis AC, AD,

AD, rectam AC, quæ eſt vicinior maximæ AB, eſſe majorem AD, quæ ab eadẽ AB eſt remotior.

Dico quarto. Duas tantum ex A ad diverſas partes duci poſſe rectas AD, AT inter ſe æquales.

Conſtructio.



EX centro G ad circumferentiam ducantur rectæ GC, GD.

Demonſtratio primæ partis.

IN triangulo AGC (*per 20. pri.*) latera AG, GC ſimul ſumpta ſunt majora latere AC: ſed (*per axio. 2.*) AG, GC ſimul ſumpta adæquant AB: ergo AB major quam AC. q. e. 1. d.

Demonſtratio ſecundæ partis.

IN triangulo GAD (*per 20. pri.*) latera GA, AD majora latere GD: ſed (*per def. circuli.*) GD = GE: ergo GA, AD majora quam GE; & ablata cõmuni quantitate GA [*per axio. 3.*] reſiduum AD majus reſiduo AE. q. e. 2. d.

Demonſtratio tertiæ partis.

DUo triangula AGC, AGD habent latera AG, GC ſingula ſingulis æqualia lateribus AG, GD angulus vero AGC (*per axio. 9.*] major eſt ang. AGD: quare (*per 24. pri.*) baſis AC major baſe AD. q. e. 3. d.

Demonſtratio quartæ partis.

PEr demonſtrata in tertia parte ex puncto A in ſemicirculo BCDE nulla recta duci poteſt æqualis ipſi GD (idem valet in ſemicirculo BTE, in quo alia recta ipſi AT æqualis duci nequit) quamobrem ſi AD eſt æqualis AT nulla alia ipſi AT æqualis erit. q. e. 4. d.

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Propoſ. VIII. Theor. VII.

Si extra circulum EDBH acceptum fuerit punctum A, a quo in cavam dati circuli periferiam cadant quæcumque rectæ lineæ AB, AC, AD, AL, in convexam vero cadant AI, AF, AE, AH.

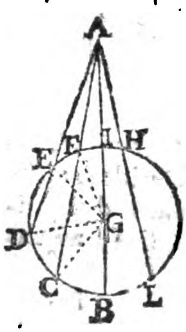
1. Dico rectarum intra circulum in cavam periferiam cadentium maximam eſſe AB per centrum G tranſeuntem.

2. Dico rectam AC maximæ AB viciniorem, majorem eſſe quam *AD* à maxima *AB* remotiorem.

3. Dico rectarum extra circulum in convexam periferiam cadentium minimam eſſe *AI*, quæ producta per centrum G tranſit.

4. Dico rectam *AF* minimæ AI viciniorem eſſe minorem quam *AE* a minima *AI* remotiorē.

5. Dico ex puncto *A*, ſive in cavam intra circulum, ſive in convexam extra circulum, duas tantum rectas æquales ad diverſas partes maximæ, vel minimæ duci poſſe.



Conſtructio.

A Centro G ducantur rectæ GC, GD, GE, GF.

Demonſtratio primæ partis.

IN triangulo *A*GC (*per 20. pri.*) latera AG, GC ſunt majora latere AC; ſed (*per axio.*

axio. 2.] AG, GC æquant AG, GB, hoc eſt AB: ergo AB major quam AC. q. e. 1. d.

Demonſtratio ſecundæ partis.

IN triangulis AGC, AGD latera AG, GC ſingula ſingulis æqualia ſunt lateribus AG, GD, angulus vero AGC major ang. AGD: quare (*per* 24. *pri.*] baſis AC, maximæ vicinior, major baſe AD remotiore. q. e. 2. d.

Demonſtratio tertiæ partis.

IN triangulo AFG [*per* 20. *pri.*] latera AF, FG majora latere AG: ablatis æqualibus GF, GI [*per axio.* 3.) erit reſiduum AI minus reſiduo AF. q. e 3. d.

Demonſtratio quartæ partis.

QVia intra triangulum AEG ab extremitatibus lateris AG ad punctum F ductæ ſunt duæ rectæ AF, GF: (*per* 21. *pri.*) AF, FG erunt minores rectis AE, EG: ablatis æqualibus GF, GE; erit AF minor AE. q. e. 4. d.

Demonſtratio quintæ partis.

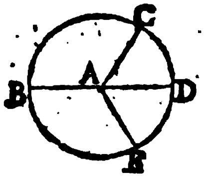
IUxta demonſtrata in ſecunda parte, omnes rectæ a puncto A intra, vel extra circulum ad partem ſemicirculi IDB [idem intelligendum de ſemicirculo IHB) omnes rectæ AC, AD, AF &c. ſunt inæquales: quare ſi ipſi AC ex alia parte ſit æqualis AL, hæc ſola dari poterit æqualis AC, cum reliquæ omnes majores ſint, vel minores; quam AL: quare AC ad aliam partem ſolam habere poterit æqualem AL, & AF; AH. q. e. 5. d.

Propoſ. IX. Theor. VIII.

Si a puncto A intra circulum BCDE accepto in circumferentiam cadant plures, quam duæ

rectæ lineæ *AC*, *AD*, *AE*, æquales.

Dico punctum *A* eſſe centrum circuli CDE.

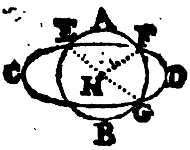


Demonſtratio.

SI punctum A non eſt centrum dati circuli CDE, a puncto A extra centrum accepto dari poterunt tres rectæ AC, AD, AE æquales: quod cum ſit contrarium demonſtratis in 4. parte prop. 7. hujus, erit A centrum circuli BCDE. q. e. d.

Propoſ. X. Theor. IX.

Circuli *AB*, *CD* in duobus tantum punctis ſe mutuò ſecant.



Conſtructio.

SUppoſito quod circuli AB, CD ſe interſecent in tribus punctis, E, F, G (*per 1. hujus*) invento centro circuli AB, quod ſit H, notentur rectæ HE, HF, HG.

Demonſtratio.

QVia punctun H (*per conſtructionem*) eſt centrum circuli *AB* (*per def. circuli.*) rectæ HE, HF, HG æquales erunt: cum autem iſtæ tres rectæ æquales cadant in circumferentiam alterius circuli CD (*per 9. hujus*) punctum *A* eſſet centrum commune circulis ſe mutuo ſecantibus *AB*, *CD*; quod cum ſit contra 5. hujus; ſequitur circulum non poſſe alium circulum ſecare in pluribus punctis quam duobus. q. e. d.

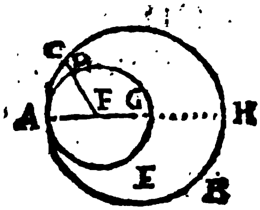
Propoſ.

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Propoſ. XI. Theor. X.

Si duo circuli *ACB*, *ADE* ſe intus contingant in *A*,
Dico rectam per talium circulorum centra F, & G ductam tranſire per contactum *A*.

Conſtructio, & Demonſtratio.

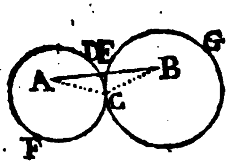


SI recta per centra G, & F ducta, atque producta verſus F non cadit in contactum *A*, ſi fieri poteſt, cadat extra, ſecetque ambas circumferentias in D, & C. Hoc ſuppoſito quia punctum F non eſt centrum circuli *A*CB (*per 2. partem 7. hujus.*) erit FC minor quam F*A*; ſed (*per def. circuli.*) FD = F*A*: quare FC minor quam FD, totum ſcilicet ſua parte minus; quod (*per axio. 9.*) eſt impoſſibile: ergo GF producta verſus F nequit cadere extra punctum A, & ideo tranſibit per punctum contactus *A*. q. e. d.

Propoſ. XII. Theor. XI.

Si duo circuli ECG, DCF ſe exterius tangant in C,
Dico rectam horum circulorum centra A, & B coniungentem tranſire per contactum C.

Conſtructio, & Demonſtratio.



SI recta AB non tranſit per contactum C, ſecabit utramque circumferentiam in punctis D, & E. A centris circulorum A, & B ad contactum C notentur rectæ AC, BC.

His ſtantibus, quia punctũ A (*per hip.*) eſt centrũ circuli FCD, erit AC = AD; pariterque in circulo CEG, erit BC = BE: quare

tota AB ſuperabit rectas AC, & BC: ergò in triangulo ACB latus AB majus erit lateribus AC, BC, (*contra demonſtrata in 20. pri.*): unde AB tranſibit per contactum C. q. e. d.

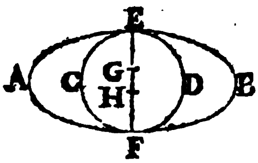
Propoſ. XIII. Theor. XII.

Si Circulus AB tangat circulum CD.

Dico eum non tangere in pluribus punctis, quam uno, ſiuè intus, ſiuè extra tangat.

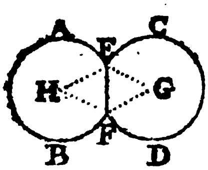
Conſtructio, & Demonſtratio.

Fig. I.



SI circuli AB, CD, (*Fig. 1.*) intus ſe tangerent in duobus punctis E, & F, quia (*per 6. bujus.*) circuli ſe intus tangentes non habent idem centrum, ſit H centrum circuli AB, & G centrum circuli CD: hoc poſito recta HG coniungens circulorum centra (*per 11. bujus*) ſi producatur, tranſibit per puncta contactus E, & F: eritque EF recta linea. Cum verò punctum G ſupponatur centrum circuli CD, & H cētrum circuli AB; erit GE = GF, & HE = HF: quare recta EF erit bifariam diviſa in punctis G, & H; quod (*per axio. 25.*) eſt impoſſibile: unde circulus CD nequit intus tangere circulum AB in pluribus punctis quam uno. q. e. d.

Fig. II.



Si poſtea dicti circuli AB, CD [*Fig. 2.*] ſe extra tangant in tota circumferentia EF (*per 11. bujus.*] lineæ centra H, & G coniungentes, atque per contactus E, & F tranſeuntes erunt lineæ rectæ ſpatium claudentes; quod cum ſit impoſſibile (*per axio. 14.*) verum erit dicere circulum AB exterius non tangere circulum CD in pluribus punctis, quam uno. q. e. d.

SCO-

SCHOLIUM.

EX *præcedentibus propoſitionibus ſatis conſtat quemcumque circulorum contactum eſſe in puncto, ſiuè intus, ſiuè extra ſe tangant.*

Propoſ. XIV. Theor. XIII.

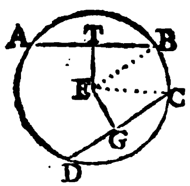
Si in circclo ABCD æquales rectæ lineæ AB, DC conſtitutæ ſint.

1. Dico illas æqualiter diſtare a centro E.

Si æqualiter a centro E diſtent,

2. Dico eſſe æquales.

Conſtructio.



A Centro E ad rectas AB, DC [*per* 12. *pri.*] perpendiculares ducantur ET, EG: pariter ab eodem centro E notentur rectæ EB, EC.

Demonſtratio primæ partis.

QVia rectæ ET, EG [*per conſtructionem*] veniunt a centro E, ſuntque ipſis AB, CD perpendiculares (*per* 2. *hujus*.) eaſdem bifariam diuident in T, & in G: cum autem [*per ſuppoſitum.*] AB ſit = DC., & erit medietas TB = medietati GC.

In triangulis rectangulis ETB, EGC (*per* 47. *pri.*)

$$\left.\begin{matrix} q.ET \\ q.TB \end{matrix}\right\} = q.EB, \& \left.\begin{matrix} q.EG \\ q.GC \end{matrix}\right\} = q.EC;$$

(*per eſſentiam circuli.*)

q. EB = q. EC: quare

$$\left.\begin{matrix} q.ET \\ q.TB \end{matrix}\right\} = \left.\begin{matrix} q.EG \\ q.GC \end{matrix}\right\} \text{Ablatis æqualib. quadratis} \left.\begin{matrix} TB \\ GC \end{matrix}\right\}$$

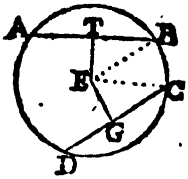
remanebit q. ET = q. EG,

ideoque recta ET = rectæ EG: igitur.

(per

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(*per def. 4. hujus.*]
rectæ AB, DC a centro E æqualiter diſtabunt. q. e. d.



Demonſtratio ſecundæ partis.

Jiſdem manentibus, ſi diſtantia ET = ſit diſtantiæ EG,
q. ET = q. EG
Ablatis his æqualibus quadratis
q. ET) & a q. EG)
q. TB) q. GC) remanebit q. TB = q. GC
unde, & recta TB = rectæ GC: ſed
rectæ TB, GC ſunt medietates rectarum AB, DC: ergo
AB = DC. q. e. d.

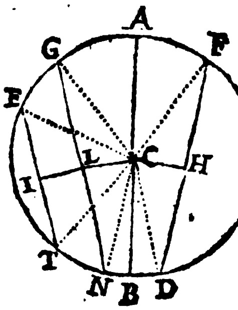
Propoſ. XV. Theor. XIV.

In circulo AEB, cujus centrum C ſi ducantur rectæ lineæ.

1. Dico maximam illarum, eſſe diametrum AB.

2. Dico aliarum illam, quæ eſt proprior centro C, eſſe remotiore majorem.

Conſtructio.



Per centrum C ducatur diameter ACB: pariter ſignentur rectæ FD, ET illa vicinior, & hæc remotior a centro C: ulterius notentur a centro C duæ rectæ CH, CI perpendiculares rectis FD, ET, quarum perpendicularium CI major erit CH, quia recta ET ſupponitur magis a centro C diſtare quam FD. Ex CI [*per 3. pri.*] auferatur CL = CH: per L ducatur GLN paral-

parallela ET; addanturque rectæ CF, CG. CE, CT, CN, CD.

Demonſtratio primæ partis.

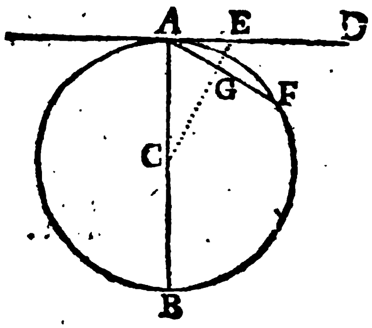
IN triangulo FCD [*per* 20. *pri.*] latera FC, CD majora ſunt latere FD: cum autem FC, & CD (*per def. circuli.*) adæquent *AB*; erit AB major quam FD. Eodem modo demonſtrabitur AB major quacumque alia linea extra centrum C in circulo ducta: quare diameter AB erit maxima omnium linearum, quæ in circulo ducuntur. q. e. 1. d.

Demonſtratio ſecundæ partis.

QVia (*per conſtructionem*) perpendicularis CH = perpendiculari CL; (*per* 14. *hujus.*) erit FD = GN. Pariter quia triangula GCN, ECT [*per def. circuli.*] habent latera GC, CN ſingula ſingulis æqualia lateribus EC, CT; angulum vero GCN majorem angulo ECT: (*per* 24. *pr.*) erit GN major quam ET: quia vero (*per conſtructionem*) perpendicularis CH = CL (*per* 14. *hujus*) eſt GN = FD: ergo FD propior centro C remotiore ET major erit. q. e. 2. d.

Propoſ. XVI. Theor. XV.

Si ab extremitate diametri AB eidem diametro perpendicularis ducatur AD.

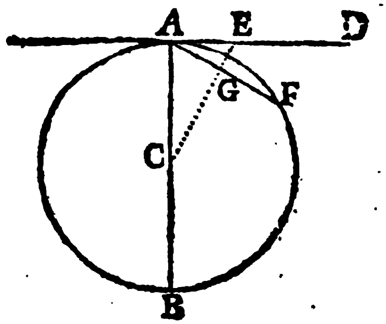


1. Dico AD totam cadere extra circulum AFB.

2. Dico intra rectam AD, & circumferentiam AF aliam rectam lineam non cadere.

3. Dico angulum ſe-

mi-



micirculi BAF majorem eſſe quocumque angulo acuto rectilineo: contactus vero angulum DAF quocumque angulo acuto rectilineo eſſe minorem.

Conſtructio, & Demonſtratio primæ partis.

IN recta AD accepto quovis puncto E notetur recta CE, eritque efformatum triangulum CAE, in quo, cum (*per hyp.*) angulus CAE ſit rectus, angulus CEA (*per 17. pri.*) erit acutus: quare (*per 18. pri.*) latus CE majus latere CA; ſed CA terminum habet in circumferentia circuli: ergo terminus rectæ CE erit extra circulum: hoc idem demonſtrari poterit de quovis alio puncto lineæ AD: unde erit verum dicere, rectam AD totam eſſe extra circulum. q. e. 1. d.

Demonſtratio ſecundæ partis.

SI intra rectam AD, & circuli peripheriam AF alia recta duci poteſt, quæ perveniat ad punctū A abſque eo quod circulum ſecet, ſit hæc recta AF, quæ cum diametro AB conſtituet angulum acutum CAF. Ex puncto C ad AF perpendicularis ducatur CG, quæ [*per corol. 17. pri.*] cadet ad partes anguli acuti CAG: quo ſtante triangulum CGA habebit angulum CGA rectum, angulum vero CAG acutum: quare [*per 18. pri.*] latus CA majus latere CG: ſed CA habet ſuum terminum A in circumferentia AF: ergo CG habebit ſuum terminum G intra circumferentiam AFB: ideoque recta AF non cadet inter circumferentiam AF, & rectam AD quare intra rectam AD, & circumferentiam AFB ad punctum A alia recta non poteſt duci, quæ circulum non ſecet. q. e. 2. d.

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Demonſtratio tertiæ partis.

CUm per demonſtrata in ſecunda parte intra rectam AB, & rectam AD omnes, quæ cadunt rectæ, circulum ſecent, una ex his rectis ſit AF: hoc poſito angulus ſemicirculi comprehenſus a diametro *AB*; & a circumferentia *AFB* erit totum, ejuſque pars erit angulus rectilineus C*AF*: quare angulus ſemicirculi major erit quocumque angulo acuto rectilineo.

Pariter cum angulus contactus comprehenſus a recta *AD*, & circumferentia *AF*, ſit pars anguli rectilinei *DAF*: ſequitur, angulum contactus ſemper exiſtere minorem quocumque angulo acuto rectilineo. q. e. 3. d.

Corollarium primum.

EX prima parte hujus theorematis colligitur, rectam lineam in ſolo puncto circulum tangere.

Corollarium ſecundum.

PAriter fit manifeſtum, rectam lineam extremitati diametri cujuſuis circuli perpendicularem, eſſe circuli tangentem, & angulum a tangente AD, & circuli periferia comprehenſum eſſe angulum CONTACTUS, de quo plura ſcripſerunt Chriſtophorus, Clavius, Peletarius, Tacquet, aliique in hoc loco, quamvis hæc ad geometrica elementa vix reduci poſſint, nam Geometria utitur ſolis angulis efformatis a lineis rectis, vel a circulorum arcubus, cum iſti ab illis menſurentur, ut ex ſphæricis elementis fit manifeſtum; quod ſanè non verificatur de angulis contactus, vel de illis angulis, qui juxta methodum ſyntheticam a curvis lineis in eodem plano exiſtētibus efformantur.

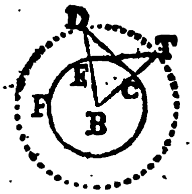
Propoſ. XVII. Prob. II.

A puncto D extra circulum dato rectam lineam ducere, quæ datum circulum CEF

tangat.

tangat:

Conſtructio.



CEntrum dati circuli [*per* 1. *bujus*.) inventum ſit B. A puncto B ad *D* notetur recta BD, quæ circulum datum CEF ſecabit in E. Centro B intervallo BD deſcribatur circulus DT, & ex E (*per* 11. *pri.*) perpendicularis ſignetur ET. ducatur BT, quæ ſecet circulum CEF in C: notentur DC.

Dico DC tangere datum circulum CEF.

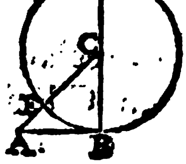
Demonſtratio.

IN triangulis BCD, BET latera EB, BT [*per def. circuli.*) ſunt ſingula ſingulis æqualia lateribus CB, B*D*, & angulus B communis utrique triangulo, quare [*per* 4. *pri.*] ang. BET = ang. BCD; ſed ang. BET [*per conſtructionem*] eſt rectus: quare & ang. BCD rectus erit: ergo [*per corol.* 2.16. *bujus*) DC erit tangens circuli CEF. q. e. f.

Propoſ. XVIII. Theor. XVI.

Si recta AB tangat circulum BDT in B. Dico rectam BT per contactum B, & centrum C ductam, tangenti AB eſſe perpendicularem.

Conſtructio, & Demonſtratio.



SI non concedatur CB tangenti AB eſſe perpendicularem, ex centro C [*per* 12. *pri.*) ducatur CEA perpendicularis tangenti AB.

His manentibus (*per conſtructionem*.] angulus CAB erit rectus, & angulus CBA (*per* 17. *pri.*)

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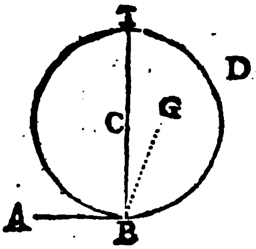
pri.) acutus: quare (*per* 18. *pri.*) latus CB majus latere CA, at CB (*per def. circuli.*) = CE: ergo CE majus CA; quod cum ſit contra [*axio. 9.*]; CA non erit ipſi AB perpendicularis: idem abſurdum demonſtrabitur de quacumque alia linea ex C ad AB ducta, ſola CB excepta; unde CB erit tangenti AB perpendicularis. q. e. d.

Propoſ. XIX. *Theor.* XVII.

Si recta AB circulum BDT tangat in B, & a puncto contactus excitata ſit BT perpendicularis tangenti AB.

Dico in BT eſſe centrum Circuli BDT.

Conſtructio, & Demonſtratio.



SI propoſiti circuli BDT centrum non concedatur in recta BT, erit extra conſtituendum, ut in G: a puncto G ad B ducatur recta GB, quæ (*per* 18. *hujs.*) erit tangenti AB perpendicularis: quare ang. GBA erit rectus; ſed (*per hyp.*) eſt etiam rectus ang. TBA: ergo (*per axio.* 12.) ang. GBA = ang. TBA; pars æqualis toto, quod fieri nequit: unde in BT erit centrum circuli BDT. q. e. d.

Propoſ. XX. *Theor.* XVIII.

Si in circulo DAB, cujus centrum C ſuper eadem baſi AB ad centrum conſtitutus ſit angulus ACB, & ad periferiam ADB.

Dico angulum ad centrum ACB duplum eſſe anguli ad periferiam ADB.

In

In hac theoremate hæc contingere poſſunt.

1. Quod latera AD, DB comprehendentia angulum ad circumferentiam intra ſe includant latera efformantia angulum ad centrum, ut in fig. 1.

2. Quod unum latus anguli ad periferiam, ut AD tranſeat per circuli centrum C, coincidatque cum latere AC, quod pertinet ad angulum ad centrum, ut in fig. 2.

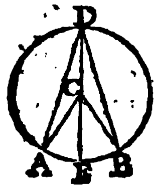
3. Quod unum ex lateribus conſtituentibus angulum D ad circumferentiam ſecet unum ex lateribus pertinentibus ad angulum ad centrum, ut in fig. 3.

Conſtructio.

PEr puncta D, & C recta ducatur DCE.

Demonſtratio primæ partis.

Fig. 1.



TRiangulum ACD Fig. 1. [*per def. circuli.*] habet latus CA = CD: quare [*per 5. pri.*] ang. CDA = ang. CAD; ſed angulus externus ACE (*per 32. pri.*) eſt æqualis ang. CDA } CAD } : ergo angulus ACE duplus ang. CDA. Eadem prorſus ratione in triangulo BCD demonſtrabitur ang. BCE duplus ang. CDB: quo ſtante angulus ad centrum ACB (*per axio. 2.*) duplus erit anguli ad periferiam ADB. q. e. 1. d.

Demonſtratio ſecundæ partis.

Fig. 2.

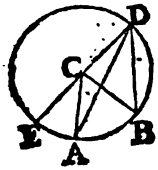


IN triangulo BCD (*Fig. 2.*) latera CB, CD (*per def. circuli*) ſunt æqualia; unde [*per 5. pri.*) ang. CBD = ang. CDB, ſed ang. ad centrum ACB (*per 32. pri.*) æqualis angulis CDB } CBD } : ergo angulus ad centrum ACB duplus anguli ad peripheriam ADB. q. e. 2. d.

De-

Demonſtratio tertiæ partis.

Fig. 3.



NOn ſolum ang. ad centrum ECB [*Fig.* 3.] per demonſtrata in ſecunda parte, eſt duplus anguli ad circumferentiam EDB, verum etiam angulus ECA duplus eſt anguli EDA: quare totus ang. ECB duplus totius ang. EDB, & ablatus ECA duplus ablati EDA: quamobrem [*per axio.* 20.] reſiduum ACB duplum reſidui ADB. q. e. 3. d.

Propoſ. XXI. Theor. XIX.

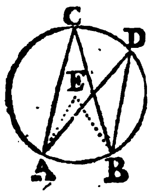
Si in eodem circuli ſegmento ACDB ſint quicumque anguli nempe ACB, ADB &c. Dico tales angulos eſſe æquales.

In hac propoſitione tria pariter contingere poſſunt.

1. Quod anguli ſint in ſegmento ſemicirculo majore.
2. Quod ſint in ſemicircu o.
3. Quod ſint in ſegmento ſemicirculo minore.

Conſtructio.

Quando anguli ſunt in ſegmento ſemicirculo majore.



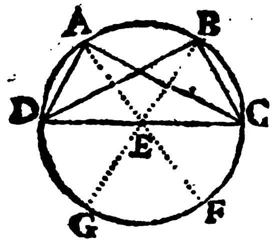
EX E circuli centro ducantur rectæ EA, EB.

Demonſtratio.

ANgulus ad centrum AEB [*per* 20. *hujus*] duplus ang. ACB, & ang. ADB: quare (*per axio.* 7. anguli C, & D, & ſi qui fuerint alij in ſegmento ACDB, erunt æquales. q. e. 1. d.

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Conſtructio
Quado anguli ſunt in ſemicirculo.

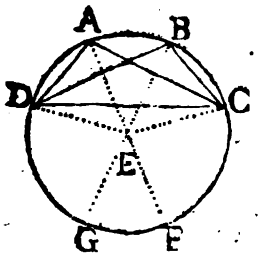


SI in ſemicirculo DABC, cujus centrum E, ſint anguli DAC, DBC ex punctis A,& B per centrum E ducantur duæ rectæ AEF, BEG.

Demonſtratio.

ANgulus *DEF* [*per 2. part. 20. hujus.*) duplus eſt anguli DAF, & angulus FEC duplus anguli FAC: quare angulus DAC erit medietas trium angulorum DEG, GEF, FEC: Pariter ab eandem rationem angulus CEG duplus anguli CBG, angulus GED duplus anguli GBD: ergo angulus CBD erit medietas trium angulorum DEG, GEF, FEC: quare [*per axio. 7.*] ang. DAC = ang. DBC, q. e. 2. d.

Conſtructio
Pro angulis in minore ſegmento exiſtentibus.



SI anguli DAC, DBC ſint in ſegmento ſemicirculo minore, ex punctis A, & B per centrum E notentur rectæ AEF, BEG, atque ex centro E ducantur rectæ EC, ED.

Demonſtratio.

ANgulus DAF [*per demonſtrata in 2. parte 20. hujus.*) eſt medietas anguli DEF, & angulus FAC eſt medietas anguli FEC: quare totus angulus DAC erit medietas trium angulorum DEG, GEF, FEC: pariter ob eandem rationem angulus DBG eſt medietas anguli DEG, atque angulus GBC medietas anguli GEC: ergo totus angulus DBG, & ipſe erit medietas trium angulorum DEG, GEF, FEC: quamobrem (*per axio. 7.*) angulus DAC = angulo DBC. q. e. 3. d.

Pro-

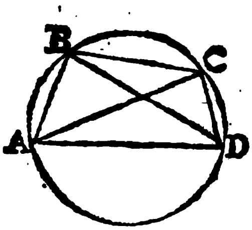
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Propoſ. XXII. Theor. XX.

Si in circulo ABCD inſcriptum ſit quadrilaterum ABCD.

Dico hujus quadrilateri oppoſitos angulos B, & D (idem valet de angulis A, & C), eſſe æquales duobus rectis.

Conſtructio.



AB angulis B, & C, ad oppoſitos D, & A ducantur rectæ BD, CA.

Demonſtratio.

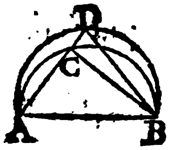
ANgulus ABD [*per 21. hujus.*] æqualis angulo ACD: pariter [*per eamdem prop.* 21.] ang. DAC = ang. DBC: ergo totus angulus ABC = duobus ang. $\left.\begin{matrix}ACD\\CAD\end{matrix}\right\}$: addito communi ang. ADC, erunt ang. $\left.\begin{matrix}ABC\\ADC\end{matrix}\right\}$ = tribus ang. $\left.\begin{matrix}ACD\\DAC\\ADC\end{matrix}\right\}$; ſed iſti tres anguli (*per* 32. *pri.*) ſunt duobus rectis æquales: ergo, & duo anguli oppoſiti in quadrilatero $\left.\begin{matrix}ABC\\ADC\end{matrix}\right\}$ duobus rectis æquales erunt. q. e. d.

Propoſ. XXIII. Theor. XXI.

Si ſuper eadem recta linea AB ad eaſdem partes conſtituta ſint duo circulorum ſegmenta ACB, ADB inæqualia.

Dico non eſſe ſimilia.

Conſtructio.



SI ſuper recta *AB* ad eaſdem partes conſtituti poſſunt duo circulorum ſegmenta inter ſe ſimilia, & inæqualia, ſint ſegmentum *ACB*, & ſegmentum *ADB*. Ducatur recta *ACD*, quæ utramque ſegmentorum circumferentiam ſecet in C, & D, addanturque rectæ CB, D*B*.

Demonſtratio.

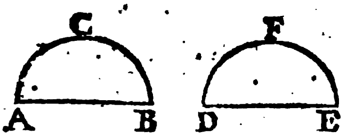
SI ſegmenta *ACB*, *ADB* ſunt ſimilia [*per def.* 10. *huius.*] ang. *ACB* æqualis ang. *ADB*, quod [*per* 16. *pri.*] fieri nequit: quamobrem ſegmenta *ACB*, *ADB* non erunt ſimilia, & inæqualia. q. e. d.

Propoſ. XXIV. Theor. XXII.

Super æqualibus rectis lineis AB, DE ſi ſimilia fuerint conſtituta circulorum ſegmenta ACB, DFE.

Dico eſſe æqualia.

Demonſtratio.



CUm rectæ *AB*, *DE* ſupponantur æquales, ſi *AB* fuerit ſuperpoſita ipſi DE (*per 1. partem axio.* 8.) congruet; manente hac congruentia, ſi ſegmentum *ACB* fuerit ſuperpoſitum ſegmento DFE, vel cadit extra, vel intra, vel erit partim intra, & partim extra ſegmentum DFE: ſed neque intra, neque extra cadere poteſt, quia ſuper eadem recta linea duo circulorum ſegmenta ſimilia, & inæqualia ad eaſdem partes conſtituerentur. (*contra demonſtrata in prop.* 23. *huius*) neque pro parte extra, & pro parte intra cadere poteſt, quia illi circuli, quorum hæc ſunt ſegmenta, in pluribus punctis

quam

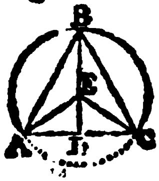
quam duobus ſe ſecarent, [*contra* 10. *hujus.*]: quare data ſegmenta erunt congruentia, ideoque [*per axio.* 8.] æqualia. q.e.d.

Propoſ. XXV. Probl. III.

Dato circuli ſegmento ABC, illum circulum, cujus eſt ſegmentum complere.

Conſtructio.

Fig. 1.



DAti ſegmenti corda, ſeu recta *AC* [*per* 10. *pri.*] biſariam diuidatur in D: ex D [*per* 11. *pri.*) notetur perpendicularis DB: ab *A* ad B. ducatur *AB*.

Ad punctum *A*, & ad rectam *AB* (*per* 23. *pri.*) conſtituatur angulus B*AE* = ang. *ABE*, ſecetque *AE* rectam BD in E; debet enim ſecare, quia in triangulo rectangulo *ADB* [*Fig.* 1.] angulus *EBA* (*per* 17. *pri.*] eſt acutus; cum autem illi factus ſit æqualis ang. *BAE*, ſequitur angulos $\frac{ABE}{BAE}$ eſſe duobus rectis minores, quare [*per axio.* 13.] *AE* ſecabit BD.

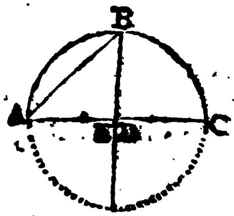
Dico punctum E eſſe centrum illius circuli, cujus ſegmentum eſt *AbC*. *A*ddatur EC.

Demonſtratio.

IN triangulo AEB (*per conſtructionem*] ang. B A E = ang. ABE: ergo [*per* 6. *pri.*] AE = EB: Rurſus in triangulis ADE, CDE duo latera AD, DE ſingula ſingulis æqualia duobus lateribus CD, DE, atque anguli ADE, CDE æquales, quia recti; ſequitur (*per* 4. *pri.*) AE = CE: ergo (*per axio.* 1.) rectæ EB, EA, EC ſunt æquales: quare [*per* 9. *hujus.*] punctum E erit centrum dati ſegmenti ABC: ſi ergo contro E interuallo EA, vel EC deſcribatur circulus ABC, hic erit circulus dati ſegmenti ABC. q. e. f.

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Fig. 2.



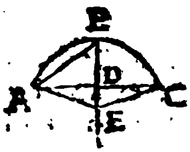
SCHOLIUM.

Quando angulus D B A (Fig. 1.) minor eſt DAB, circuli centrum E eſt intra ſegmentum, quia ſegmentum majus ſemicirculo.

Si angulus DBA (Fig. 2.) ſit æqualis angulo DAB, centrum eſt in recta AC, quia ſegmentum eſt ſemicirculus.

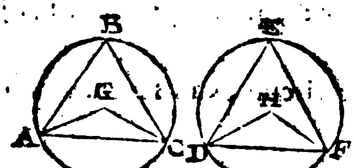
Demum ſi angulus D B A major ſit angulo DAB (Fig. 3.) circuli deſcribendi centrum E, erit extra ſegmentum: quia ſegmentum minus eſt ſemicirculo.

Fig. 3.



Propoſ. XXVI. Theor. XXIII.

Si in circulis æqualibus ABC, DEF, quorum centra G, & H ſint ad centra æquales anguli AGC, DHF, & ad periferias ſit ang. B = ang. FE. Dico tales angulos inſiſtere æqualibus periferiis AC, DF.



Demonſtratio.

Anguli B, & E ad periferias (*per hyp.*) ſunt æquales: quare (*per def. 10. hujus.*) ſegmenta ABC, DEF erunt ſimilia. Rurſus (*per hyp.*) non ſolum latera GA, GC = lateribus HD, HF, verum etiam ang. G = ang. H: ergo [*per 4. pri.*] AC = DF.

His ſtantibus circulorum ſegmenta ABC, DEF erunt ſuper æquales rectas AC, DF conſtituta: quare (*per 24. hujus.*) dicta ſegmenta ABC, DEF erunt æqualia: quia vero (*per hyp.*) æquales ſunt circuli ABC, DEF, ſi a circulis auferatur dicta ſegmenta (*per axio. 3.*) erit ſegmentum AC = ſegmento DF, & conſequenter periferia AC = periferiæ DF. q. e. d.

Pro-

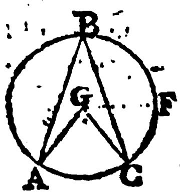
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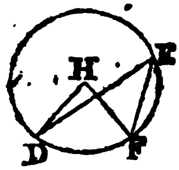
Propoſ. XXVII. Theor. XXIV.

Si in circulis æqualibus ABC, DEF, quorum centra G, & H, anguli ad centra AGC, DHF, vel ad periferias ABC, DEF inſiſtant æqualibus periferiis *AC*, *DF*.

Dico ang. G = ang. H, & ang. *B* = ang. E.

Conſtructio, & Demonſtratio.





SI angulus DHF non eſt æqualis ang. AGC ſit major: ad punctum G, & ad rectam AG [*per* 23. *pri.*] conſtituatur angulus AGF = ang. DHF: quare (*per* 26. *hujus.*) periferia DF = periferiæ ACF: ſed periferia AC ſupponitur æqualis periferiæ DF: ergo periferia AC = peripheriæ ACF: pars æqualis toti: quod repugnat: quare ang. DHF = ang. AGC: q. e. d.

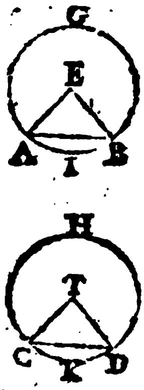
Eodem modo demonſtrabitur æqualitas angulorum ad peripherias E, & B, ſuppoſita æqualitate circumferentiarum DF, AC, quibus inſiſtunt; quia demonſtrata æqualitate angulorum ad centra, ſequitur æqualitas angulorum ad circumferentias, cum illi [*per* 20. *hujus.*] ſint iſtorum dupli. q. e. d.

Propoſ. XXVIII. Theor. XXV.

Si in circulis æqualibus *AGB*, C*HD* ſit recta *AB* = rectæ *CD*.

Dico majorem periferiam *AGB* = maiori peripheriæ C*HD*, minoremque *AIB* = minori *CKD*.

Conſtructio.



EX centris E, T, rectæ ducantur EA, EB, TC, TD.

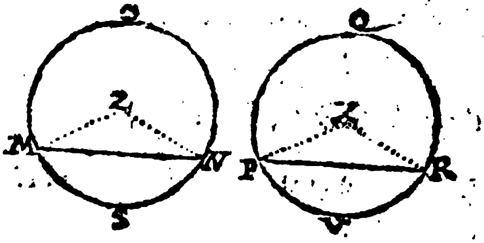
Demonſtratio.

Circuli AGBI, CHDK (*per hyp.*) ſunt æquales: ergo (*per def. 1. hujus.*) rectæ EA, EB = rectis TC, TD: cum autem, & baſis AB ſupponatur æqualis baſi CD; (*per 8. pri.*) ang. E = ang. T: quare (*per 26. hujus*) peripheria AIB = peripheriæ CKD; his ab æqualibus circulis ablatis, erit etiam periferia AGB = peripheriæ CHD. q. e. d.

Propoſ. XXIX. Theor. XXVI.

Si in circulis æqualibus MON, PQR ſint æquales arcus MSN, PVR.

Dico arcuum æqualium ſubtenſas MN, PR eſſe æquales.



Conſtructio.

EX centris Z, & X notentur rectæ ZM, ZN, XP, XR.

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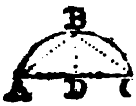
Demonſtratio.

OB ſuppoſitam æqualitatem circulorum MON, PQR (*per def. pri. hujus.*) rectæ ZM, ZN ſingulæ ſingulis erunt æquales rectis XP, XR. Pariter angulus Z (*per 27. hujus.*) = ang. X: quare in triangulis MZN, PXR (*per 4. pri.*) MN = PR. q. e. d.

Propoſ. XXX. Prob. IV.

Datam circuli circumferentiam *ABC* biſariam diuedere.

Conſtructio.



DUcatur corda dati arcus, quæ ſit AC, quæ (*per 10. pri.*) biſariam diuidatur in D: ex D ipſi AC perpendicularis excitetur DB.

Dico datum arcum ABC eſſe biſariam diviſum in B.

Conſtructio.

Ducantur duæ rectæ BA, BC.

Demonſtratio.

DVo triangula ADB, CDB habent latera AD, DB ſingula ſingulis æqualia lateribus CD, DB, & angulos ADB, CDB æquales, quia rectos: ergo (*per 4. pri.*) baſis AB = baſi CD: quare (*per 28. hujus.*) arcus AB = arcui BC. q. e f.

Propoſ. XXXI. Theor. XXVII.

In circulo *ABDE*.

1. Dico in ſemicirculo angulum *ADE* eſſe rectum.

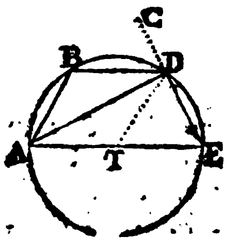
2. Di-

2. Dico in majore ſegmento DEA angulum *DEA* eſſe acutum.

3. In minore ſegmento ABD anguſum ABD eſſe obtuſum.

4. Dico Majoris ſegmenti angulum ADE eſſe recto majorem, minoris vero ſegmenti angulum *ADB* recto minorem exiſtere.

Conſtructio.



DAti circuli ABDE (*per 1. hujus.*) inveniatur centrum T. Per centrum ducatur recta ATE. In ſemicirculo ABDE conſtituatur angulus ADE. Pariter in minore ſegmento ABD ponatur angulus ABD. Notetur DT, producaturque ED in C.

Demonſtratio primæ partis.

REcta TD (*per def. circuli.*) æqualis TE: quare (*per 5. pri.*) angulus TDE = ang. TED; eadem ratione ang. TD*A* = ang. T*A*D: ergo totus ang. *A*DE = ang. $\left.\begin{matrix}TED\\ TAD\end{matrix}\right\}$; ſed (*per 32. pri.*) angulus externus *A*DC = ang. $\left.\begin{matrix}TED\\ TAD\end{matrix}\right)$: igitur [*per axio. 1.*] ang. *A*DC = ang. *ADE*: quare (*per 10. def. 1.*) ang. *ADE* erit rectus, q. e. 1. d.

Demonſtratio ſecundæ partis.

IN triangulo *AED* angulus *ADE* [*per demonſtrata in prima parte*) eſt rectus: igitur *per 17. pri.* angulus *AED* in majore ſegmento conſtitutus, erit acutus q. e. 2. d.

De-

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Demonſtratio tertiæ partis.

IN quadrilatero ABDE in circulo deſcripto (*per 22. hujus.*) anguli oppoſiti B, & E ſunt æquales duobus rectis; ſed angulus E in ſecunda parte demonſtratus eſt acutus: ergo angulus B in ſegmento minore erit obtuſus. q. e. 3. d.

Demonſtratio quartæ partis.

MAjoris ſegmenti angulus comprehenſus a corda *AD*, & a circumferentia DE eſt totum reſpectivè ad angulum rectilineum ADE: ſed angulus rectilineus *ADE* [*per primam partem hujus propoſitionis.*) eſt rectus: ergo *ADE* majoris ſegmenti angulus erit major recto.

Quia vero minoris ſegmenti angulus comprehenſus a corda *AD*, & a periferia *BD* eſt pars anguli recti *ADC*: erit *ADB* minoris ſegmenti angulus, minor recto. q. e. 4. d.

Corollarium.

EX hac propoſitione fit manifeſtum, quod in triangulo, dum unus angulus eſt reliquis duobus angulis æqualis, ille eſt angulus rectus.

Propoſ. XXXII. Theor. XXVIII.

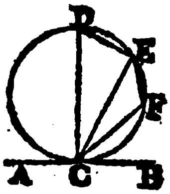
Si recta AB circulum tangat in C, & CE ex contactu ducta circulum ſecet.

Dico angulos ACE. BCE a tangente, atque ſecante efformatos æquales eſſe illis angulis, qui in alternis circuli ſegmentis CFE, CDE conſiſtunt.

In hac propoſitione poteſt contingere, quod ſecans tranſeat per circuli centrum, vel extra.

Con-

Conſtructio.



QVando ſecans ex puncto contactus C ducta tranſit per circuli centrum, ut CD; ad punctum E ducantur duæ rectæ DE, CE.

Demonſtratio hujus partis.

SI (*ut ſupponitur*) CD per centrum tranſit; ſequitur (*per* 31. *hujus*) angulum CED eſſe rectum: ſed (*per* 18. *hujus*) recti ſunt, & anguli DCB, DCA: ergo anguli facti a tangente ACB, & a ſecante CD æquales erunt angulis factis in alternis circuli ſegmentis, cum omnes ſint recti: quare ang. DCA = ang. DEC conſtituto in alterno ſegmento DEC. q. e. 1. d.

Conſtructio.

QVando ſecans CE tranſit extra centrum in circumferentia ſegmenti CFE accipiatur punctum F, ad quod ducantur recta EF, CF.

Demonſtratio hujus partis.

IN triangulo CED angulus E (*per* 31. *hujus.*) eſt rectus: quare (*per* 32. *pri.*) ejuſdem trianguli CED reliqui anguli $\left.\begin{matrix}\text{EDC}\\ \text{ECD}\end{matrix}\right\}$ erunt æquales uni recto: cum autem (*per* 18. *hujus.*) rectus etiam ſit angulus DCB: erunt ang. $\left.\begin{matrix}\text{EDC}\\ \text{ECD}\end{matrix}\right\}$ = ang. DCB: ablato communi ang. DCE; erit ang. EDC, in alterno ſegmento conſtitutus, æqualis angulo BCE a tangente BC, & a ſecante CE efformatus.

Pariter cum in quadrilatero DEFC in circulo deſcripto [*per* 22. *hujus.*] anguli oppoſiti D, & F ſint duobus rectis æquales, pariterque anguli ECB, ECA (*per* 13. *pri.*) ſint, & ipſi æquales duobus rectis: erunt ang. $\left.\begin{matrix}\text{D}\\ \text{F}\end{matrix}\right\}$ = ang. $\left.\begin{matrix}\text{ECB}\\ \text{ECA}\end{matrix}\right\}$

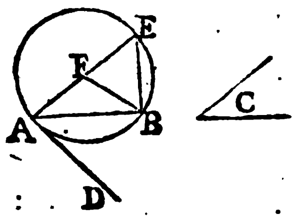
Sed

Sed (*per demonſtrata in prima parte.*)
ang. D = ang. ECB: hoc ablato
erit angulus ECA comprehenſus a tangente AC, & a ſecante
CE æqualis angulo F in alterno ſegmento conſtituto. q. e. 2. d.

Propoſ. XXXIII. Probl. V.

Super data Recta linea AB circuli ſegmentum deſcribere, quod capiat angulum æqualem dato angulo C.

Conſtructio.



AD punctum A, & ad rectam AB [*per* 23. *pri.*] conſtituatur angulus DAB = ang. dato C. Ex A (*per.* 11. *pri.*) ducatur AE ipſi AD perpendicularis, quæ cum AB efformabit ang. EAB.

Ad punctum B, & ad rectam AB efformetur angulus ABF = ang. BAE, producaturque BF quousque ſecet AE in F.

Demum centro F intervallo FA, vel FB; ſunt enim (*per* 6. *pri.*) latera æqualia notetur ſegmentum circuli AEB.

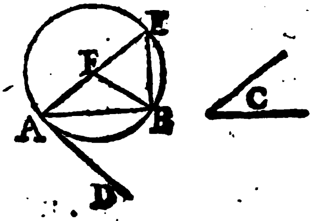
Dico hoc eſſe illud ſegmentum, quod capit angulum æqualem dato angulo C D, ducatur BE.

Demonſtratio.

ANgulus BAD (*per conſtructionem*) æqualis angulo dato C: ſed ang. BAD (*per* 32. *hujus*) æqualis ang. E: quare angulus E = ang. C; cum autem angulus E ſit in ſegmento AEB; hoc erit illud ſegmentum, quod capit angulum æqualem dato angulo C. q. e. f.

Propoſ.

Propoſ. XXXIV. Probl. VI.



Ex dato circulo ABE abſcindere ſegmentum, quod capiat angulum æqualem dato angulo C.

Conſtructio.

Vtcumque [*per 17. hujus.*] ducatur recta DA, quæ datum circulum ABE tangat in *A*: ad punctum contactus A, & ad tangentem *DA* [*per 23. pri.*] conſtituatur angulus *DAB* æqualis dato angulo C.

Dico ſegmentum *AEB* eſſe illud, quod capit angulum æqualem dato angulo C.

Per centrum F, & contactum *A* notetur *AFE*; addaturque *BE*.

Demonſtratio.

Angulus datus C (*per conſtructionem.*) æqualis angulo *DAB*: pariter (*per 32. hujus*) angulus E in ſegmento *AEB* poſitus, æqualis angulo *DAB*: ergo angulus *E* = dato ang. *C*: quare ſegmentum circuli *AEB* capit angulum æqualem angulo dato C. q. e. f.

Propoſ. XXXV. *Theor.* XXIX.

Si in circulo duæ rectæ AB, CD ſe utcumque ſecent in E.

Dico CE x ED = AE x EB.

Circa hanc linearum ſectionem hæc contingere poſſunt.

1. Quod ambæ lineæ per centrum tranſeant.

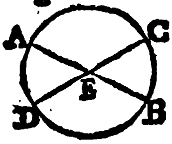
2. Quod una tantum tranſeat per centrum, aliamque non per centrum tranſeuntem bipharian ſecet.

3. Quod per centrum tranſiens aliam non per centrum tranſeuntem

untem non bipharíam dividat.

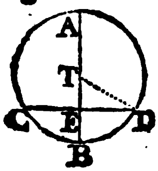
4 Quod neutra illarum linearum per centrum ducatur.

Fig.1.



Quoad primam hujus propoſitionis partem hoc per ſe patet; quia æqualium ſegmentorum rectangula *AE* × *EB*, *DE* × *EC* (*Fig.* 1.) ſunt æqualia quadrata, ob æqualitatem linearum *AE*, *DE* &c.

Fig.2.



2. Si recta *AB* (*Fig.*2.) tranſiens per centrum T ſecet CD extra centrum ductam bipharíam in E.

Dico *AE* × *EB* = *CE*; × *ED*.

Conſtructio.

Ducatur TD.

Demonſtratio.

AB eſt bipharíam diviſa in T, & non bipharíam in E: quare (*per* 5. 2.)

AE × *EB* + quad.ET = quad TB, ſeu TD:

(Sed [*per* 47. *pri.*]

quad.TD = quad.ED + quad.ET: ergo

AE × *EB* + quad. *ET* = quad. *ED* + quad.*ET*.

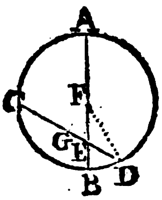
Dempto communi quad. *ET* remanebit

AE × *EB* = quad.ED.

Quia verò CD eſt bipharíam diviſa in E.

quad.ED erit CE × ED: unde ſequitur

AE × *EB* = *CE* × *ED*. q. e. 2. d.



3. Si recta *AB* tranſiens per centrum F ſecet rectam CD extra centrum ductam non bifariam in E.

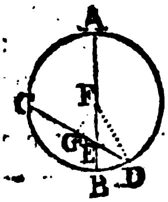
Dico *AE* × *EB* = *CE* × *ED*.

Conſtructio.

A Centro F rectæ *CD* [*per* 12. *pri.*] perpendicularis ducatur FG, addaturque FD.

De-

Demonſtratio.



Recta *AB* eſt bifariam diviſa in F, & non bifariam in *E*:

quare *per* 5. *ſecundi.*)

$AE \times EB +$ quad. $FE =$ quad. FB, hoc eſt q. FD.

Sed (*per* 47. *primi.*

q. $FE =$ q. $FG +$ q. GE, atq; q. $FD =$ q. $FG +$ q. GD:

ergo

$AE \times EB +$ q. $FG +$ q. $GE =$ q. $FG +$ q. GD.

Dempto communi q. FG: erit

$AE \times EB +$ q. $GE =$ q. GD.

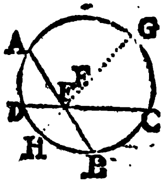
Pariter quia recta CD (*per* 3. *hujus.*) eſt bifariam diviſa in G, & non biphariam in E.

(*per* 5. *ſecundi.*) erit

$CE \times ED +$ q. $GE =$ q. GD: ergo

$AE \times EB +$ q. $GE = CE \times ED +$ q. GE.

Ablato communi q. GE: erit



$AE \times EB = CE \times ED$: hoc eſt rectangulum partium *AE*, *EB* æquale rectangulo facto a partibus *CE*, *ED*, q. e. 3. d.

4. Tandem ſi neutra rectarum *AB*, *DC* per centrum tranſeat, ſe autem ſecent in *E*.

Dico $AE \times EB = DE \times EC$.

Conſtructio.

Per circuli centrum F, & per interſecationis punctum *E* notetur recta GH.

Demonſtratio.

Per demonſtrata in tertia parte hujus theorematis

$AE \times EB = GE \times EH$: atque $DE \times EC = GE \times EH$.

Quare

$AE \times EB = DE \times EC$: hoc eſt rectangulum a ſegmentis *AE*, *EB* factum æquale rectangulo a ſegmentis *DE*, *EC* comprehenſo. q. e. 4. d.

Propoſ.

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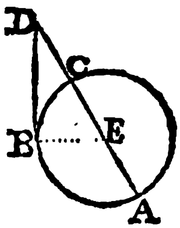
Propoſ. XXXVI. Theor. XXX.

Si extra circulum accipiatur punctum D, a quo ducatur recta DA, quæ circulum ſecet atque recta BD, quæ circulum tangat.

Dico $AD \times DC = q.\ DB$.

Primo. Secans *DA* poteſt tranſire per circuli centrum.
Secundo. Poteſt tranſire extra centrum *E*.

Conſtructio, quando ſecans DA tranſit per centrum E.



A centro E ad contactum B ducatur recta EB.

Demonſtratio.

Recta CA eſt bifariam diviſa in E, & illi in rectum addita CD,

Quare [*per 6. ſecundi.*)

$AD \times DC + q.\ EC$, hoc eſt $q.\ EB = q.\ ED$.

Sed (*per 47. pri.*)

$q.\ ED = q.\ EB + q.\ DB$: ergo

$AD \times DC + q.\ EB = q.\ EB + q.\ DB$.

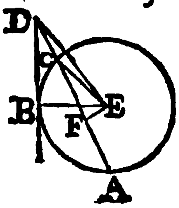
Dempto communi q. EB erit

$AD \times DC = q.\ DB$.

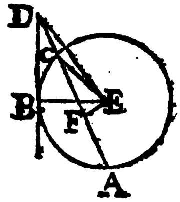
Quare rectangulum factum a ſecante DA, & DC eſt æquale quadrato tangentis DB. q. e. p. d.

Si ſecans DA extra centrum E ſit ducta.

Conſtructio.



Recta CA (*per 10. pri.*) bifariam dividatur in F: ex centro E ad F recta ſignetur EF. Pariter notentur rectæ EB, EC, ED.



Demonſtratio.

QVia a puncto contactus B per centrum E ducta eſt BE (*per 18. hujus,*) angulus EBD eſt rectus. Pariter quia CA eſt bifariam diviſa in F (*per 3. hujus.*) angulus EFD rectus eſt.

His manentibus recta CA [*per conſtructionem*] eſt bifariam diviſa in F & illi in rectum addita CD; Quare (*per 6. ſecundi.*)

$AD \times DC + q.FC = q.FD.$

Addito communi q. FE, erit

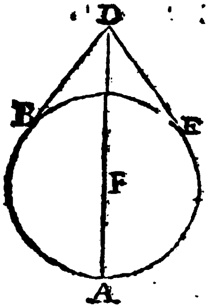
$AD \times DC + \left.\begin{matrix} q.FC \\ q.FE \end{matrix}\right\} = q.FD + q.FE:$

Sed (*per 47. pri.*)

$\left.\begin{matrix} q.FC \\ q.FE \end{matrix}\right\} = q.EC$, ſeu EB: & $\begin{matrix} q.FD \\ q.FE \end{matrix} = q.ED$: quare $AD \times DC + q.EB = q.ED$: ſed (*per 47. pri.*)

$q.ED = \left.\begin{matrix} q.EB \\ q.DB \end{matrix}\right\}$ ergo $AD \times DC + q.EB = \left.\begin{matrix} q.EB \\ q.DB \end{matrix}\right\}$.

Dempto communi q. EB: erit $AD \times DC = q.DB$. q. e. d.



Corollarium.

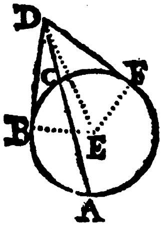
SI ab eodem puncto *D* duæ rectæ *DB*, *DE* circulum tangant: dico illas eſſe æquales: quia tam quadratum *DB*, quam quadratum *D*E [*per hanc prop.*) æquant $AD \times DC$: quare quad. DB = quad. DE; ideoque $DB = DE$.

Pro-

Propoſ. XXXVII. Theor. XXXI.

Extra circulum accepto puncto D, ab eoque ducta ſit ſecans DA, & incidens DB; ſi rectangulum DA, DC æquale ſit qnadrato DB.

Dico DB circulum tangere.



Conſtructio,

A Puncto *D* (*per* 17. *bujus.*) ducatur *DF* tangens circulum in F.

Ex centro E notentur rectæ EB, *ED*, *EF*.

Demonſtratio.

CUm a puncto D extra circulum accepto ducta ſit ſecans DA, & tangens DF.

(*per* 36. *bujus.*] erit

AD X DC = quad. DF.

Sed (*per hyp.*)

AD X DC = quad. DB: ergo

[*per axio.* 1.)

quad DB = quad. DF: quare DF = DB.

Quo ſtante in triangulis DF*E*, DB*E* erunt duo latera DF, FE ſingula ſingulis æqualia lateribus DB, B*E*, baſiſque D*E* communis utrique triangulo: quare (*per* 8. *pri.*] ang. D F *E* = ang. *DBE*: ſed angulus DF*E* [*per* 18. *bujus.*] eſt rectus: ergo, & angulus DB*E* rectus erit: igitur [*per corol. prop.* 16. *bujus.*] incidens DB erit tangens circuli ABF. q. e. d.

Elementi Tertij Finis.

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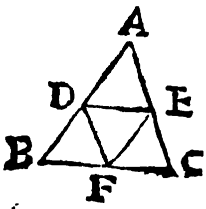
LIBER IV.

EUCLIDIS ELEMENTUM QVARTUM.

DEFINITIONES.

Def. I.

FIgura rectilinea DEF in figura rectilinea ABC INSCRIBI dicitur, cum ſinguli ejus figuræ, quæ inſcribitur, anguli D, E, F, ſingula latera AB, BC, CA ejus, in qua inſcribitur, tangunt.



SI omnes tres anguli figuræ DEF tangant tria latera figuræ ABC, tunc figura DEF dicitur INSCRIPTA figuræ ABC.

Def. II.

Figura ABC circa figuram DEF DESCRIBI dicitur, cum ſingula latera figuræ ABC tangunt angulos D, E, F, illius figuræ, circa quam deſcribitur.

Def. III.

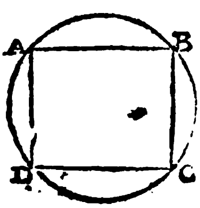


Figura rectilinea ABCD dicitur in circulo ABCD INSCRIPTA, quan-

quando omnes anguli figuræ inſcriptæ tangunt circumferentiam circuli ABCD.

Def. IV.

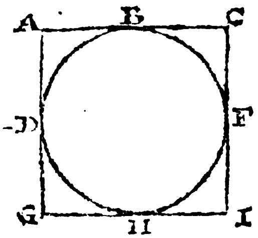
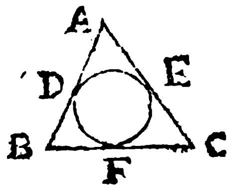


Figura rectilinea ACIG circa circulum BFHD deſcribi dicitur, cum ſingula latera figuræ circumſcriptæ AC, CI, IG, GA tangunt circumferentiam illius circuli BFHD, cui circumſcribitur.

Def. V.

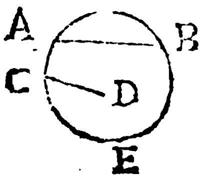


Circulus DEF in figura rectilinea *ABC* inſcribi dicitur, cum circuli periferia DEF tangit omnia latera figuræ *ABC*, cui inſcribitur.

Def. VI.

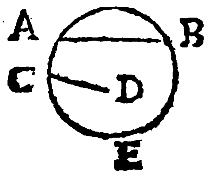
Circulus circa figuram rectilineam deſcribi dicitur, cum circuli periferia tangit omnes angulos illius figuræ, cui circumſcribitur.

Def. VII.



ACCOMODARI, ſeu COAPTARI dicitur in circulo recta linea *AB*, cum ejus extrema, *A*, & *B*

fuerint in circuli periferia.

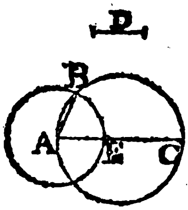


Quia extrema, ſeu termini rectæ AB ſunt in periferia circuli ABE, recta AB dicitur in circulo ABE accomodata non ſic recta CD, cum punctum, ſeu extremum D non ſit in periferia ABC.

Propoſ. I. Probl. I.

In dato circulo *ABC* accomodare rectam æqualem datæ rectæ D, quæ dati circuli diametro non ſit major.

Conſtructio.



DAti circuli A B C ducatur diameter AC: ſi data recta CD fuerit æqualis diametro AC, factum eſt, quod quæritur.

Si data D minor ſit diametro AC (*per 3. pri.*) auferatur AE = D. Centro A, & diſtantia AE notetur circulus BE; ducatur AB.

Dico rectam AB eſſe in circulo accomodatam, & æqualem ipſi D.

Demonſtratio.

REcta *AE*[*per conſtructionem*] æqualis ipſi D: *AB*[*per def.circ.*) æqualis ipſi *AE*: ergo (*per axio.2.*] *AB* = *D*. q. e. f.

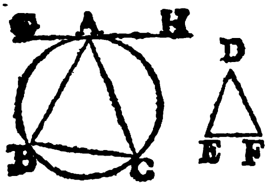
Propoſ. II. Probl. II.

In circulo *ABC* deſcribere triangulum dato triangulo DEF æquiangulum.

DAti circuli BAC[*per 17.tertii.*] ducatur tangens GH, quæ circulum tangat in A. Ad punctum A, & ad rectam GA

(*per*

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[*per* 23. *pri.*] conſtituatur angulus GAB æqualis angulo F; pariter ad idem punctũ A, & ad rectã AH ponatur angulus HAC æqualis angulo *E*, cadetq; recta AC intra rectas AH, & AB, quia duo anguli *E*, & F (*per* 17. *pri.*) ſunt duobus rectis minores; duo vero anguli GAB, HAB [*per* 13. *pri.*) duobus rectis æquales: quare angulus HAC minor angulo HAB; ideoque AC cadet intra AH, & AB. Ducatur BC.

Dico triangulum ABC eſſe æquiangulum triangulo DEF.

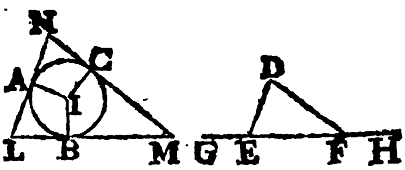
Demonſtratio.

ANgulus GAB (*per conſtructionem.*) æqualis ang. F.
Ang. C [*per* 32. *tertij.*] æqualis ang. GAB: ergo
Ang. C = ang. F.
Pariter angulus HAC (*per conſtruct.*) = ang. E.
Ang. HAC [*per* 32. *tertij.*] = ang. B: ergo
Ang. B = ang. E.
Quare (*per* 32. *pri.*)
Ang. BAC = ang. D:
Ideoque in circulo BAC deſcriptum eſt triangulum BAC æquiangulum dato triangulo DEF. q. e. f.

Propoſ. III. Probl. III.

Circa datum circulum *A*BC deſcribere triangulum dato triangulo DEF æquiangulum.

Conſtructio.



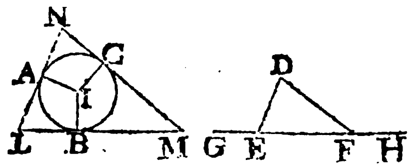
DAti trianguli *EDF* unum latus, ut *EF*, producatur in G, & H.

Ex I centro dati circuli utcumque recta ducatur IA.

Ad punctum I, & ad rectam AI (*per* 23. *pri.*) conſtituatur angulus AI*B* = ang. *DEG*. Pariter ad idem punctum I, & ad eamdem rectam AI, ponatur angulus *A*IC = ang. DFH.

 ctis

In hac conſtructione recta IC ſemper cadit extra rectas *AI*, *BI*. & hoc quia duo anguli AIB, AIC (utpote æquales) duobus angulis DEG, DFH ſunt quatuor rectis minores: cum autem anguli circa punctum I [*per corol. 1. prop. 15. pri.*] ſint quatuor rectis æquales; erit IC extra *AI*, BI.



Per tria puncta *A*, C, *B* (*per 11. pri.*) rectis *AI*, CI, BI ducantur perpendiculares LN, LM, MN, quæ [*per corol. 2. prop. 16. tertii.*] erunt tangentes circuli *ACB*.

Quia vero talium tangentium duæ, uti ſunt AN, CN (idem valet de reliquis) efficiunt cum eadem recta duos angulos duobus rectis minores (nam ſi conceperis rectam ab A ad C ductam, duo anguli CAN, ACN erunt minores duobus rectis); debent concurrere in aliquo puncto N, (*per axio. 13.*) & ſimili ratione debent tres dictæ tangentes concurre etiam in punctis L, & M; erit igitur efformatum triangulum NLM circa datum circulum ACB, quod

Dico dato triangulo EDF æquiangulum.

Demonſtratio.

IN quadrilatero AIBL (*per corol. 5. prop. 32. pri.*) quatuor anguli ſunt quatuor rectis æquales; & quia duo anguli IAL, IBL [*per conſtructio.*) ſunt recti, ideoque duobus rectis æquales: ſequitur duos angulos AIB, ALB exiſtere duobus rectis æquales.

Cum autem [*per 13. pri.*] & duo anguli DEG, DEF ſint duobus rectis æquales: erunt

$$\text{ang.}\left.\begin{matrix}AIB\\ ALB\end{matrix}\right\} = \text{ang.}\left.\begin{matrix}DEG\\ DEF\end{matrix}\right\}$$

Sed (*per conſtructionem*,)

ang. AIB = ang. DEG: ergo ang. ALB = ang. DEF.

Ob eandem rationem ang. N demõſtrabitur æqualis ang. DFE:

Unde (*per 32. pri.*)

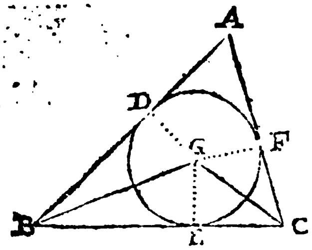
ang. M = ang. *D*: ergo triangulum NLM circa datum circulum ACB deſcriptum, erit æquiangulum dato triangulo *DEF*. q. e. f.

Pro-

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Propoſ. IV. Probl. IV.

In dato triangulo ABC circulum deſcribere.



Conſtructio.

EX tribus angulis dati trianguli, duo, uti ſunt anguli B, & C (*per* 9. *pri.*) biſariam dividantur rectis BG, CG concurrentibus in G.

Dico circulum deſcriptum centro G, diſtantia GD, eſſe deſcriptum in triangulo ABC.

Ex G ad tria latera trianguli ABC (*per* 12. *pri.*) perpendiculares ducantur GD, GE, GF.

Demonſtratio.

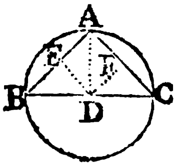
IN triangulis BGE, BGD (*per conſtruct.*) duo anguli GBE, GEB ſunt ſinguli ſingulis æquales angulis GBD, GDB: & latus GB commune: quare (*per* 26. *pri.*) GE = GD.

Ob eamdem rationem in triangulis GEC, GFC, recta GE = GF: ergo tres rectæ GD, GE, GF erunt æquales.

Ideoque centro G diſtantia GD, deſcriptus circulus, tranſibit per F, & E: eruntque GD, GF radii ejuſdem circuli, quibus cum ſint perpdndicularia latera trianguli AB, BC, CA: erunt [*per* 16. *tertij.*] hæc latera circulum tangentia; quare erit dictus circulus deſcriptus in triangulo ABC. q. e. f.

Propoſ. V. Probl. V.

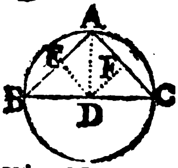
Circa datum triangulum BAC circulum deſcribere.



Conſtructio.

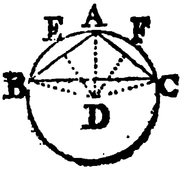
SI datum triangulum ſit rectangulum, vel obtuſangulum, ut in Fig. I. & II, [*per* 10. *pri.*] latera AB, AC comprehendentia angulum re-

Fig. I.



rectũ, vel obtuſum A bifariam dividantur in punctis E, & F: A punctis E, & F [*per 11. pri.*] lateribus AB, AC ſignentur perpendiculares ED, FD, quæ concurrant in D: debent enim concurrere, quia ſi intelligatur recta EF, anguli, DEF, DFE ſunt duobus rectis minores.

Fig. II.



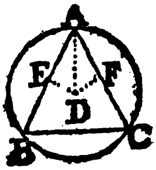
Dico punctum *D* eſſe centrum circuli deſcribendi circa datum triangulum ABC.

In Figura I. ducatur recta DA.

In Figura II. & III. ducantur rectæ DB, DA, DC.

Demonſtratio.

Fig. III.



IN triangulis DEB, DEA (*per conſtructionem.*) latera DE, EB ſingula ſingulis æqualia ſunt lateribus DE, EA: & anguli a dictis lateribus comprehenſi DEB, DEA æquales, quia recti: quare (*per 4. pri.*) DB = DA.

Eodem modo demonſtrabitur CD = DA: ideoque rectæ DB, DA, DC æquales: igitur ſi centro D intervallo DB circulus deſcribatur, tranſibit per puncta B, A, C: unde erit deſcriptus circa datum triangulum BAC. q. e. f.

SCHOLIUM.

1. *QUando datum triangulum BAC* (Fig. I.) *eſt rectangulum, centrum circuli deſcribendi circa datum triangulum, eſt in uno ex lateribus trianguli; quia* (*per 31. tertij*) *anguli in ſemicirculo ſunt recti.*

2. *Quando triangulum eſt obtuſangulum* [Fig. II.] *centrum D eſt extra triangulum, quia angulus obtuſus eſt in ſegmento ſemicirculo minore.*

3. *Quando triangulum eſt acutangulum* (Fig. III.) *centrum D eſt intra triangulum, quia anguli acuti ſemper ſunt in ſegmento ſemicirculo majore.*

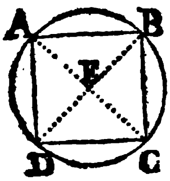
Propoſ.

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Propoſ. VI. Probl. VI.

In dato circulo ABCD quadratum deſcribere.

Conſtructio,



DUcantur duæ diametri AC, DB ſecantes ſe ad angulos rectos in centro E: iungantur rectæ AB, BC, CD, DA.

Dico ABCD eſſe quadratum in dato circulo inſcriptum.

Demonſtratio.

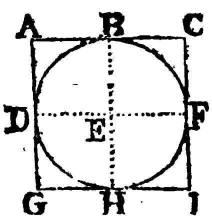
TRiangula AEB, BEC, CED, DEA (*per def. circuli.*) habent duo latera duobus lateribus æqualia, anguloſque AEB, BEC &c. æquales, quia rectos: ergo (*per 4. pri.*) baſes AB, BC, CD, DA erunt æquales.

Rurſus quia anguli DAB, ABC, BCD, CDA ſunt in ſemicirculo [*per 31. tertij.*] ſunt recti, ideoque æquales: quare *per def. 31. pri.*] figura ABCD erit quadratum dato circulo inſcriptum. q. e. f.

Propoſ. VII. Probl. VII.

Circa datum circulum BFHD quadratum deſcribere.

Conſtructio.



IN dato circulo *BFHD* ducantur duæ diametri ſe ſecantes ad rectos angulos BH, DI: ab extremitatibus harum diametrorum ducantur tangentes AC, CI, IG, GA, quæ concurrent in punctis A, C, I, G.

Dico figuram AI a dictis tangentibus comprehenſam eſſe quadratum circa datum datum circulum BFHD deſcriptum. De-

Demonſtratio.

ANguli CFE, ADE [*per conſtructionem.*] ſunt recti: ergo [*per* 28. *pri.*] rectæ AG, CI erunt parallelæ.

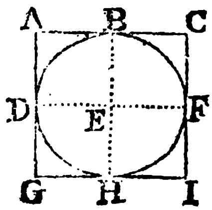
Ob eandem rationem non ſolum AC parallela GI, verum etiam BH parallela AG, & CI, atque DF parallela rectis AC, & GI: quare (*per def.* 40. *pri.*) figura AI erit parallelogrammum, in quo (*per* 34. *pri.*) AC = GI, atque AG = CI: ulterius AC = DF, & AG = BH. Cum autem DF adæquet BH; erit & AC = AG. Quare in parallelogrammo AI omnia latera erunt æqualia.

Ulterius cum in parallelogrammo AE (*per conſtructionem*) angulus BE*D* ſit rectus, etiam angulus A rectus erit: ergo in parallelogrammo AI [*per corol. prop.* 29. *pri.*] anguli erunt recti: quare AI erit quadratum circa datum circulum BFHD deſcriptum. q. e. f.

Propoſ. VIII. Probl. VIII.

In quadrato ACIG circulum deſcribere.

Conſtructio.



DAti quadrati omnia latera AC, CI, IG, GA, ut in antecedenti figura, (*per* 10. *pri.*) biſariam dividantur punctis B, *D*, H, F; ducanturque rectæ BH, DF ſe interſecantes in puncto E. Centro E intervallo EB, ſignetur circulus B*D*HF.

Dico hunc eſſe circulum in dato quadrato inſcriptum.

Demonſtratio.

CUm dati quadrati latera AC, GI ſint æqualia, & parallela, etiam eorum medietates AB, GH erunt æquales, & parallelæ: quare (*per* 33. *pri.*) AG = BH. Eadem ratione, quia

BH

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BH = CI: AC = *DF*, atque DF = GI: talium rectarum æqualium medietates æquales erunt.

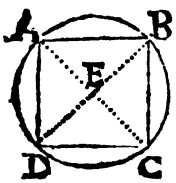
Cum igitur *AB* ſit parallela, & æqualis ipſi *DE* (*per 33. pri.*) *AE* erit parallelogrammum: ob eamdem rationem parallelogramma erunt C*E*, IE, GE: quare (*per 34. pri.*) BE = AD, atque DE = AB; ſed AD = AB: ergo DE = EB.

Eodem modo demonſtrabitur EB = EF, atque *E*F = EH: igitur centro E, atque diſtantia unius æqualium rectarum, ut *EB* deſcriptus circulus tranſibit per puncta D, H, F: unde erit inſcriptus dato quadrato AI. q. e f.

Propoſ. IX. Probl. IX.

Circa datum quadratum ABCD circulum deſcribere.

Conſtructio.



Dati quadrati ABCD ducantur duæ diametri AC, BD ſe ſecantes in E.

Dico circulum ex centro E, & diſtantia EA ſignatum eſſe dato quadrato ABCD circumſcriptum.

Demonſtratio.

In triangulo BAD (*per hyp.*) BA = AD, & angulus *BAD* rectus: quare (*per 5. pri.*) anguli ABD, ADB erunt ſemirecti. Hoc idem demonſtrabitur de reliquis angulis ad puncta B, C, D, conſtitutis, videlicet quod ſint omnes ſemirecti.

Quo ſtante, cum in triangulo AEB, duo anguli EAB, EBA (*per demonſtrata*) ſint æquales, quia ſemirecti: ſequitur (*per 6. pri.*) EA = EB. Hoc idem demonſtrabitur de rectis EB, EC &c.: quare rectæ *EA*, *EB*, *EC*, *ED* erunt æquales.

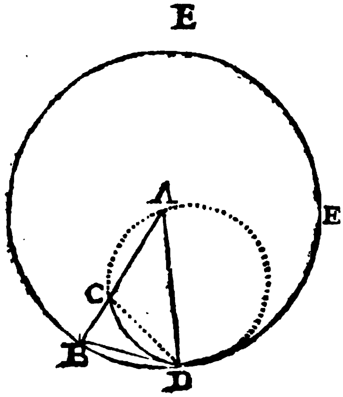
Si ergo centro *E* intervallo *EA* fuerit deſcriptus circulus, per puncta *B*, *C*, *D* tranſibit; unde erit circa datum quadratum deſcriptus. q. e. f.

Propoſ.

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Propoſ. X. Prob. X.

Iſoſceles triangulum efformare, cujus quilibet angulorum ad baſim ſit reliqui anguli duplus.



Conſtructio,

Accipiatur quævis recta AB, quæ (*per* 11. *ſecundi*) taliter dividatur in C, ut $AB \times BC =$ q. CA. Centro A, & diſtantia *AB* notetur circulus *BDE*. In hoc circulo (*per* 1. *huius*. accomodetur recta $BD = CA$: Ducatur AD.

Dico triangulum *DAB* illud eſſe, quod quæritur.

Ducatur *CD*. Circa triangulum *ACD* [*per* 5. *hujus*.] deſcribatur circulus *ACD*.

Demonſtratio.

Recta AB [*per conſtructionem*] eſt taliter diviſa in C, ut $AB \times BC =$ q. CA. *BD* facta eſt æqualis CA: quare
$AB \times BC =$ q. *BD*.

Cum autem punctum *B* ſit extra circulum *ACD*, & recta *BA* ſit ſecans circulum, & BD ſit incidens in periphæriam.
(*per* 37. *tertij*.)
recta *BD* in *D* tanget circulum *ACD*.
ergo (*per* 32. *tertij*.)
angulus $BDC =$ ang. *A*.
addito communi angulo *CDA*, erit
angulus *BDC* + ang. *CDA* = ang. *A* + ang. *CDA*.
hoc eſt ang. *BDA* = ang. *A* + ang. *CDA*:
ſed (*per* 32. *pri*.)
ang. *BCD* = $\left. \begin{matrix} \text{ang. } A \\ \text{ang. CDA} \end{matrix} \right\}$: quare
Angulus *BCD* = *BDA*.

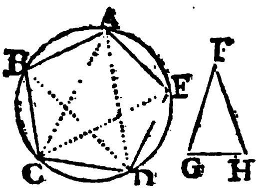
quia

quia vero [*per hyp.*]
angulus *BDA* = ang. *DBA*: erit
angulus *BCD* = ang. *DBA*.
unde [*per 6. pri.*]
latus CD = D*B*.
cum autem DB (*per conſtruct.*) ſit æquale CA, erit
latus *C*D = CA.
igitur (*per 5. pri.*)
angulus CDA æqualis ang. A.
Cum igitur angulus *B*D*A* demonſtratus ſit
æqualis ang. CDA + ang. A: erit
angulus *B*D*A* duplus ſolius anguli A
Demum, quia (*per hyp.*)
angulus *DBA* = ang. *B*D*A*: erit &
angulus D*BA* duplus anguli *A*: ergo
iſoſceles triangulum *BA*D habet utrumque angulorum ad
baſim duplum reliqui. q. e: f.

Propoſ. XI. Probl. XI.

In dato circulo ABCDE pentagonum æquilaterum, & æquiangulum inſcribere.

Conſtructio.



PEr antecedens problema efformetur triangulum iſoſceles FGH, cujus uterque angulorum ad baſim G, & H ſit duplus reliqui anguli F.

In dato circulo [*per 2. hujus.*] inſcribatur triangulum *A*C*D* æquiangulum triangulo FGH.

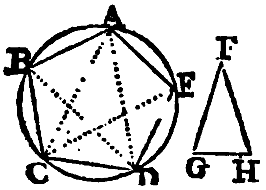
*A*nguli *A*CD, *A*DC (*per 9. pri.*) bifariam dividantur rectis C*E*, D*B*.

Ducantur rectæ *AB*, BC, CD, D*E*, *EA*,

Dico pentagonum *ABCDE* eſſe æquilaterum, & æquiangulum.

De-

Demonſtratio.



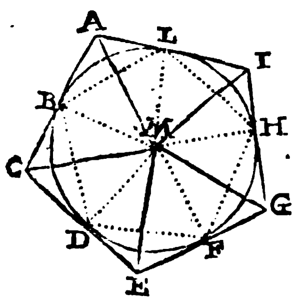
QVia (*per conſtructionem*) quilibet angulorum *ACD*, *ADC* eſt duplus anguli C*AD*, ſuntque dicti anguli à rectis C*E* D*B* bifariam diviſi: quare anguli C*DB*, *BDA*, *DCE*, *ECA* ſinguli æquales angulo C*AD*: ergo (*per 26. terii.*) arcus, quibus inſiſtunt æquales; ideoque [*per 29. tertij.* rectæ C*B*, *BA*, *AE*, ED *D*C æquales.

Ulterius cum demonſtrati ſint æquales quinque arcus C*B*, *BA* &c. ex his tres quomodocum ;accepti,erunt tribus æquales.His manentibus quia anguli *BAE*, A*ED* &c. inſiſtunt tribusarcubus (*per 27.tertii.*)dicti anguli æquales erunt:ergo figura rectilinea, *ABCDE*, cum habeat quinque latera æqualia, & quinque angulos æquales, erit pentagonum æquilaterum, & æquiangulum dato circulo inſcriptum. q. e. f.

Propoſ. XII. Prob. XII.

Circa datum circulum BLHFD deſcribere pentagonum æquilaterum, & æquiangulum.

Conſtructio.



IN dato circulo *BLHFD* [*per 11. hujus*] deſcribatur,pentagonum æquilaterum, & æquiangulum *BLH FD*. Ex punctis *B*, *L*, *H*, *F*, D ad centrum M ducantur rectæ *BM*, *LM*, *HM*, *F M*, D M, ad quas notentur perpendiculares C*A*, *AI*, *IG*, *GE*, *E*C, quæ concurrent in punctis C,*A*,*I*, *G*, *E*; quia anguli C*DB*, C*BD* ſunt duobus rectis minores.

Dico pentagonum *ACEGI* circa datum circulum *BDFHL* de-

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deſcriptum eſſe æquilaterum, & æquiangulum.
Ducantur rectæ *AM*, CM &c.

Demonſtratio.

IN triangulo rectangulo *EDM* (*per 47. pri.*)

q. *ED* } = q. *EM*.
q. *DM* }

Pariter in triangulo rectangulo EFM [*per eamdem 47.*)

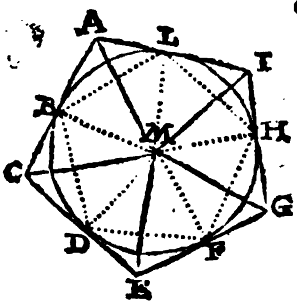
quad. E F } = quad. EM: quare
quad. FM }

(*per axio. 1.*)

quad. E D) = quad. E F)
quad. DM) = quad. FM)

demptis æqualibus quad. DM } : erit
quad. F M }

quad. ED = quad. EF: quare ED = EF. hoc demonſtrato.



In triangulis DME, FME latera DM, ME ſingula ſingulis æqualia lateribus FM, ME, baſiſque ED = baſi EF: ergo

[*per 8. pri.*]

angulus DME = ang. FME, atque angulus DEM = ang. FEM: igitur angulus DMF duplus ang. EMF.

Eodem modo demonſtrabitur ang. FMH [*idem valet de reliquis*] duplus ang. FMG.

quia vero (*per 27. tertii.*)

angulus DMF = ang. FMH, horum medietates, hoc eſt angulus EMF, & ang. GMF, erunt æquales.

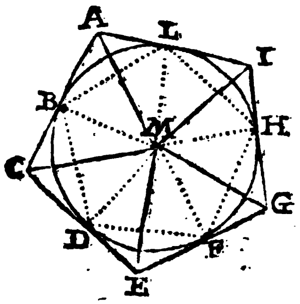
Hoc demonſtrato

In triangulis EMF, GMF, anguli FEM, FME ſinguli ſingulis æquales erunt angulis FGM, FME, latuſque FM commune: quare

(*per 26. pri.*)

EF = FG: ideoque EG dupla EF.

Eodem modo demonſtrabitur



EC dupla ED, & CA dupla CB &c.

Cum autem ſuperius fuerit demonſtratum

EF = ED, erit & EG = EC.

Conſimili modo demonſtrabitur EC = CA; CA = AI &c.

Quamobrem pentagonum ACEGI circulo circumſcriptum, erit æquilaterum.

Quod poſtea ſit, & æquiangulum, ita demonſtratur.

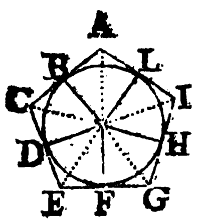
Anguli MEF, MGF ſuperius non ſolum ſunt demonſtrati æquales, verum etiam medietates angulorum DEF, FGH: quare

Angulus DEF = ang. FGH: quod cum de reliquis angulis HIL, LAB, BCD etiam demonſtrari poſſit; pentagonum erit etiam æquiangulum. q. e. f.

Propoſ. XIII. Probl. XIII.

In dato pentagono ACEGI æquilatero, & æquiangulo circulum inſcribere.

Conſtructio.



DAti pentagoni, duo, propinqui anguli, videlicet IAC, ACE (*per 9. pri.*) bifariam dividantur rectis lineis AO, CO concurrentibus in O, quod punctum O dico eſſe intra datum pentagonum: nam ſi ductæ ſint rectæ OE, OG, OI demonſtrabitur eas eſſe æquales, quia (*per hyp.*) in triangulis ACO, ECO latera AC, CO ſingula ſingulis æqualia ateribus EC, CO, & anguli ACO, ECO (*per conſtructionem*) æquales: quare [*per 4. pri.*] baſis AO = baſi EO, & ang. CAO = ang. CEO.

(*per*

Cum (*per hyp.*)

angulus CAI = ang. CEG, & ang. CAO medietas ang. CAI; erit æqualis ang. CEO medietas ang. CEG: quare etiam ang. CEG erit biſariam diviſus a recta EO.

Hoc modo poterit demonſtrari dati pentagoni reliquos angulos EGI, GIA eſſe biſariam diviſos a rectis GO, IO.

His demonſtratis

In triangulis ACO, ECO latera AC, CO ſingula ſingulis æqualia lateribus EC, CO, & angulus ACO = ang. ECO: ergo (*per* 4. *pri.*) baſis AO = baſi EO. Eadem demonſtratione adhibita, erunt rectæ AO, CO, EO, GO, IO æquales: quare punctum O erit intra pentagonum.

Hoc probato a puncto O ad ſingula pentagoni latera ducantur perpendiculares OB, OD, OF, OH, OL.

Demonſtratio.

IN triangulis ABO, ALO duo anguli BAO, OBA ſinguli ſingulis æquales ſunt angulis LAO, OLA, latuſque AO commune: quare (*per* 26. *pri.*) BO = LO.

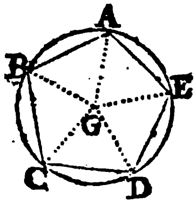
Quod dictum eſt de perpendicularibus BO, LO, etiam de reliquis OH, OF, OD intelligendum. Cum igitur iſtæ perpendiculares ſint æquales, ſi centro O intervallo OB fuerit deſcriptus circulus tranſibit per puncta B, D, F, H, L;

Vlterius quia [*per conſtructionem*] rectæ OB, OD &c. ſunt pentagoni lateribus perpendiculares, dicta latera (*per corol.* 16. *tertij.*) tangent circulum: unde circulus BDFHL, erit dato pentagono inſcriptus. q. e. f.

Propoſ. XIV. Probl. XIV.

Circa datum pentagonum ABCDE æquilaterum, & æquiangulum, circulum deſcribere.

Conſtructio.



DUo propinqui anguli *BAE*, *ABC* (*per 9. pri.*) biſariam dividantur rectis *AG*, *BG*, quæ (*per demonſtrata in prop.* 13. *hujus.*] intra pentagonum cõcurrunt in G: ducantur rectæ *GE*, *GD*, *GC*.

Demonſtratio.

DAti pentagoni omnes anguli (*per conſtructionem*, & *per demonſtrata in antecedente prop.*]ſunt biſariam diviſi a rectis GA, GB, GC, GD, GE. Cum autem (*per ſuppoſitum*) datum pentagonum ABCDE ſit æquiangulum; talium angulorum medietates erunt æquales: quare.

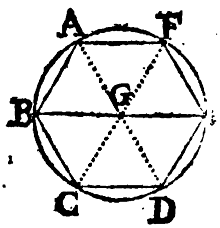
In triangulo AGB (idem de reliquis triangulis intelligendum) angulus GAB = ang. G B A: ergo (*per 6. pri.*) GA = GB.

Si igitur omnes rectæ a puncto G ad pentagoni angulos ductæ ſunt æquales, circulus centro G, atque diſtantia G A deſcriptus, tranſibit per puncta A, B, C, D, E: ideoque erit dato pentagono circumſciptus. q. e. f.

Propoſ. XV. Probl. XV.

In dato circulo A E C exagonum æquilaterum, & æquiangulum inſcribere.

Conſtructio.



DAti circuli (*per 2. tertij.*] inveniatur centrum G. ducatur diameter B G E. In circulo (*per 1. hujus.*) accomodentur rectæ BA, BC ſingulæ æquales ſemidiametro BG. Notentur rectæ ACD, CGF; addanturque rectæ CD, DF, FF: FA.

Dico figuram ABCDEF eſſet exagonum æquilaterum, & æquiangulum dato circulo inſcriptum. *De-*

Demonſtratio.

DUo triangula BAG, BCG (*per conſtructionem*) ſunt æquilatera, ideoque (*per corol. prop. 5. pri.*) æquiangula. Cum autem (*per 32. pri.*) in triangulo tres anguli ſint duobus rectis æquales; angulus BGA erit tertia pars duorum rectorum: hoc idem dicendum de angulo BGC: quare angulus BGA, = ang. BGC, atque ſimul æquabunt duas tertias partes duorum rectorum.

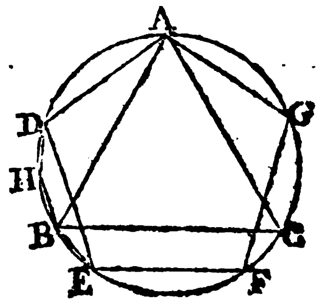
Inſuper quia (*per 13. pri.*) anguli CGA, CGD ſunt æquales duobus rectis, & angulus CGD tertia pars duorum rectorum: conſequenter tres anguli BGA, BGC, CGD inter ſe æquales.

Cum autem (*per 15. pri.*) dictis angulis æquales ſint anguli DGE, EGF, FGA; ſex anguli ad centrum G conſtituti, erunt æquales: quo ſtante in triangulis BGA, BGC &c. (*per 4. pri.*) baſes AB, BC, CD, DE, EF, FA, æquales erunt: ergo Hexagonum erit æquilaterum.

Inſuper cum æquilatera ſint triangula AGB, BGC &c. (*per corol. 5. pri.*) talium triangulorum anguli erunt æquales: ex quo ſequitur, angulum ABC (idem valet de reliquis) adæquare angulum BCD, cum ambo ex æqualibus angulis componantur: quare Hexagonum dato circulo inſcriptum, erit æquilaterum, & æquiangulum. q. e. f.

Corollarium.

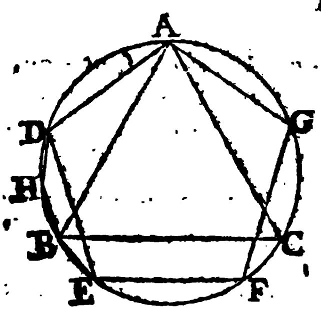
HInc manifeſtum eſt Hexagoni latus æquale circuli ſemidiametro; quia BA Hexagoni latus [*per demonſtrata*] æquat BG.



Propoſ. XVI. Probl. XVI.

In dato circulo ABC deſcribere quintidecagonum æquilaterum, & æquiangulum.

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Conſtructio.

IN dato circulo [*per 2.huius.*] deſcribatur triangulum æquilaterum ABC. Rurſus in eodem circulo (*per* 11.*hujus.*) deſcribatur pentagonum ADEFG æquilaterum, & æquiangulum, cujus unus angulus ſit ad punctum A conſtitutus. Arcus DB (*per* 11.*tertij.*) biſariam dividatur in H.

Dico rectam DH eſſe latus quintidecagoni in dato circulo deſcribendi.

Demonſtratio.

QVia triangulum ABC (*per conſtructionem*) eſt æquilaterum (*per corol.* 5.*pri.*) erit æquiangulum: quare (*per* 28.*tertij.*) arcus AB, BC, GA æquales erunt.

Ob eamdem cauſam (*per conſtructionem*) ADEFG eſt pentagonum æquilaterum ergo (*per* 28.*tertij*) quinque arcus AD, DE, EF, FG, GA æquales erunt.

His ſtantibus qualium partium quindecim eſt tota circumferentia ABC, talium partium quinque erit arcus AB, & tres erit arcus *AD*: quare iſtarum partium duas continebit arcus DB: cum autem arcus DB (*per conſtructionem*) ſit biſariam diviſus in H, erit arcus DH pars decimaquinta totius circumferentiæ *ABC*, unde recta DH erit corda, ſiuè ſubtenſa decimæquintæ partis integri circuli *ABC*: ſi igitur in dato circulo (*per* 1.*hujus.*) accomodentur rectæ ipſi DH æquales, in circulo erit efformatum quintidecagonum æquilaterum, cujus omnes anguli cum inſiſtant treſdecim ex dictis arcubus æqualibus [*per* 27. *tertij.*) erunt inter ſe æquales: quamobrem in dato circulo deſcriptum erit quintidecagonum æquilaterum, & æquiangulum. q.e.f.

Elementi Quarti Finis.

LIBER

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LIBER V. EUCLIDIS ELEMENTUM QUINTUM.

DEFINITIONES.

Definitio I.

PARS dicitur magnitudo magnitudinis, minor majoris, quando minor metitur majorem.



SI AB ſit magnitudo magnitudinis AC, atque minor quam AC, & ipſam AC metiatur, tunc PARS vocatur.

Quia verò minor magnitudo poteſt majorem adæquatè, & inadæquatè metiri, ideo pars divitur in ALIQUOTAM, & ALIQUANTAM. Pars ALIQUOTA dicitur illa, quæ aliquoties ſumpta adæquatè metitur ſuum totum: talis eſt linea trium palmorum, quæ ter ſumpta adæquatè metitur lineam novem palmorum.

Pars ALIQUANTA dicitur illa, quæ inadæquatè metitur ſuum totum, talis eſt linea 4. palmorum reſpectivè ad lineam 10. palmorum: nam linea 4. palmorum bis accepta deficit, ter verò excedit lineam 10. palmorum.

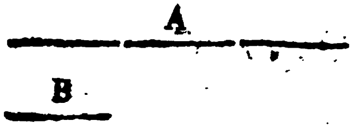
SCHOLIUM.

SUperius data Euclidis definitio videtur intelligenda de ſola parte ALIQUOTA, quia hæc aliquoties accepta metitur ſuum totum, non ſic verò pars ALIQUANTA, quæ aliquoties ſumpta vel excedit, vel deficit a ſuo toto. Addo inſuper, partem ALIQUANTAM ab Euclide in 7. Elem. lib., non partem, ſed partes

appellari, & rectè, quia linea 4. palmorum relativè ad lineam 6. palmorum non partem, ſed duas tertias partes continet.

Def. II.

MULTIPLEX dicitur major minoris, quando minor metitur majorem.



Si B aliquoties ſumpta metiatur A, tunc A vocatur multiplex ipſius B.

Def. III.

RATIO appellatur duarum magnitudinum ejuſdem generis mutua ſecundum quantitatem habitudo.

QUotieſcumque duæ magnitudines ejuſdem generis, uti ſunt duo numeri, duæ lineæ, duæ ſuperficies, vel duo ſolida ad invicem comparantur ſecundum quantitatem quatenus ſcilicet una quantitas ſit major, minor, vel æqualis alteri; talis comparatio, ſeu reſpectus dicitur RATIO, & a nonnullis PROPORTIO.

Nota. Illa quantitas, quæ ad aliam comparatur vocatur ANTECEDENS, illa verò alia, ad quam comparatur, dicitur CONSEQUENS.

Si magnitudo A comparetur ad B; tunc A dicitur antecedens, & B conſequens: Si poſtea B referatur ad A, erit B antecedens, & A conſequens.

SCHOLIUM.

De rationum diviſione.

RATIO ſuperius definita univerſaliter accepta partitur in rationem rationalem, ſeu exponibilem, & irrationalem, ſeu ineffabilem.

Primo. Ratio rationalis illa dicitur, quæ numeris exprimi poteſt: talis eſt illa ratio, quæ vadit inter magnitudines A, & B, dummodo exprimi poſſit numeris 20. & 10.; qui numeri indicant, rationem A ad B eſſe duplam. Se-

Secundo. *Ratio irrationalis illa vocatur, quæ numeris exhiberi nequit: talis eſt illa ratio, quæ cadit inter diametrum quadrati, & latus ejuſdem quadrati, uti ab Euclide in prop. 117, lib. 10. fuit demonſtratom.*

Alij vero Scriptores rationalem rationem exponunt, dicendo, eam eſſe, quæ cadit inter quantitates commenſurabiles, nempè illas, inter quas datur communis pars aliquota: propterea numerus ad numerum rationalem ſemper habebit rationem, quia ſaltem unitas ſemper eſt communis menſura omnium numerorum; uti patet in his numeris 7, & 5, quorum uterque ab unitate menſuratur.

Irrationalem poſtea rationem exponunt, dicendo, illam reperiri inter quantitates, quarum nulla pars communis aliquota reperitur, uti evenit in diametro, & latere quadrati, cum hæ duæ quantitates nullam habeant communem partem aliquotam, quæ tam diametrum, quam latus quadrati valeat menſurare; quemadmodum ab Euclide in prop. 117. lib. 10. fuit demonſtratum.

Nota. Numerus ad numerum ſemper habet rationem rationalem, ideoque Authores, ut oſtendant inter magnitudines cadere rationem rationalem, communiter dicunt habere rationem cadentem inter numerum, & numerum.

Tertio. Rationalis ratio denuo ſubdividitur in rationem æqualitatis, & inæqualitatis. Æqualitatis ratio vocatur illa, quæ inter æquales magnitudines reperitur; uti eſt illa, quæ inter 4, & 4; 10, & 10. reperitur.

Inæqualitatis ratio appellatur illa, quæ inter inæquales magnitudines reperitur, uti eſt illa, quæ habetur, dum numerus 20. ad 10, vel ad 7. comparatur.

Quarto. Hæc inæqualitatis ratio iterum ſubdividitur in rationem majoris, atque minoris inæqualitatis. Majoris inæqualitatis ratio dicitur, dum major magnitudo ad minorem refertur: ex adverſo illa ratio, quæ habetur, dum minor quantitas ad majorem refertur, minoris inæqualitatis ratio nuncupatur.

Quinto. Utraque harum rationum in quinque ſpecies partiri ſolet, quæ his nominibus appellantur.

Ratio Multiplex
Superparticularis.
Superpartiens.
Multiplex ſuperparticularis.

Mul-

Multiplex Superpartiens.

Tria priora genera dicuntur ſimplicia; duo vero poſteriora compoſita appellantur.

Sexto. Ratio MULTIPLEX habetur, dum major quantitas ad minorem comparata, eam pluries continet, uti evenit, dum numerus 4. ad 2. comparatur.

Quo ad multiplicium ſpecies, quando major quantitas ad minorem comparata, eam bis comprehendit, dicitur DUPLA: ſi ter TRIPLA; ſi centies CENTUPLA, ut in ſequenti Tabula.

Ratio	4	—	2	*Dupla.*
Ratio	6	—	2	*Tripla.*
Ratio	8	—	2	*Quadrupla.*
Ratio	10	—	2	*Sextupla &c.*

Septimo. SUPERPARTICULARIS ratio dicitur, quãdo major quantitas minorem ſemel continet, & id quod remanet, eſt pars aliquota minoris; uti cernere licet, dum numerus 6. ad numerum 4. refertur; ſiquidem 6. ſemel continet 4., atque reſiduum 2. eſt pars aliquota numeri minoris 4.

Octavo. Quantum ad ſpecies rationum ſuperparticularium eſt obſervandum, quod quando major quantitas minorem ſemel continet, & inſuper reſiduum eſt medietas minoris, dicitur ratio SESQUIALTERA: Si vero contineat ſemel, & reſiduum ſit tertia pars minoris, appellatur ratio SESQUITERTIA; quod de reliquis venit intelligendum, ut patet ex appoſita tabula.

Ratio	3	—	2.	*Seſquialtera.*
Ratio	8	—	6.	*Seſquitertia.*
Ratio	10	—	8.	*Seſquiquarta.*
Ratio	12	—	10.	*Seſquiquinta. &c.*

Nono. Ratio SUPERPARTIENS dicitur, quando major quantitas relata ad minorem, eam ſemel continet, & id quod reſtat collectim ſumptum eſt pars Aliquanta minoris. Hoc clarè conſtat, dum numerus quinarius ad ternarium refertur; ſiquidem numerus quinarius ſemel continet ternarium, quique reſtat binarius, eſt pars Aliquanta numeri ternarij.

Sub hoc rationis genere innumeræ continentur ſpecies, nam quæ remanet pars aliquanta poteſt eſſe duæ tertiæ partes minoris: pari-

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riter poteſt eſſe tres quartæ partes ejuſdem minoris ; quatuor quintæ ; tres quintæ &c.

Si major quantitas ſemel contineat minorem, atque reſiduum ſit duæ tertiæ partes minoris quātitatis, tunc dicitur SUPERBIPARTIENS TERTIAS. Si reſiduū ſit tres quartæ partes minoris, appellatur SUPERTRIPARTIENS QUARTAS, ut in ſequēti Tabula.

Ratio 5 — 3. Superbipartiens tertias.
Ratio 7 — 4. Supertripartiens quartas.
Ratio 9 — 5. Superquadripartiens quintas.
Ratio 9 — 7. Superbipartiens ſeptimas.

Hæc de ſimplicium rationum generibus.

De Compoſitis.

Decimo. Ratio MULTIPLEX SUPERPARTICULARIS vocatur, dum major quantitas pluries minorem continet, inſuperque partem aliquotam minoris. Si enim A bis contineat B, & inſuper partem aliquotam ipſius B, hujuſmodi ratio dicenda erit MULTIPLEX SUPERPARTICULARIS, ut in hac tabula.

Ratio 10 — 4. Dupla ſeſquialtera.
Ratio 13 — 3. Quadrupla ſeſquitertia.
Ratio 26 — 8 Tripla ſeſquiquarta.
Ratio 11 — 5 Dupla ſeſquiquinta.

Undecimo. Ratio MULTIPLEX SUPERPARTIENS dicitur, quando major quantitas pluries continet minorem, & inſuper partem aliquantam minoris: Si enim major quantitas A pluries contineat minorem B, & reſiduum ſit pars aliquāta ipſius B; talis ratio dicē da erit MULTIPLEX SUPERPARTIENS, ut in his exemplis.

Ratio 8 — 3. Dupla ſuperbipartiens tertias.
Ratio 11 — 4. Dupla ſupertripartiens quartas.
Ratio 19 — 5. Tripla ſuperquadripartiens quintas.
Ratio 19 — 7. Dupla ſuperquintupartiens ſeptimas.

Hæc de rationibus majoris inæqualitatis.

Duodecimo. Sub illo rationum genere, quod minoris inæqualitatis.

Multiplex Superpartiens.

Tria priora genera dicuntur ſimplicia; duo vero poſteriora compoſita appellantur.

Sexto. Ratio MULTIPLEX *habetur, dum major quantitas ad minorem comparata, eam pluries continet, uti evenit, dum numerus 4. ad 2. comparatur.*

Quo ad multiplicium ſpecies, quando major quantitas ad minorem comparata, eam bis comprehendit, dicitur DUPLA: *ſi ter* TRIPLA; *ſi centies* CENTUPLA, *ut in ſequenti Tabula.*

Ratio	4 —	2	*Dupla.*	
Ratio	6 —	2	*Tripla.*	
Ratio	8 —	2	*Quadrupla.*	
Ratio	10 —	2	*Sextupla*	*&c.*

Septimo. SUPERPARTICULARIS *ratio dicitur, quãdo major quantitas minorem ſemel continet, & id quod remanet, eſt pars aliquota minoris; uti cernere licet, dum numerus 6. ad numerum 4. refertur; ſiquidem 6. ſemel continet 4., atque reſiduum 2. eſt pars aliquota numeri minoris 4.*

Octavo. Quantum ad ſpecies rationum ſuperparticularium eſt obſervandum, quod quando major quantitas minorem ſemel continet, & inſuper reſiduum eſt medietas minoris, dicitur ratio SESQVIALTERA: *Si vero contineat ſemel, & reſiduum ſit tertia pars minoris, appellatur ratio* SESQVITERTIA; *quod de reliquis venit intelligendum, ut patet ex appoſita tabula.*

Ratio	3 —	2.	*Seſquialtera.*	
Ratio	8 —	6.	*Seſquitertia.*	
Ratio	10 —	8.	*Seſquiquarta.*	
Ratio	12 —	10.	*Seſquiquinta.*	*&c.*

Novo. Ratio SUPERPARTIENS *dicitur, quando major quantitas relata ad minorem, eam ſemel continet, & id quod reſtat collectim ſumptum eſt pars Aliquanta minoris. Hoc clarè conſtat, dum numerus quinarius ad ternarium refertur; ſiquidem numerus quinarius ſemel continet ternarium, quique reſtat binarius, eſt pars Aliquanta numeri ternarij.*

Sub hoc rationis genere innumeræ continentur ſpecies, nam remanet pars aliquanta poteſt eſſe duæ tertiæ parte-

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riter poteſt eſſe tres quartæ partes ejuſdem minoris ; quatuor quintæ ; tres quintæ &c.

Si major quantitas ſemel contineat minorem, atque reſiduum ſit duæ tertiæ partes minoris quātitatis, tunc dicitur SUPERBIPARTIENS TERTIA. *Si reſiduū ſit tres quartæ partes minoris, appellatur* SUPERTRIPARTIENS QUARTAS, *ut in ſequēti Tabula.*

Ratio 5 — 3. *Superbipartiens tertias.*
Ratio 7 — 4. *Supertripartiens quartas.*
Ratio 9 — 5. *Superquadripartiens quintas.*
Ratio 9 — 7. *Superbipartiens ſeptimas.*

Hac de ſimplicium rationum generibus.

De Compoſitis.

Decimo. Ratio MULTIPLEX SUPERPARTICULARIS *vocatur, dum major quantitas pluries minorem continet, inſuperque partem aliquotam minoris. Si enim A bis contineat B, & inſuper partem aliquotam ipſius B, hujuſmodi ratio dicenda erit* MULTIPLEX SUPERPARTICULARIS, *ut in hac tabula.*

Ratio 10 — 4. *Dupla ſeſquialtera.*
Ratio 13 — 3. *Quadrupla ſeſquitertia.*
Ratio 26 — 8 *Tripla ſeſquiquarta.*
Ratio 11 — 5 *Dupla ſeſquiquinta.*

Undecimo. Ratio MULTIPLEX SUPERPARTIENS *dicitur, quando major quantitas pluries continet minorem, & inſuper partem aliquantam minoris: Si enim major quantitas A pluries contineat minorem B, & reſiduum ſit pars aliquāta ipſius B; talis ratio dicēda erit* MULTIPLEX SUPERPARTIENS, *ut in his exemplis.*

Ratio 8 — 3. *Dupla ſuperbipartiens tertias.*
Ratio 11 — 4. *Dupla ſupertripartiens quartas.*
...io 19 — 5. *Tripla ſuperquadripartiens quintas.*
... 19 — 7. *Dupla ſuperquintupartiens ſeptimas.*

...tionibus majoris inæqualitatis.

D... Sub illo rationum genere, quod minoris in...

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tis dicitur, dum ſcilicet minor quantitas ad majorem refertur, tot rationum ſpecies continentur, quot ſub ratione majoris inæqualitatis, ut paulo ante explicavimus. Iſtæ minoris inequalitatis rationes ſola particula SUB *indicantur; ut in his exemplis.*

Ratio 4 — 6. *Sub ſeſquialtera.*
Ratio 5 — 7. *Sub ſuperbipartiens quintas.*
Ratio 8 — 18. *Sub dupla ſeſquiquarta.*
Ratio 5 — 18. *Sub tripla ſrpertripartiens quintas.*

Defin. IV.

PROPORTIO dicitur rationum ſimilitudo.

QUando quantitates ad invicem comparatæ ſimiles obtinent rationes, uti ſunt rationes, cadentes inter 12., & 4.: & inter 9., & 3. talis rationum ſimilitudo, ſiuè uniformitas PROPORTIO appellatur. *A* Græcis dicitur *ANALOGIA*, & à quam plurimis Latinis *PROPORTIONALITAS* vocatur.

De varijs Proportionum ſpeciebus.

SCriptores tres aſſignant Proportionum ſpecies, quæ ſunt
Proportio GEOMETRICA.
Proportio ARITHMETICA.
Proportio ARMONICA, ſiuè MUSICA.

Proportio GEOMETRICA dicitur, dum quantitates ad alias comparatæ eamdem habent rationem; ſi enim quantitates *A*, & B comparatæ ad C, & D uniformes habeant rationes: in tali caſu hæc erit dicenda proportio GEOMETRICA: Si *A* bis contineat C, pariterque B bis contineat D; talis rationum ſimilitudo componit proportionem GEOMETRICAM.

Hæc GEOMETRICA proportio partitur in CONTINVAM, & DISCRETAM

CONTINVA vocatur illa, in qua intermediæ quantitates taliter accipiuntur, ut ſint antecedentia, & conſequentia rationum, ut in hoc exemplo.

Pro-

Proportio Geometrica continua.

A — B — C — D — E
2 — 6 — 18 — 54 — 162 &c.

In hac enim proportione numerus B (*idem valet de reliquis.*) non ſolum eſt conſequens reſpectiuè ad A, verum etiam antecedens relatiuè ad C; qua de cauſa hæc proportio dicitur continuata.

DISCRETA proportio Geometrica appellatur, dum intermediæ quantitates, ſeu termini ſemel tantum accipiuntur, ita ut fiat veluti rationum ſeparatio: ut in hoc exemplo.

Poportio Geometrica diſcreta.

A — B — C — D
2 — 6 — 20 — 60

In hac proportione B tantummodo eſt conſequens ipſius A; non autem eſt antecedens ipſius C, uti evenit in proportione continua.

Arithmetica proportio dicitur, quando tres, vel plures numeri eamdem obtinent differentiam, uti ſunt iſti numeri.

4 — 7 — 10 — 13 — 16 &c.

Pariter hæc proportio ARITHMETICA in CONTINVAM, atque DISCRETAM partitur.

CONTINVA dicitur, in qua inter medios terminos eadem eſt differentia, ut in ſuperiore exemplo in quo inter 4., & 7., & inter 7., & 10. tres mediant unitates.

DISCRETA proportio Arithmetica vocatur, quãdo inter terminos intermedios eadem non eſt differentia, ut in hoc exemplo.

A. — B. — C. — D.
2 — 4. — 20 — 22

In hac enim proportione illa differentia, quæ eſt inter 2., & 4. non eſt illa, quæ inter 4., & 20. reperitur, ideoq; DISCRETA proportio dicitur.

Pro-

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Proportio Arithmetica continua.

2 — 4 — 6 — 8 — 10 — 12 &c.

Proportio Arithmetica diſcreta.

2 — 4: 20 — 22: 25 — 27 &c.

HARMONICA, ſiuè MUSICA Proportio dicitur, quando tribus datis quantitatibus, prima ad tertiam eamdem habet rationem quam differentia inter primam, & ſecundam, ad differentiam inter ſecundam, & tertiam.

Si enim propoſiti ſint iſti tres numeri A. B. C. 3 — 4 — 6, quia eadem eſt ratio primi A ad tertium C, ac differentiæ inter A, & B ad differentiam inter B, & C, hi numeri dicendi erunt in proportione HARMONICA, ſiuè MUSICA.

Talis autem proportio HARMONICA, atque MUSICA appellatur, quia Muſicæ conſonantiæ hac proportione exprimuntur: datis enim his numeris harmonicè proportionalibus

6 — 8 — 12 — 24

Ratio 8. ad 6. Seſquitertia. DIATESSERON, ſiuè quartam conſtituit,

Ratio 12. ad 8. ſeſquialtera cõtinet conſantiam DIAPENTEN, ſiuè quintam.

Ratio 12. ad 6. dupla indicat conſonantiam DIAPASON, ſiuè octauam.

Ratio 24. ad 8. tripla oſtendit conſonantiam DIAPASON, & DIAPENTEN, hoc eſt duodecimam.

Ratio 24. ad 6. quadrupla exhibet conſonantiam DISDIAPASON, ſiuè decimamquintam.

Hæc de Proportionum diviſione.

Def. V.

Illæ magnitudines inter ſe rationem habere dicuntur, quæ multiplicatæ poſſunt ſeſe invicem ſuperare.

Eucli-

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EUclides (*in Def. 3.*) duarum magnitudinum ejuſdem generis, ſecundum quantitatem habitudinem RATIONEM vocavit; in hac vero (*Def. 5.*) exponit quid requiratur ad hoc, ut duæ quantitates ejuſdem generis rationem habere dicantur, dum inquit, illas tantummodo quantitates dici inter ſe rationem habere, quarum utravis multiplicata ita augeri poſſit, ut alteram ſuperet.

Illæ etenim quantitates, quarum una quomodocumque multiplicata nequit alteram ſuperare, & e contra, nullam habere poſſunt rationem: ex quo colligitur, lineam finitam ad infinitam nullam poſſe dici habere rationem, quia finita linea quomodocumque multiplicata, nequit infinitam lineam ſuperare.

Def. VI.

Prima magnitudo ad ſecundam, & tertia ad quartam eamdem rationem habere dicuntur, cum primæ, & tertiæ æqualiter multiplicia à ſecundæ, & quartæ pariter multiplicibus (ſecundum quamcumque multiplicationem) utrumque ab vtroque, vel æqualia ſunt, vel una excedunt, vel una deficiunt, ſi prout inter ſe reſpondent, ita ſumpta fuerint.

SEnſus hujus definitionis eſt, quod poſitis quatuor magnitudinibus A. B. C. D.

ſumptiſque primæ A, & tertiæ C pariter multiplicibus, ſecundum quamcumque multiplicationem: pariterque acceptis aliis æquemultiplicibus ſecundæ B, & quartæ D, juxta quamcumque multiplicationem, ſi inter hæc pariter multiplicia talis ordo ſemper inveniatur videlicet, quod

Quando multiplex primæ A ſuperat multiplicem ſecundæ B, etiam multiplex tertiæ C ſuperet multiplicem quartæ D.

Quando multiplex primæ A minor eſt multiplice ſecundæ B, etiam multiplex tertiæ C minor ſit multiplice quartæ D.

Quan-

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Quando multiplex primæ A æquat multiplicem ſecundæ B, etiam multiplex tertiæ C æquet multiplicem quartæ D.
Quibuſcumq; acceptis pariter multiplicibus; tunc quatuor propoſitæ magnitudines A. B. C. D. dicendæ erunt in eadem ratione, hoc eſt quod

Ratio primæ A ad ſecundam B ſimilis ſit rationi tertiæ C ad quartam D.

Poſitis quatuor numeris ſumptiſque pariter multiplicibus primi A, & tertii C, ſecundi B, & quarti D, ſecundum quamlibet multiplicationem; ſi, ut in hac tabula, quando multiplex primi A ſuperat multiplicem ſecundi B, etiam multiplex tertii C ſuperet multiplicem quarti D.

A.	B.	C.	D.
6	4	3	2
24:	28:	12:	14.
15:	8:	9:	4.
36:	36:	18:	18.

Quando multiplex primi A minor eſt multiplice ſecundi B, etiam multiplex tertii C minor ſit multiplice quarti D.
Quando multiplex primi A æqualis multiplici ſecundi B, etiam multiplex tertii C æqualis multiplici quarti D.
Hoc ſervato ordine inter hæc pariter multiplicia in quacumque multiplicatione: quatuor numeri 6 — 4 — 3 — 2 dicentur habere eamdem rationem, hoc eſt
Ratio 6 ad 4 eadem ſiuè ſimilis rationi 3 ad 2.

SCHOLIUM.

NOnnulli ex modernis Scriptoribus contendunt, hanc Euclidis definitionem potius inter propoſitiones demonſtrandas, quam inter definitiones eſſe reponendam; quorum ratio potiſſima eſt, quia nimis obſcura eſt illa neceſſaria connexio, quæ cadit inter ordinem pariter multiplicium ſuperius ab Euclide propoſitum, & illam rationum ſimilitudinem, quam habent illæ quantitates, quarum ſunt æque multiplicia. Et ſanè, ſi veritatem fateri volumus, etiam ſup-

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ſuppoſito, quod in quacumque multiplicatione, quando multiplex primæ magnitudinis ſuperat multiplicem ſecundæ, etiam multiplex tertiæ ſuperet multiplicem quartæ: quando æquale, æquale, & quando minor, minor, ut in Euclidiana definitione; non apparet ſufficiens ratio ad dicendum illas magnitudines eamdem habere rationem, licet talium magnitudinum æquemultiplicia ſervent ordinem ſuperioritatis, inferioritatis, & æqualitatis ab Euclide præſcriptum: quamobrem dicti Authores volunt hanc veritatem eſſe demonſtrandam, non autem ſine demonſtratione tanquam definitionem admittendam.

Quamobrem illi Scriptores, qui Euclidianam definitionem recipere nolunt, ſequentem proponunt, dicendo

Datis quatuor magnitudinibus A. B. C. D. ſi quemadmodum prima A continet, vel continetur in ſecunda B; ita tertia C contineat, vel contineatur in quarta D; datæ magnitudines dicendæ erunt in eadem ratione.

Ad oſtendendum poſtea, quando nam una magnitudo toties, ſeu ita contineat, vel contineatur in alia, quoties alia aliam continet, vel in alia continetur, ad ALIQUOTAS PARTES SIMILES recurrunt, illas ſcilicet, quæ æqualiter ſua tota menſurant, uti evenit in partibus 2. & 5. reſpectivè ad tota 8., & 20., cum 2. æqualiter menſuret 8., ac 5. menſurat 20. quod idem eſt ac dicere.

Quando prima magnitudo æqualiter continet ſecundam, ac tertia contineat quartam inſuperque eamdem partem aliquotam, vel eaſdem partes aliquantas, illæ magnitudines dicendæ ſunt proportionales, hoc eſt eamdem rationem habentes: uti cernere licet in his numeris 12. 5. 24. 10. primus enim numerus 12. bis continet ſecundum 5., & inſuper duas partes quintas ipſius ſecundi: pariter tertius numerus 24. bis continet quartum numerum 10., & inſuper duas quintas partes ipſius numeri 10: quare ſecundum hanc definitionem ratio 12. ad 5. eadem erit ac ratio 24. ad 10.

Hac in re ſatis dubia, an ſcilicet in definiendis magnitudinibus eamdem rationem habentibus accipiendus ſit ordo æquemultiplicium, uti docet Euclides, vel recurrendum ſit ad partes aliquotas ſimiles, uti volunt aliqui Modernorum, placet addere ſequentia.

Quia Euclides voluit definire magnitudines eamdem rationem, tam rationalem, quam irrationalem habentes, coactus fuit recurrere ad illum pariter multiplicium ordinem, quem ſupra explicavimus,

mus, quia nil notius habuit, quo exponeret magnitudines eamdem rationem, tam rationalem, quam irrationalem habentes.

Moderni Scriptores, qui eamdem magnitudinum rationem in aliquotarum partium ſimilitudinẽ refundunt, ſolas magnitudines eamdem rationalem rationem habentes comprehendunt, non autem irrationalem, quia irrationalis ratio nullam admittit partem aliquotam, vel aliquantam, ſecus non eſſet ratio irrationalis.

His ſtantibus videtur dicendum Euclidianam proportionalium magnitudinum definitionem, tam rationales, quam irrationales rationes comprehendere; Modernorum verò definitionem ad ſolas rationes rationales ſe extendere, non autem ad irrationales.

Neque ſat eſt opponere contra Euclidis definitionem dicendo omnino deliteſcere connexionem cadentem inter multiplicium ordinem, & identitatis rationem; quia ſatis manifeſtum eſt haud neceſſarium eſſe rationem reddere, cur ſcilicet res uno, vel alio modo definiantur, & in apertum ponere connexionem cadentem inter definitionem, & rem definitam; cum ſatis ſit quod nunquam res definita de aliqua re affirmetur, niſi prius fuerit demonſtratum definitionem eidem rei convenire: qui definitionum rationes inquirunt, eas cum propoſitionibus confundunt.

Euclides in 10. defin. lib. primi Elementorum, angulum rectum illum dicit, qui fit a recta linea ſuper rectam cadente, dum angulos deinceps æquales facit. De hac definitione rationem quærere, cur ſcilicet Euclides taliter angulum rectum definiverit, ridiculum foret, quia definitio omnem rationem excludit, ſecus non eſſet definitio. Quod de anguli recti definitione dictum eſt, de hac ſexta definitione venit intelligendum.

Addo inſuper aſſerentes definitionem Euclidis tamquam illegitimam, aliamque proponentes, ut propoſitiones demonſtrent, aliqua ponere axiomata, circa quæ maior eſt difficultas, quam circa definitionem ab Euclide propoſitam, ut ſedulo conſiderantibus clarè apparebit.

Cum igitur per datam Euclidis definitionem ſecluſo quocumque axiomate, haud ſatis tuto, omnes huius Elementi Propoſitiones demonſtrentur, ſatius duxi Euclidianam definitionem recipere, quam propoſitiones demonſtrandas tamquam axiomata affirmare.

Defin.

Def. VII.

Illæ magnitudines, quæ eamdem habent rationem, PROPORTIONALES vocentur.

Signa quatuor magnitudinum proportionalitatem indicantia, hæc ſunt

A: B:: C: D

Quibus ſignis ſemper indicabitur rationem A ad B eſſe illam, quam obtinet C ad D; quod eſt idem ac dicere.

uti A ad B, ita C ad D.

Def. VIII.

Prima magnitudo ad ſecundam dicitur habere majorem rationem, quam tertia ad quartam, cum ſuppoſitis pariter multiplicibus primæ, & tertiæ, atque ſecundæ, & quartæ, in aliqua multiplicatione multiplex, primæ excedit multiplicem ſecundæ, at multiplex tertiæ non ex cedit multiplicem quartæ.

Poſitis quatuor numeris, quia contingere poteſt, quod ſumptis pariter multiplicibus A, & C, atque B, & D, dum multiplex primi A ſuperat multiplicem ſecundi B, multiplex tertii C non ſuperet multiplicem quarti D: dicendum erit A ad B majorem habere rationem, quam C ad D.

A.	B.	C.	D.
10.	4.	6.	3.
30:	28:	18:	21.

Def. IX.

PROPORTIO in tribus terminis pauciſſimis conſiſtere dicitur.

QVia ad habendam proportionem duæ ſaltem requiruntur rationes, & quælibet ratio duos habet terminos, nempè Antecedens, & Conſequens; inde ſequitur proportionem in tribus tantum pauciſſimis terminis poſſe conſiſtere, quia idem terminus poteſt eſſe conſequens unius rationis, & antecedens alterius: uti cernere licet in hac proportione reperta in numeris 2. 4. 8., in qua medius terminus 4. eſt conſequens rationis 2. ad 4, & antecedens alterius rationis 4. ad 8.

Def. X.

Si tres magnitudines A. B. C. fuerint proportionales, prima A ad tertiam C dicitur habere rationem DUPLICATAM ejus, quam habet ad ſecundam B.

SI quatuor magnitudines D. E. F. G. fuerint proportionales, prima D ad quartam G dicitur habere triplicatam rationem ejus, quam prima D habet ad ſecundam E.

Hocque deinceps ſeruandum uno amplius, quam diu proportio extiterit.

Def. XI.

HOMOLOGÆ, ſiue ſimiles ratione magnitudines dicuntur, antecedentes quidem cum antecedentibus, & conſequentes cum conſequentibus.

SI fuerint quatuor magnitudines proportionales A. B. C. D. antecedentes A, & C, dicuntur HOMOLOGA; quemadmodum

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modum etiam conſequentia B, & D HOMOLOGA appellatur.

Def. XII.

ALTERNA ratio dicitur comparatio antecedentis ad antecedentem, & conſequentis ad conſequentem.

NOta. In hac, & in ſequentibus definitionibus Euclides oſtendit ſex arguendi modos, quibus uti poſſumus in magnitudinibus proportionalibus: qui ſequentibus nominibus appellantur.

1. Alterna ratio, ſeu permutata ratio.
2. Inverſa ratio, ſeù è contrario.
3. Compoſitio rationis, ſeù coniuncta ratio.
4. Diviſio rationis, ſeù diſiuncta ratio.
5. Converſio rationis, ſive euerſa ratio.
6. Ex æqualitate, ſeù æqua ratio, quæ etiam ex æquo dicitur.

Def. XIII.

INVERSA RATIO dicitur ſumptio conſequentis ceu antecedentis ad antecedentem, velut ad conſequentem.

Def. XIV.

COMPOSITIO RATIONIS appellatur ſumptio antecedentis cum conſequente, ut una magnitudo ad ipſum conſequens.

HÆc compoſitio rationis duplex eſt, quarum una dicitur compoſitio rationis CONVERSA, dum ſcilicet antece-

dens, & conſequens, ut unum, comparantur ad antecedens: altera vero dicitur Compoſitio rationis CONTRARIA, quæ evenit, dum antecedens refertur ad antecedens, & ad conſequens, ut unum.

Def. XV.

DIVISIO rationis vocatur, dum exceſſus, quo antecedens ſuperat conſequentem, refertur ad ipſum conſequentem.

ETiam in diviſione rationis datur diviſio rationis CONVERSA,& diviſio rationis CONTRARIA. CONVERSA habetur, dum conſequens comparatur ad exceſſum, quo conſequens ſuperat antecedens. Diviſio rationis CONTRARIA evenit, dum antecedens confertur cum exceſſu, quo conſequens ſuperat antecedens.

Def. XVI.

CONVERSIO rationis appellatur, dum antecedens comparatur ad exceſſum, quo antecedens ſuperat ipſum conſequentem.

EXemplum harum argumentationum habemus in hac tabula ſuppoſitis quatuor magnitudinibus proportionalibus.

	A.	B.	C.	D.
	6.	4.	3.	2
Alterna ratio —	6:	3:	4:	2
Inverſa ratio —	4.	6.	2.	3
Compoſitio rationis —	10.	4.	5.	2

Com-

Compoſitio rationis Converſa —	10:	6:	5:	3
Compoſitio rationis contraria —	6:	10:	3:	5
Diviſio rationis —	2.	4.	1.	2
Diviſio rationis Converſa —	4.	2.	2.	1
Diviſio rationis contraria —	2.	1.	4.	2
Converſio rationis —	6.	2.	3.	1

Nota. In prima, atque ſecunda ſpecie diviſionis rationis, antecedens debet ſuperare conſequens: In tertia vero ſpecie, hoc eſt in diviſione rationis contraria, conſequens debet ſuperare antecedens.

Def. XVII.

Ex æqualitate ratio dicitur, ſi plures duabus ſint quantitates A, B, C, & aliæ ipſis numero æquales D. E. F; quæ binæ, & in eadem ratione ſumantur, inferturque, ut in primis quantitatibus A ad C, ita in ſecundis quantitatibus D ad F.

SCHOLIUM.

*Q*Uoniam duobus modis, licet ex æqualitate arguere, nempè ex æqualitate ORDINATA, & PERTURBATA, Euclides in ſequentibus definitionibus quid ſit ordinata, & quid perturbata ratio exponit.*

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Def. XVIII.

ORDINATA proportio dicitur, cum fuerit quemadmodū antecedens ad conſequens ex una parte, ita antecedens ad conſequens ex alia, & ut conſequens ad aliud, ita conſequens ad aliud.

A. B. E. C. D. F. SI ratio A ad B ſit illa quæ C ad D; paritérque ratio B ad E eadem ſit ac ratio D ad F, tunc Magnitudines numero pares A. B. E. C. D. F. dicendæ ſunt in ratione ordinata; & dum infertur rationem A ad E eſſe rationem C ad F, dicitur argumentum ex æqualitate, ſiue ex æquo in ratione ordinata, demonſtraturque ab Euclide ad Prop. 22. hujus Elementi.

Def. XIX.

PERTURBATA ratio appellatur, cum poſitis tribus magnitudinibus G. H. L., aliiſque numero æqualibus M. I. K., uti G ad H, ita I ad K, & uti H ad L, ita M ad I.

G. H. L. M. I. K. HOc ſuppoſito, dum infertur, rationem G ad L eſſe illam, quam habet M ad K, dicitur argumentum ex æqualitate, ſiuè ex æquo in ratione perturbata; quod legitimum eſſe ad Propoſ. 23. hujus Elementi demonſtrat Euclides.

Axioma Unicum.

DAtis quibuſcumque tribus magnitudinibus, quarta datur proportionalis.

Prop.

Propoſ. I. Theor. I.

Si quotcumque magnitudines A. B. C. ſint quotcumque magnitudinum numero æqualium D. E. F. ſingulæ ſingularum æque multiplices.

Dico quam multiplex eſt A ipſius D, tam tiplices eſſe omnes A. B. C. omnium D. E. F.

Demonſtratio.

A. B. C.
D. E. F.

Cum magnitudines A. B. C. (*per hyp.*) ſint æque multiplices magnitudinum D. E. F. [*per def. 2. hujus*] quoties D menſurat A, toties E menſurabit B, & F menſurabit C: ſupponamus D bis ſumptam menſurare A; E bis acceptam menſurare B, & F, C: & ideo quoties D menſurat A, toties D.E.F menſuabunt A. B. C. ergo (*per def. 2 hujus*) quam multiplex eſt A ipſius D, tam multiplices erunt omnes A. B. C. omnium D. E. F. q. e. d.

SCHOLIUM.

HÆc veritas in omni ratione, etiam irrationali demonſtrabitur in Propoſ. 12. hujus Libri.

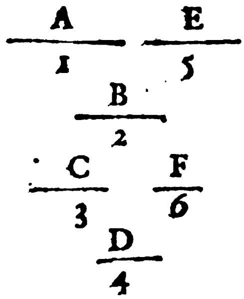
Propoſ. II. Theor. II.

Si prima magnitudo A fuerit tam multiplex ſecundæ B, quam multiplex eſt tertia C, quartæ D: pariterque ſi quinta E æquemultiplex fuerit ſecundæ B, ac ſexta F quartæ D.

Dico compoſitam ex prima A, & quinta E tam eſſe multiplicem ſecundæ B; quam multiplex eſt compoſita ex tertia C, & ſexta F, quartæ D.

De-

Demonſtratio.



CUM [*per hyp.*] A ſit tam multiplex B, quam C ipſius D: erunt in A tot partes æquales ipſi B, quot ſunt in C æquales ipſi D. Pariter quia (*per ſuppoſitum*) E eſt æquemultiplex ipſius B, ac F ipſius D; quot partes ſunt in E æquales ipſi B, tot erunt in F æquales ipſi D. Hoc ſtante ſi partibus numero æqualibus ipſarum A, & C addantur partes numero æquales ipſarum E, & F; tot partes erunt in A, & E æquales ipſi B, quot ſunt in C, & F æquales ipſi D: quare prima A cum quinta E tam multiplices erunt ſecundæ B, quam multiplices ſunt tertia C cum ſexta F, quartæ D. q. e. d.

SCHOLIUM.

HÆc propoſitio univerſaliter de quacumque ratione demonſtrabitur in Propoſ. 24. hujus Libri.

Propoſ. III. Theor. III.

Si prima magnitudo A ſit tam multiplex ſecundæ B, quam multiplex eſt tertia C quartæ D. Pariter ſi E eſt æquemultiplex primæ A, quam F tertiæ C.

Dico quod ex æquo E eſt tam multiplex ſecundæ B, quam multiplex eſt F quartæ D.

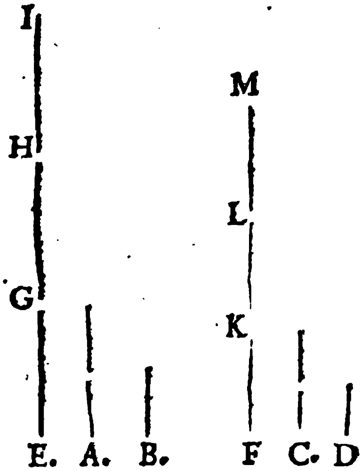
Conſtructio.

MAgnitudo E dividatur in partes EG, GH, HI æquales ipſi A, pariterque F reſolvatur in partes FK, KL, LM æquales ipſi C.

De-

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Demonſtratio.



Quia (*per ſuppoſitum.*) E eſt tam multiplex ipſius A, quam F ipſius C; erunt in E tot partes æquales ipſi A, quot ſunt in F æquales ipſi C. Quia verò A ſupponitur tam multiplex B, quam C ipſius D, erit & EG = ipſi A tam multiplex B, quam multiplex FK = C ipſius D. Idem intelligendum de reliquis partibus GH, HI reſpectivè ad B, & de partibus KL, LM in ordine ad D.

His manentibus erit prima EG tam multiplex ſecundæ B, quam multiplex eſt tertia FK, quartæ D: pariter quinta GH erit tam multiplex ſecundæ B, quam multiplex eſt ſexta KL quartæ D: ergo (*per 2. hujus*) EH compoſita ex prima, & quinta; erit tam multiplex ſecundæ B, quam multiplex eſt FL cumpoſita ex tertia, & ſexta quartæ D. Denuo quia prima EH eſt tam multiplex ſecundæ B [*ex demonſtratis*), quam multiplex eſt tertia FL quartæ D: quinta pariter HI tam multiplex ſecundæ B, quam multiplex eſt ſexta LM quartæ D: erit [*per 2. hujus.*] EI compoſita ex prima, & quirta tam multiplex ſecundæ B, quam multiplex eſt FM compoſita ex tertia, & ſexta, quartæ D: unde ex æquo tam multiplex eſt E ipſius B, quam multiplex eſt F ipſius D. q.e.d.

SCHOLIUM.

Hoc Theorema in Prop. 22. hujus Libri de quacumque ratione demonſtrabitur.

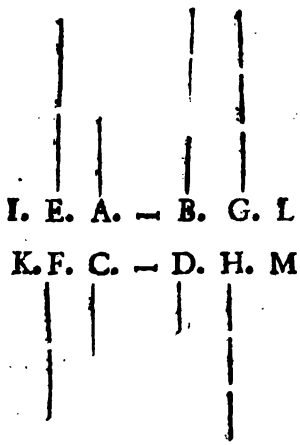
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Propoſ. IV. Theor. IV.

Si A ad B eamdem habuerit rationem, quam habet C ad D, ſintque antecedentium A, & C æqualiter multiplicia E, & F; & conſequentium B, & D alia pariter multiplicia G, & H ſecundum quamcumque multiplicationem.

Dico E ad G eamdem habere rationem, quam habet F ad H.



Conſtructio.

MAgnitudinum E, & F accipiantur pariter multiplicia I, & *K*, atque magnitudinum G, & H aliæ æque multiplicia L, & M ſecundum quamcumque multiplicationem.

Demonſtratio.

QVoniam [*per conſtructionem.*] E eſt tam multiplex ipſius A, quam F ipſius C: itemque I tam multiplex ipſius E, quam *K* ipſius F: ex æquo [*per 3. hujus.*] erit I tam multiplex ipſius A, quam K ipſius C. Eadem ratione demonſtrabitur L tam multiplex ipſius B, quam M ipſius D.

Quia verò (*per ſuppoſitum*) A : B : : C : D
ſequitur (*per 6. def. hujus.*) quod
Quando I ſuperat L, etiam K ſuperat M,
Quando I = L, etiam K = M,
Quando I minor L, etiam *K* minor M.
Inſuper quia (*per conſtructionem.*
I eſt tam multiplex E, quam K multiplex F, &
L eſt tam multiplex G, quam M multiplex H.
erit [*per 6. def. hujus.*]
E : G : : F : H. q. e. d.

Corol-

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Corollarium.

Ex modo dictis demonſtratur inverſa ratio explicata in Def. 12, hujus libri, videlicet.

Si A : B :: C : D. dico invertendo
B : A :: D : C.

Demonſtratio.

Cum E, & F (*per hyp.*) ſint æquemultiplicia primæ A, & tertiæ C, atque G, & H æquemultiplicia ſecundæ B, & quartæ D.

(*per def. 6. hujus*) [

Quando E major eſt quam G, etiam F major quam H,
Quando E = G, etiam F = H,
Quando E minor G, etiam F minor H:

Quare è contrario.

Quando G major eſt quam E, etiam H major, quam F,
Quando G = E, etiam H = F,
Quando G minor E, etiam H minor F.

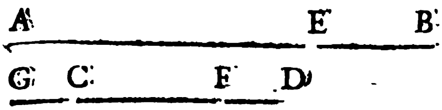
Ergo (*per def. 6. hujus*.) invertendo
B : A :: D : C. q. e. d.

Propoſ. V. Theor. V.

Si magnitudo AB fuerit tam multiplex magnitudinis CD, quam multiplex eſt ablata AE ablatæ CF.

Dico reliquam EB tam eſſe multiplicem reliquæ FD, quam multiplex eſt tota AB totius CD.

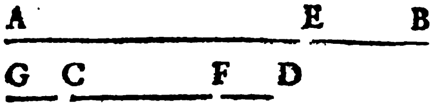
Conſtructio.



Quam multiplex eſt AE ipſius CF, ſit EB ipſius GC.

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Demonſtratio.



Cum (*per conſtructionem.*) AE ſit tam multiplex ipſius CF, quam EB ipſius GC.

(*per pri. hujus.*) erit
AB tam multiplex ipſius GF, quam multiplex eſt AE ipſius CF,
Sed (*per hyp.*)
AB eſt tam multiplex ipſius CD, quam AE ipſius CF:
Igitur AB tam multiplex ipſius GF, quam ipſius CD.
ergo (*per axio. 6.*) GF = CD
communi parte CF ablata, erit
GC = FD: quare
EB erit tam multiplex ipſius FD quam ipſius GC:
Sed (*per hyp.*
EB eſt tam multiplex GC, quam AE multiplex ipſius CF,
hoc eſt, ut tota AB, totius CD: ergo
reliqua EB erit tam multiplex reliquæ FD, quam tota AB totius CD. q. e. d.

SCHOLIUM.

Hoc Theorema de quacumque ratione demonſtrabitur ad Prop. 19. hujus Libri.

Propoſ. VI. Theor. VI.

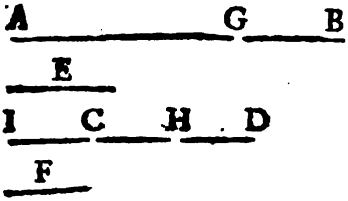
Si magnitudines AB, CD ſint magnitudinum E, F ſingulæ ſingularum æquemultiplices, ſintque detractæ AG, CH earumdem E, F pariter multiplices.

Dico reſidua GB, HD eſſe vel æqualia ipſarum E, & F, vel æqualiter multiplicia.

Con-

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Conſtructio.



Primæ partis, quādo reſiduū GB = E.

Ponatur IC = F.

Demonſtratio primæ partis.

QVia [*per hyp. & per conſtructionem.*] Prima magnitudo AG eſt tam multiplex ſecundæ E, quam multiplex eſt tertia CH quartæ F: pariter quinta GB eſt æqualis ſecundæ E, ſextaque IC æqualis quartæ F.

(*per 2. hujus.*] erit

AB compoſita ex prima, & quinta, tam multiplex ſecundæ E, quam multiplex eſt IH compoſita ex tertia, & ſexta quartæ F.

Sed (*per hyp.*)

CD eſt tam multiplex ipſius F, quam AB ipſius E: ergo IH, & CD æque multiplices ipſius F: quare

[*per axio. 6.*)

IH = CD.

ablata communi parte CH, erit

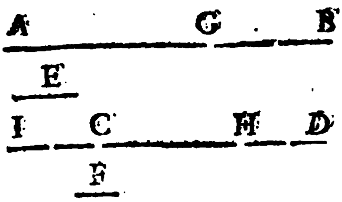
IC = HD.

Cum autem IC (*per conſtructionem.*) ſit æqualis ipſi F erit, & HD æqualis ipſi I. q. e. i. d.

Conſtructio ſecundæ partis, in qua reſiduum GB eſt multiplex ipſius E.

Ponatur IC tam multiplex ipſius F, quam multiplex eſt GB ipſius E.

Demonſtratio ſecundæ partis.



QVemadmodum in prima parte dictum eſt (*per 2. hujus.*) erit AB tam multiplex ipſius E, quam multiplex IH ipſius F,

Sed *per hyp.*)

Quam multiplex eſt AB ipſius E, tam mutiplex CD ipſius F.

ergo IH, & CD, erunt æquemultiplices ipſius F.

ergo (*per axio. 6.*

IH = CD. Dempta communi parte CH; erit.

IC = HD

Sed (*per conſtructionem.*)

GB eſt tam multiplex ipſius E, quam IC ipſius F: ergo

GB tam multiplex ipſius E, quam HD, ipſius F. q. e. 2. d.

SCHOLIUM.

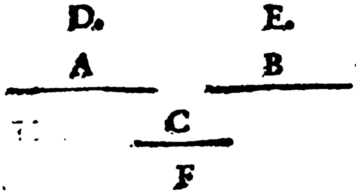
IN omni rationum genere hoc demonſtrabitur ad Propoſ. 24. hujus Libri.

Propoſ. VII. Theor. VII.

Si ſint æquales magnitudines A, & B, & alia quæcumque C.

Dico A, & B eamdem habere rationem ad C: atque viciſſim C eamdem habere rationem ad A, & B.

Conſtructio,



ÆQualium magnitudinum A, & B ponantur æquemultiplicia D, & E; pariterque accipiatur F quomodocumque multiplex ipſius C.

Demonſtratio.

QVoniam (*per hyp.*) A = B, atque (*per conſtructionem.*) D, & E ſunt ipſorum A, & B æquemultiplicia; etiam D = E: quotieſcumque igitur D ſuperat F, etiam E ſuperabit F.

Quando D = F, etiam E = F,

Quando D minor F, etiam E minor F.

Si

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Si ergo ſtatuatur A prima magnitudo; C ſecunda; B tertia, & C quarta, erit verum dicere.

Quando D multiplex primæ A ſuperat F multiplicem ſecundæ C; etiam E multiplex tertiæ B ſuperat F multiplicem quartæ C. &c. Unde (*per 6. def. hujus.*)

A : C :: B : C.

Quare æquales magnitudines A, & B eamdem rationem habebunt ad C. q. e. 1. d.

Quod poſtea viciſſim magnitudo C eamdem rationem habeat ad magnitudines æquales A, & B, ita demonſtratur.

Cum [*per demonſtrata in prima parte.*] æqualia ſint pariter multiplicia *D*, & *E*: ſequitur quod quando F major eſt quam D, eſt etiam F major quam E: quando F = D, etiam F = E, & quando F minor D, etiam F minor E.

Ponatur C prima magnitudo, A ſecunda, C tertia, & *B* quarta. Hoc ſuppoſito, quia pariter multiplicia primæ, & tertiæ comparata ad æquemultiplicia ſecundæ, & quartæ, ſervant ordinem præſcriptum in def. *6.* hujus Libri: erit

C : A :: C : B.

Quare magnitudo C ad æquales A, & B comparata eamdem habet rationem. q. e. 2. d.

Propoſ. VIII. Theor. VIII.

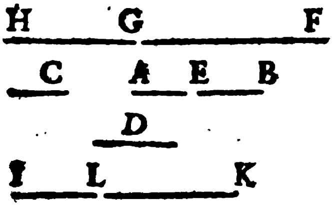
Si ſint inæquales magnitudines AB major, & C minor; aliaque magnitudo D.

1. Dico majorem magnitudinem AB ad D majorem habere rationem, quam minor magnitudo C habeat ad eamdem D.

2. Dico e converſo D ad majorem AB, minorem habere rationem, quam ad minorem C.

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Conſtructio.



Ex majore magnitudine AB auferatur AE = C.

Sumantur ipſarum AE, EB æqualiter multiplicia HG, GF, cum hac lege, videlicet quod GF multiplex ipſius EB major ſit quam D: HG verò multiplex ipſius AE non ſit minor *D*, at ſit æqualis, vel major.

Ulterius accipiatur IK multiplex ipſius D proximè major quam HG: noteturque LK = D.

Demonſtratio.

Quia [*per conſtructionem*] FG, GH ſunt pariter multiplicia ipſarum AE, EB; [*per* 1. *hujus.*] erit HF tam multiplex ipſius AB, quam multiplex eſt HG ipſius AE, ſeu ipſius C.

Cum autem IK (*per conſtructionem.*) ſit multiplex ipſius D proximè major, quam HG, ſi ex IK auferatur LK = D, non erit IL major quam HG, aliter IK multiplex ipſius D non foret proximè major quam HG, at HG ſemper æqualis, vel major quam IL; & hoc quia (*per conſtructionem*) non ſolum IK facta eſt multiplex ipſius D proximè major HG, verum etiam quia HG poſita eſt non minor quam D.

His manentibus, quia FG (*per conſtructionem*) facta eſt major quam *D*, & LK = D: erit FG major LK. Rurſus, quia (*ex demonſtratis.*) HG nequit eſſe minor IL; tota FH major erit quam IK.

Cum igitur FH ſit tam multiplex primæ AB, quam multiplex eſt HG tertiæ C: IK verò æquemultiplex D ſecundæ, & D quartæ, in hujuſmodi æquemultiplicibus contingere poterit, quod ſcilicet,

FH multiplex primæ AB major ſit quam IK multiplice ſecundæ D: HG verò multiplex tertiæ C, non ſit major IK multiplice quartæ *D*.

Quare [*per def.* 8. *hujus.*)

major erit ratio primæ AB ad ſecundam D, quam tertiæ C, ad quartam D. q. e. p. d.

Quo

Quo ad alteram partem hujus prop., videlicet, quod e contrario magnitudo D ad majorem AB minorem habeat rationem, quam ad minorem C, ita demonſtratur.

Accipiatur D tanquam prima magnitudo, C ſecunda, D tertia, & AB quarta.

(*per ſuperius demonſtrata.*)

IK multiplex primæ D major quam HG multiplex ſecundæ C
IK multiplex tertiæ D minor quam FH multiplex quartæ AB:

quare (*per def.* 8. *hujus.*)

Major erit ratio primæ D ad ſecundam C, quam tertiæ *D.* ad quartam AB. q. e. 2. d.

Propoſ. IX. Theor. IX.

Si magnitudines A, & B eamdem rationem habeant ad C.

Dico A = B: & viciſſim.

Dico quod ſi C eamdem habeat rationem ad A, & B. A = B.



A C B

Demonſtratio primæ partis.

SI A non æquat B, erit major, vel minor quam B. Ponatur A major quam B.

Si A major quam B [*per* 8. *hujus.*] major erit ratio A ad C, quam B ad C, contra ſuppoſitum.

Si A minor quam B (*per* 8. *hujus.*) minor erit ratio A ad C, quam B ad C. Pariter contra ſuppoſitum: ergo A = B.

Demonſtratio ſecundæ partis.

SI A ponatur major quam B [*per ſecundam partem* 8. *hujus.*] C ad A majorem, habebit minorem rationem, quam C ad B minorem: quod cum ſit contra ſuppoſitum: erit A = B. q.e.2.d.

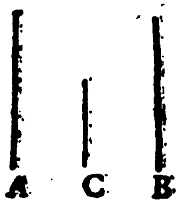
Propoſ. X. Theor. X.

Si A ad C majorem habeat rationem, quam B ad eamdem C.

Dico A majorem eſſe, quam B. Viciſſim

Si C ad B majorem rationem habeat quam C ad A.

Dico B minorem eſſe quam A.



Demonſtratio primæ partis.

SI A non eſt major quam B erit vel æqualis, vel minor; ſi æqualis, [*per 7. hujus.*] ratio A ad C erit eadem ac ratio B ad C: contra ſuppoſitum,

Si A minor quam B [*per 8. hujus.*) A ad C minorem habebit rationem, quam B ad eamdem C: quod eſt pariter contra ſuppoſitum: Quare A major erit quam B. q. e. 1. d.

Demonſtratio ſecundæ partis.

SI B non eſt minor quam A, erit æqualis, vel major. Si æqualis (*per 7. hujus.*) C ad B eamdem habebit rationem quam C ad A: contra ſuppoſitum.

Si major (*per 2. partem 8. hujus.*) C minorem rationem haberet ad B quam ad A, quod pariter eſt ſuppoſito contrarium, quare B = A. q. e. 2. d.

Propoſ. XI. Theor. XI.

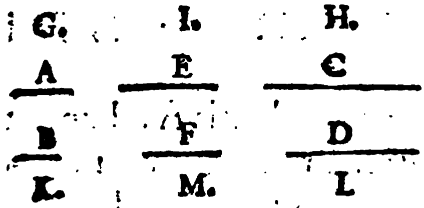
Si ratio A ad B ſit eadem ac ratio E ad F: pariterque ratio C ad D eadem ſit ac ratio E ad F.

Dico rationem A ad B eſſe eamdẽ, ac C ad D.

Con-

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Conſtructio.



ANtecedentium A. E. C. accipiantur æquemultiplicia G. I. H. Pariter aſſumantur conſequentium B. F. D. pariter multiplicia K. M. L. ſecundum quamcumque multiplicationem.

Demonſtratio.

QVIA (*per hyp.*) A : B : : E : F, ſintque G, & I antecedentium A, & E æquemultiplicia, item K, & M pariter multiplicia conſequentium B, & F. (*per def. 6. hujus.*)

Quando G major K, etiam I major M.
Quando G = K, etiam I = M.
Quando G minor K, etiam I minor M.

Eodem proſus modo demonſtrabitur, quod

Quando I major M, etiam H major L,
Quando I = M, etiam H = L
Quando I minor M, etiam H minor L: ergo

Quando G major K, etiam H major L.
Quando G = *K*, etiam H = L
Quando G minor K, etiam H minor L.

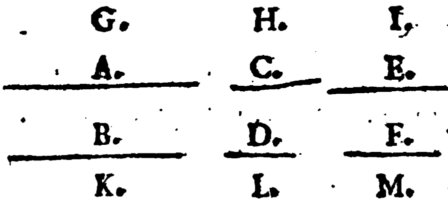
Hoc demonſtrato, ſi A ſit prima magnitudo, B ſecunda, C tertia, & D quarta, æquemultiplicia primæ, & tertiæ comparata ad æque multiplicia ſecundæ, & quartæ ſervant ordinem præſcriptum in def. 6. hujus libri: quare
A: B; : C; D. q. e. d.

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Propoſ. XII. Theor. XII.

Si ratio A ad B ſit ratio C ad D, & ratio C ad D ſit ratio E ad F.

Dico rationem unius antecedentis A ad unum conſequens B, eſſe rationem omnium antecedentium A.C.E. ad omnia conſeq. B. D. F.



Conſtructio.

Antecedentium A. C. E. ponantur æquemultiplicia G. H. I. conſequentium pariter B. D. F. ponatur pariter multiplicia K. L. M.

Demonſtratio.

Quia (*per hyp.*) A : B : : C : D. ſumptaq; ſunt æquemultiplicia primæ A, & tertiæ C., quæ ſunt G, & H; uti pariter facta ſunt æqualiter multiplicia ſecundæ B, & quartæ D, hoc eſt K, & L.

(*per def. 6. hujus.*)
Quando G major K, etiam H major L.
Quando G = K, etiam H = L.
Quando G minor K, etiam H minor L.

Denuo quia [*per hyp.*] C : D : : E : F. idem poterit demonſtrari de æquemultiplicibus primæ, & tertiæ; atque ſecundæ, & quartæ, quæ ſunt H. I., & L. M. nempe quod

Quando H major L, etiam I major M.
Quando H = L, etiam I = M.
Quando H minor L, etiam I minor M.
Quare ex hoc ſequitur,

Quan-

Quando G majus K, etiam G. H. I. majora eſſe K. L. M.
Quando G = K, etiam G. H. I. = K. L. M.
Quando G minus K, etiam G. H. I. minora K. L. M.

Quia vero (*per conſtructionem*) G. H. I. ſunt pariter multiplicia antecedentium A. C. E: (*per 1. hujus*) quam multiplex eſt G ipſius A, tam multiplices erunt G. H. I. ipſorum A. C. E.

Eadem ratione K erit tam multiplex ipſius B, quam multiplices ſunt K. L. M. ipſorum B. D. F.

Hoc ſtante, ſi A ponatur prima magnitudo, B ſecunda, A. C. E. tertia, & B. D. F. quarta: ſequitur,

Primæ A, & tertiæ A. C. E. æquemultiplicia eſſe G, & G.H.I.

Item ſecundæ B, & quartæ B. D. F. pariter multiplicia erunt K, & K. L. M.

Cum autem [*per demonſtrata.*] hæc multiplicia ſervent ordinem propoſitum in def. 6. hujus libri (*per eamdem def.*)

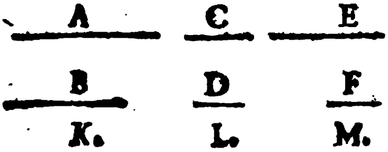
Quemadmodum antecedens A ad conſequens B, ita omnia antecedentia A. C. E. ad omnia conſequentia B. D. F. q. e. d.

Propoſ. XIII. Theor. XIII.

Si A prima magnitudo ad ſecundam B eamdem habuerit rationem, quam tertia C, ad quartam D: pariterque, ſi ratio tertiæ C ad quartam D major ſit quam ratio quintæ E ad ſextam F.

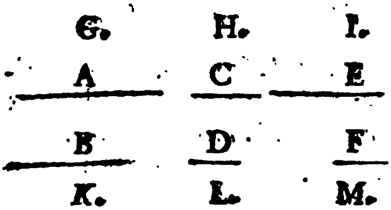
Dico majorem eſſe rationem primæ A ad ſecundam B, quam quintæ E ad ſextam F.

Conſtructio.



Antecedentium A.C.E. accipiantur æquemultiplicia G. H.I. Conſequentium pariter B. D. F. ſumantur pariter multiplicia K. L. M.

Demonſtratio.



Quia (*per hyp.*) A:B :: C: D.: ſequitur [*per def. 6. hujus.*]

Quando G major *K*. etiam H major L.
Quando G = *K*, etiam H = L.
Quando G minor *K*, etiam H minor L.

Rurſus, quia [*per ſuppoſitum*] ratio C ad D major ratione E ad F (*per def. 8. hujus.*) poterit evenire, quod

Quando H ſuperat L, I non ſuperat M: ergo
Quando G ſuperat K cõtingere poteſt quod I nõ ſuperet M.

Quare (*per def. 8. hujus.*)
Major erit ratio primæ A ad ſecundam B, quam quintæ E ad ſextam F. q. e. d.

Propoſ. XIV. Theor. XIV.

Si prima magnitndo A ad ſecundam B eamdem habuerit rationem, quam tertia C, ad quartam D.

1. Dico quando prima A major eſt tertia C, etiam ſecunda B major erit quarta D,

2. Si prima A æquat tertiam C, etiam ſecunda B æquabit quartam D.

3. Si prima A minor eſt tertia C, etiam ſecunda B minor erit quarta D.

Demonſtratio primæ partis.

SI A ſit major quam C, [*per 8. hujus.*) major erit ratio A ad B, quam C ad eamdem B.

ſed



ſed [*per hyp.*]
A : B :: C : D : ergo ratio C ad D major quam C ad B.
Quare (*per 2. partem 8. hujus.*)
Secunda B major quam D. q. e. 1. d.

Demonſtratio ſecundæ partis.

SI A = C (*per 7. hujus.*) ratio A ad B æqualis erit rationi C ad B.
Sed (*per hyp.*)
Ratio A ad B eadem, ac ratio C ad D : ergo C : B :: C : D.
Quare (*per 9. hujus.*)
B ſecunda æqualis quartæ D. q. e. 2. d.

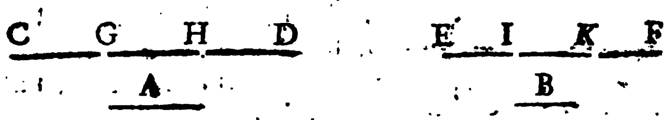
Demonſtratio tertiæ partis.

SI A minor ſit quam C (*per 8. hujus.*)
Major erit ratio C ad B, quam A ad B.
Sed (*per hyp.*)
Ratio A ad B eadem eſt ac C ad D : ergo
Ratio C ad B major, quam ratio C ad D.
Quare (*per 10. hujus.*)
B ſecunda minor, quam quarta D. q. e. 3. d.

Propoſ. XV. Theor. XV.

Si partium A, & B ſint pariter multiplicia CD, EF.

Dico A ad B, uti CD ad EF.



Conſtructio.

CD multiplex ipſius A reſoluatur in partes CG, GH, HD æquales ipſi A.

Pari-

Pariter EF multiplex ipſius B dividatur in partas EI, IK, KF æquales ipſi B.

Demonſtratio.

Cum (*per hyp.*) CD ſit tam multiplex ipſius A, quam EF ipſius B; erunt in CD tot partes æquales ipſi A, quot ſunt in EF æquales ipſi B. Si igitur in CD ſunt tres partes æquales A, tres etiam erunt in EF æquales B.

quare { CG : EI :: A : B.
GH : IK :: A : B.
HD : KF :: A : B.

Ergo (*per 11. hujus.*)
CG : EI :: GH : IK.
GH : IK :: HD : KF.

Igitur [*per 12. hujus.*]

Omnia antecedentia CG, GH, HD ad omnia conſequentia EI, IK, KF, uti CG ad EI: ſed

Quemadmodum CG ad EI, ita A ad B: ergo

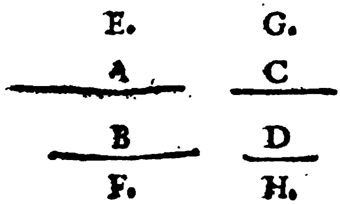
Multiplex CD ad multiplicem EF eſt ut pars A ad partem B. q. e. d.

Propoſ. XVI. Theor. XVI.

Si fuerit A : B :: C : D.

Dico viciſſim, ſiuè alternando A : C :: B : D.

Conſtructio.



Accipiantur non ſolum magnitudinum A, & B æquemultiplicia E, & F, verum etiam magnitudinum C, & D, ſintque G, & H.

De-

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Demonſtratio.

QVia E, & F (*per conſtructionem*) ſunt pariter multiplicia A, & B (*per* 15. *bujus.*) A : B : : E : F.

Ob eamdem rationem C : D : : G : H :

Cum autem [*per hyp.*] A : B : : C : D.

(*per* 11. *bujus.*)

E : F : : G : H.

Quare [*per* 4. *bujus.*]

Quando E ſuperat G, etiam F ſuperat H.
Quando E = G, etiam F = H.
Quando E minor G, etiam F minor H.

Statuatur A prima, C ſecunda, B tertia, D quarta : primæ A, & tertiæ B (*per conſtructionem*) æquemultiplicia ſunt E, & F. Secundæ C, & quartæ D æquemultiplicia ſunt G, & H.

De iſtis pariter multiplicibus demonſtratus eſt ordo ſuperioritatis, æqualitatis, minoritatis præſcriptus in def. 6. hujus Libri : quare (*per def.* 6. *bujus.*)

A : C : : B : D. q. e. d.

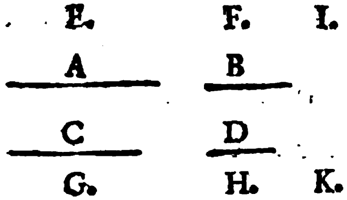
In hoc Theoremate demonſtrata eſt ALTERNA ratio ſuperius in def. 12. hujus libri propoſita.

Propoſ. XVII. Theor. XVII.

Si compoſitæ magnitudines A + B ad B eamdem habeant rationem, quam C + D ad D. Dico dividendo A : B : : C : D.

Com-

Conſtructio.



SUmantur tam magnitudinum A, & B, quam C. & D. pariter multiplicia E. F. G. H. pariter magnitudinum B, & D alia accipiantur æquemultiplicia I, & K ſecundum quamcumque multiplicationem.

Demonſtratio.

QVia E, & F (*per conſtructionem*) ſunt pariter multiplicia A, & B; atque G, & H æquemultiplicia C, & D.

(*per* 1. *hujus.*)

Quam multiplex eſt E + F ipſius A + B, tam multiplex eſt E ipſius A.

Ob eamdem Propoſitionem, quam multiplex G + H ipſius C + D, tam multiplex G ipſius C. Sed

)*per conſtructionem.*)

Quam multiplex E ipſius A, tam multiplex G ipſius C.

Ergo (*per* 11. *hujus.*)

Quam multiplex E + F ipſius A + B, tam multiplex G + H ipſius C + D.

His ſtantibus, quia prima F eſt tam multiplex ſecundæ B, quam tertia H quartæ D: item quinta I tam multiplex ſecundæ B, quam ſexta K, quartæ D.

(*per* 2. *hujus*:) erit

F + I tam multiplex B, quam multiplex H + K ipſius D.

Quia vero [*per hyp.*]

A + B ad B, uti C + D ad D, ſumptaque ſunt pariter multiplicia primæ, & tertiæ E + F, & G + H, atque ſecundæ, & quartæ F + I, & H + K:

Sequitur (*per def.* 6. *hujus.*)

Quando E + F major F + I, etiam G + H major H + K.
Quando E + F = F + I, etiam G + H = H + K.
Quando E + F minor F + I, etiam G + H minor H + K.

Dem-

Demptis communibus F, & H,

Quando E minor I, etiam G minor K,
Quando E = I, etiam G = K.
Quando E major I, etiam G major K.

Cum autem [*per conſtructionem*] primæ A, & tertiæ C ſumpta ſint æquemultiplicia E, & G, pariterque ſecundæ B, & quartæ D æquemultiplicia I, & K, cumque de iſtis æquemultiplicibus fuerit oſtenſum, quod

Quando E ſuperat I, etiam G ſuperat K
Quando E = I, etiam G = K.
Quando E minor I, etiam G minor K.

Sequitur (*per def. 6. hujus.*)
A : B :: C : D.
Si ergo A + B ad B, ita C + D ad D, erit dividendo
A : B :: C : D. q. e. d.

SCHOLIUM.

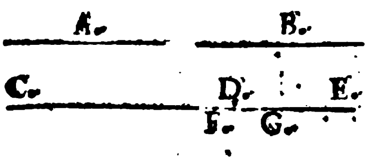
HOc Theoremate demonſtratur illud argumentum, quod Euclides def. 15. hujus, diviſionem rationis appellavit.

Propoſ. XVIII. Theor. XVIII.

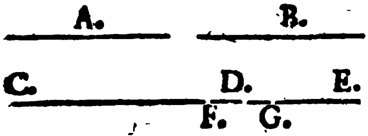
Si in diviſis magnitudinibus ratio A ad B ſit illa C ad D.

Dico per compoſitionem rationis A + B ad B, ſicuti C + D ad D.

Conſtructio, & Demonſtratio.



SI ratio A + B ad B non eſt ratio CD + DE ad DE: fiat A + B : B :: CD + DE : ad quartum, quod majus, vel minus erit



erit quam DE. Sit hæc quartum GE, minus quam DE.

Quia (*per conſtructionem.*) A + B: B :: CD + DE: GE: erit dividendo [*per 17. hujus.*]

A : B :: CG : GE.

Sed [*per hyp.*]

A : B :: CD : DE,

Quare [*per 11. hujus.*]

CD : DE :: CG : CE.

Cum autem CD minor ſit quam CG, etiam [*per 14. hujus.*] DE minor quam GE; quod fieri nequit, quia DE eſt totum, & GE pars. Eadem ratione demonſtrabitur non poſſe GE majorem eſſe quam DE: quare A + B ad B nequit habere eamdem rationem, quam habeat CD + DE ad aliam magnitudinem minorem, vel majorem ipſa DE: ergo

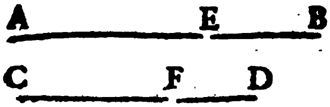
A + B : B :: CD + DE : DE. q. e. d.

Propoſ. XIX. Theor. XIX.

Si totum AB fuerit ad totum CD, quemadmodum ablatum AE ad ablatum CF.

Dico reliquum EB ad reliquum FD eſſe, ut totum AB ad totum CD.

Demonſtratio.



Quia (*per hyp.*) AB : CD :: AE : CF,

permutando (*per 16. hujus*) erit

AB : AE :: CD : CF:

dividendo [*per 17. hujus.*] erit

BE : EA :: DF : FC.

denuo permutando: erit

BE : DF :: EA : FC.

Sed (*per hyp.*)

EA : FC :: AB : CD.

ergo [*per 11. hujus.*]

BE : DF :: AB : CD. q. e. d.

Propoſ.

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Propoſ. XX. Theor. XX.

Si fuerint tres magnitudines A. B. C. & aliæ ipſis numero æquales D. E. F; ſintque ut prima A. ad ſecundam B. ex una parte, ita prima D. ad ſecundam E ex alia parte: pariterque, ut ſecunda B ad tertiam C, ita ſecunda E ad tertiam F.

1. Dico ſi prima A major fuerit tertia C, & prima D. major tertia F.

2. Si prima A = tertiæ C, & prima D = tertiæ F,

3. Si prima A minor tertia C, & prima D minor tertia F.

Demonſtratio primæ partis.



Cum (*per hyp.*) A ſit major quam C,
(*per* 8. *hujus*.)
Major erit ratio A ad B, quam C ad B.
Sed (*per hyp.*)
A : B : : D : E : ergo
(*per* 13. *hujus*.)
Major ratio *D* ad E, quam C. ad B.
Sed in vertendo (*per corol.* 4. *hujus*.)
C : B : : F : E.
ergo [*per* 13. *hujus*.]
Major ratio D, ad E, quam F ad E:
Quare (*per* 10. *hujus*.)
D major quam F. q. e. 1. d.

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Demonſtratio ſecundæ partis.



SI A = C (*per 7. hujus.*) erit A : B : : C : B.
Sed (*per hyp.*)
A : B : : D : E, & invertendo
C : B : : F : E. quare (*per 11. hujus.*)
D : E : : F : E. quare (*per 9. hujus.*)
D = F. q. e. 2. d.

Demonſtratio tertiæ partis.



SI A minor ſit quam C : [*per 8. hujus.*]
Minor erit ratio A ad B, quam C ad B,
Sed (*per hyp.*)
A : B : : D : E. & invertendo
C : B : : F : E.
Quare (*per 13. hujus.*)
Minor erit ratio D ad E, quam F ad E,
Ergo [*per 10. hujus.*]
D minor quam F. q. e. 3. d.

Propoſ. XXI. Theor. XXI.

Si fuerint tres magnitudines A. B. C. ex una parte, & aliæ tres D. E. F. ex alia parte, quæ binæ in eadem ratione perturbata ſumptæ ſint: hoc eſt, ut A ad B, ita E ad F, & ut B ad C, ita D ad E.

1. Dico ſi prima A ſit major tertia C, etiam primam D majorem eſſe tertia F.
2. Dico ſi A = C, etiam D = F.
3. Dico ſi A minor C, etiam D minorem F.

De-

Demonſtratio primæ partis.



SI A major eſt C (*per* 8. *hujus*.)
Major ratio A ad B, quam C ad *B*.
Sed (*per hyp.*)
A: B:: E: F, & invertendo.
C: B:: E: D.
Ergo (*per* 13. *hujus*.]
Major ratio *E* ad *F*, quam E ad B.
Quare [*per* 10. *hujus*.]
D. major quam *F*. q. e. 1. d.

Demonſtratio ſecundæ partis.



SI A = C (*per* 7. *hujus*.) A: B:: C: B.
Sed (*per hyp.*]
A: B:: E: F., & invertendo C: B:: E: D.
Ergo [*per* 11. *hujus*.]
E: F:: E: D.
Quare [*per* 9. *hujus*.)
D = F. q. e. 2. d.

Demonſtratio tertiæ partis.



SI A minor quam C (*per* 8. *hujus*.)
Minor ratio A ad *B*, quam C ad *B*.
Sed (*per hyp.*)
A: *B*:: E: F, atq; invertendo C: *B*:: E: D,
Ergo [*per* 13. *hujus*.]
Minor ratio E ad F, quam E ad D,
Unde (*per* 8. *hujus*.]
D. minor quam F. q. e. 3. d.

Propoſ. XXII. Theor. XXII.

Si fuerint quotcumque magnitudines A.B.C, & aliæ ipſis numero æquales D. E, F. quæ binæ in eadem ratione ordinata ſumantur.

Dico ex æqualitate A : C : : D : F.

Conſtructio.



A. B. C. D. E. F.

G. I. L. H.K. M.

Magnitudinum A, & D ſint pariter multiplicia G, & H.

Magnitudinum B, & E ſint æqualiter multiplicia I, & K.

Magnitudinum C, & F ſint æquemultiplicia L, & M.

Demonſtratio.

Quia [*per hyp.*] A : B : : D : E, ſumptaque ſint (*per conſtructionem*) antecedentium G, & H pariter multiplicia; atque I, & K æquemultiplicia conſequentium

erit (*per 4. hujus.*)

G : I : : H : K.

Pariter quia [*per hyp.*] B : C : : E : F, ſintque accepta antecedentium I, & K pariter multiplicia, & L, & M pariter conſequentium multiplicia.

Erit (*per 4. hujus.*)

I : L : : K : M.

Sunt ergo tres magnitudines G. I. L, & aliæ ipſis numero æquales H. K. M, quæ binæ, & binæ in eadem ordinata ratione ſumptæ ſunt.

Quare [*per 20. hujus.*]

Quando G ſuperat L, etiam H ſuperat M.
Quando G = L, etiam H = M.
Quando G minor L, etiam H minor M.

Cum

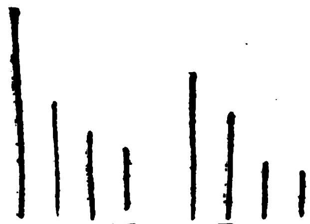
Cum autem G, & H ſint æquemultiplicia antecedentium A, & D: L, & M verò æquemultiplicia conſequentium C, & F. Erit (*per def. 6. hujus.*)

A : C :: D : F. q. e. d.

Si vero ex una parte fuerint magnitudines plures tribus A. B. C. N, & ex altera numero pares D. E. F. O, quæ in eadem ratione ſumptæ ſint,

Dico A : N :: D : O.

Conſtructio.



A. B. C. N. D. E. F. O.

G. I. L. P. H. K. M. Q.

Magnitudinum N, & O accipiantur pariter multiplicia P, & Q.

Demonſtratio.

Ex ſuperius demonſtratis in tribus magnitudinibus A. B. C: D. E. F.

A : C :: D : F.

Quare in tribus magnitudinibus A. C. N, & alijs numero æqualibus D. F. O. (*per hanc prop.*) A : N :: D : O.

Si ergo plures tribus ſint magnitudines, & aliæ ipſis numero æquales: erit A ad N, uti D ad O. q. e. d.

Propoſ. XXIII. Theor. XXIII.

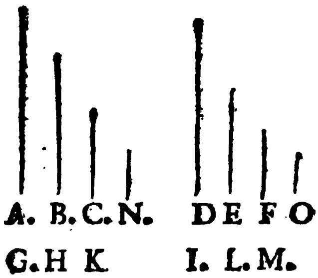
Si fuerint quotcumque magnitudines A. B. C., aliæque numero æquales D. E. F., quæ binæ, & binæ ſint in eadem ratione perturbata, videlicet ut A ad B, ita E ad F, & ut B ad C, ita D ad E.

Dico ex æqualitate A : C :: D : F.

Conſtructio.

MAgnitudinum A.B,& D ſumantur æquemultiplicia G.H.I. Magnitudinum vero C. E, & F. æqualiter multiplicia ſint K. L. M.

Demonſtratio,



QVia (*per conſtructionem*) G, & H ſunt pariter multiplicia A, & *B*, pariterque L, & M æquemultiplicia E, & *F*.

(*per 15. hujus.*) erit

Ut A ad *B*, ita G ad H, & ut E ad F, ita L ad M.

Sed (*per hyp.*)

A : *B* : : E : F,

Ergo (*per 11. hujus.*)

G : H : : L : M.

Ulterius quia [*per hyp.*] *B* : C : : D : E ; atque (*per conſtructionem.*] antecedentium *B*, & D poſita ſunt æqualiter multiplicia H, & I : pariterque conſequentium C, & E facta ſunt pariter multiplicia K : & L (*per 4. hujus.*)

H : K : : I : L.

His demonſtratis, ex una parte erunt tres magnitudines G. H. K, & aliæ numero æquales I. L. M ex alia, quæ binæ, & binæ in eadem ſunt ratione perturbata ; videlicet,

G : H : : L : M : : atque H : K : : I : L.

Sequitur [*per 21. hujus.*]

Quando G ſuperat K, etiam I ſuperat M.
Quando G = K, etiam I = M.
Quando G minor K, etiam I minor M.

Cum autem [*per conſtructionem.*) primæ A, & tertiæ D pariter multiplicia ſint G, & I : ſecundæ vero C, & quartæ F æque multiplicia ſint K, & M.

Ex æquo ſequitur (*per def. 6. hujus.*)

A : C : : D : F. q. e d.

Si

Si poſtea ſint ex una parte quatuor magnitudines A. B. C. N. ex alia vero numero pares D. E. F. O, ut in præcedente propoſitione demonſtrabitur.

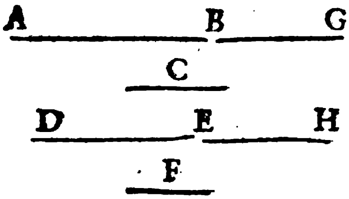
Ex æquo A : N :: D : O.

Propoſ. XXIV. Theor. XXIV.

Si prima magnitudo AB ad ſecundam C ſe habuerit, ut tertia DE ad quartam F : & quinta BG ad ſecundam C eam rationem habeat, quam ſexta EH ad quartam F.

Dico primam AB compoſitam cum quinta BG, eamdem habere rationem ad ſecundam C, ac habeat tertia DE compoſita cum ſexta EH ad quartam F.

Demonſtratio.



CUm (*per hyp.*) BG : C :: EH : F. erit

Convertendo (*per corol. 4. hujus.*)

C : BG :: F : EH.

Quoniam igitur (*per hyp.*)

AB : C :: DE : F.

Ex æqualitate [*per* 22. *hujus.*] erit

AB : BG :: DE : EH.

Componendo (*per* 18. *hujus.*) erit

AG : BG :: DH : EH.

Cum autem (*per hyp.*) BG : C :: EH : F.

Ex æquo (*per* 22. *hujus.*)

AG : C :: DH : F.

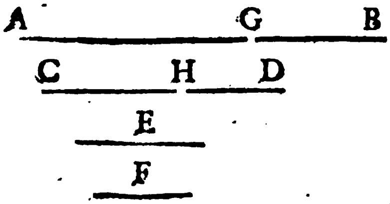
Quare AG compoſita ex prima AB, & quinta BG, erit ad ſecundam C, uti DH compoſita ex tertia DE, & ſexta EH ad quartam F. q. e. d.

Propoſ. XXV. Theor. XXV.

Si AB: CD:: E:F.

Dico maximam AB, cum minima F majores eſſe reliquis duabus CD, & E.

Conſtructio.



Ex maxima AB auferatur AG = E: pariter ex CD, dematur CH = F.

Demonſtratio.

Quia [*per conſtructionem.*] AG = E, & CH = F; erit

AG: CH:: E: F,

Sed [*per hyp.*]

E: F:: AB: CD.

Ergo [*per* 11. *hujus.*]

AG: CH:: AB: CD.

Cum igitur ſit totum AB ad totum CD, ut ablatum AG, ad ablatum CH: (*per* 19. *hujus.*) erit

Reſiduum GB ad reſiduum HD, ut totum AB ad totum CD.

Hoc eſt AB: CD:: GB: HD.

Quia vero AB prima ſupponitur omnium maxima, major erit quam CD ſecunda; quare (*per* 14. *hujus.*) & tertia GB major quarta HD.

Cum autem (*per hyp.*] ſit AG = E ſi, æquales addantur CH, & F: erit

AG + F = CH + E.

Denuo ſi ipſi AG + F addatur major GB, ipſique CH + E addatur minor HD; erit

AG + F + GB major CH + E + HD.

Quare AB compoſita ex AG, & GB + F, major erit quam CD compoſita ex CH, & HD + E: ideoque maxima AB cum minima F ſuperabit intermedias CD, & E. q. e. d.

No-

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Nota. Quando quantitates proportionales forent æquales: Nulla eſſet in eis maxima, aut minima.

SCHOLIUM.

Hæc eſt ultima propoſ. Lib. V. Elementorum Euclidis. Campanus, & alij ſequentes propoſitiones adiunxerunt, quibus ſepiſſime utuntur Scriptores, uti videre licet apud Archimedem, Apollonium, & alios magni nominis Scriptores, eaſque, ut Euclidis, recipiunt; quare, ut mos eſt noſtris temporibus, oportunum duxi hujuſmodi propoſitiones quidem Elementares, in hoc loco demonſtrare.

Propoſ. XXVI. Theor. XXVI.

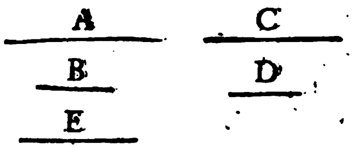
Si A prima ad B ſecundam majorem habuerit rationem, quam C tertia ad D quartam.

Dico convertendo B ad A minorem habere rationem, quam D ad C.

Conſtructio.

Ponatur E ad B, ut C ad D.

Demonſtratio.



Quia (*per hyp.*) ratio A ad B maior, quam ratio C ad D: ratio autem C ad D (*per conſtructionem:*) eadem ac E ad B:

Quare

Ratio A ad B major ratione E ad B:

Ergo (*per* 10. *hujus.*)

A major quam E.

Unde [*per* 8. *hujus.*)

Minor ratio B ad A, quam B ad E:

Sed invertendo [*per corol.* 4. *hujus.*)

B : E :: D : C : ergo

Ratio *B* ad A minor, quam D ad C. q. e. d.

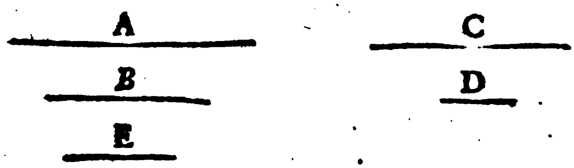
Propoſ. XXVII. Theor. XXVII.

Si A prima ad *B* ſecundam majorem habuerit rationem, quam tertia C ad quartam D. Dico viciſſim, ſiue alternando, primam A ad tertiam C majorem habere rationem quam ſecunda *B* ad quartam D.

Conſtructio.

Ponatur E ad *B*, ut C ad D.

Demonſtratio.



QVia ratio A ad *B* [*per hyp.*] major eſt ratione C ad D, ratio vero C ad D (*per conſtructionem*) æqualis rationi E ad *B*,

Erit [*per 13. bujus.*)

Ratio A ad *B* major ratione E ad *B*,

Quare [*per 10. bujus.*)

A major, quam E.

Cum igitur A ſit major quam E

(*per 8. bujus.*)

Major ratio A ad C, quam E ad eamdem C.

Sed (*per 16. bujus.*) alternando

E : C : : *B* : D.

Ergo alternando.

Major ratio A ad C, quam *B* ad D. q. e. d.

Propoſ.

Propoſ. XXVIII. *Theor.* XXVIII.

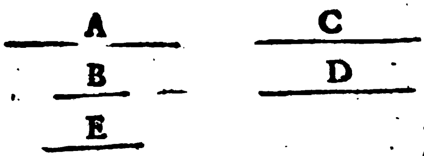
Si prima magnitudo A ad ſecundam B majorem rationem habeat, quam tertia C ad quartam D.

Dico componendo majorem rationem habere A + B ad B, quam C + D ad D.

Conſtructio.

Ponatur uti C ad D, ita E ad B.

Demonſtratio.



QVia (*per hyp.*) major ratio A ad B, quam C ad D;

Ratio autem C ad D [*per conſtructionem.*] æqualis rationi E ad B:

[*per* 13. *hujus.*] erit

Major ratio A ad B, quam E ad B: ergo

(*per* 10. *hujus.*)

A major quam E.

Hoc ſtante A + B major quam E + B, quare

(*per* 8. *hujus*))

Major ratio A + B ad B, quam E + B ad B:

Sed [*per* 18. *hujus.*] componendo,

E + B ad *B*, ita C + D ad D:

Ergo pariter componendo,

Major ratio A + *B* ad *B*, quam C + D ad D, q. e. d.

Propoſ. XXIX. Theor. XXIX.

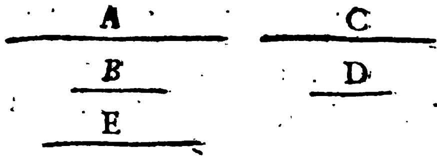
Si compoſita ex A prima, & B, ſecunda ad B ſecundam, majorem habuerit rationem, quam compoſita ex C tertia, & ex D quarta, ad D quartam.

Dico dividendo A ad B majorem habere rationem, quam C ad D.

Conſtructio.

Ponatur ut C + D, ad D, ita E + *B* ad *B*.

Demonſtratio.



QVia [*per hyp.*) major eſt ratio *A* + *B*, ad *B*, quam C + D ad D;

Ratio autem C + D ad D (*per conſtructionem.*) eadem ac E + *B*, ad *B*:

Erit [*per* 16. *hujus.*]

Major ratio *A* + *B* ad *B*, quam E + *B* ad *B*: ergo *A* + *B* major quam *EB*: Dempta communi *B*; erit

A major quam E.

Igitur dividendo

Major ratio *A* ad *B*, quam E ad *B*.

Cum autem dividendo (*per* 17. *hujus.*) ſit

E : *B* :: C : D: ſequitur

Rationem A ad *B* majorem eſſe ratione C ad D. q. e. d.

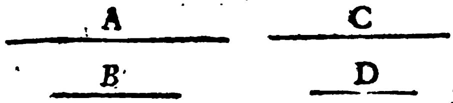
Propoſ. XXX. *Theor.* XXX.

Si compoſita A + B ad B majorem habuerit rationem, quam compoſita C + D ad D.

Dico

Dico per converſionem rationis B ad A minorem habere rationem quam D ad C.

Demonſtratio.



CUm (*per hyp.*) major ſit ratio A + *B* ad *B*, quam C + D ad D: Dividendo (*per* 29. *hujus*) erit

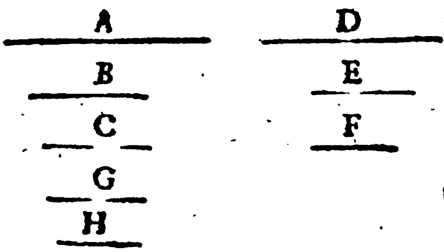
Major ratio A ad *B*, quam C ad D.
Convertendo [*per* 26. *hujus.*] erit
Minor ratio *B* ad A, quam D ad C.
Igitur componendo [*per* 28. *hujus.*]
Minor ratio A + *B* ad A, quam C + D ad C. q.e.d.

Propoſ. XXXI. *Theor.* XXXI.

Si fuerint tres magnitudines A. B. C. ex una parte, aliæque tres D. E. F. ex alia parte, ſitque major ratio primæ A ad ſecundam B, quam primæ D ad ſecundam E: pariterque major ſit ratio ſecundæ B, ad tertiam C, quam ſecundæ E, ad tertiam F.

Dico ex æquo majorem eſſe rationem primæ A ad tertiam C, quam primæ D ad tertiam F.

Conſtructio.



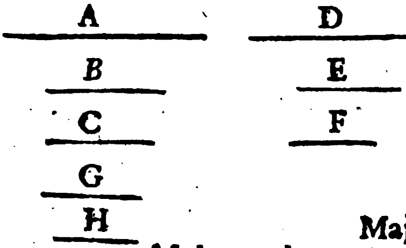
POnatur G ad C uti E ad F, atque H ad G, uti D ad E.

Demonſtratio.

QUia (*per hyp.*) major eſt ra-

ratio *B* ad C, quam E ad F;

Ratio autem E ad F (*per conſtructionem*) eadem ac ratio G ad C.



[*per* 13. *bujus.*] erit

Major ratio B ad, C quam G ad C;

Quare (*per* 10. *bujus.*)

B major quam G:

Ergo [*per* 8. *bujus.*)

Major ratio A ad G, quam A ad B:

Sed (*per hyp.*)

Major ratio A ad B, quam D ad E: ergo

Multo major ratio A ad G, quam D ad E.

Pariter quia [*per conſtructionem.*]

D: E:: H: G: erit etiam

Major ratio A ad G, quam H ad G:

Ergo [*per* 10. *bujus.*)

A major quam H:

Unde [*per* 8. *bujus.*]

Major ratio A ad C, quam H ad C:

Sed ex æquo [*per* 22. *bujus.*]

H: C:: D: F:

Ergo ex æquo

Major ratio primæ A ad tertiam C, quam primæ D ad tertiam F. q. e. d.

Propoſ. XXXII. Theor. XXXII.

Si ex una parte fuerint tres magnitudines A. B. C., treſque D. E. F ex alia; ſit autem major ratio primæ A ad ſecundam B, quam ſecundæ E ad tertiam F: pariterque major ſit ratio ſecundæ *B* ad tertiam C, quam primæ D ad ſecundam E.

Dico, ex æqualitate perturbata majorem rationem habere primam A ad tertiam C, quam primam D ad tertiam F.

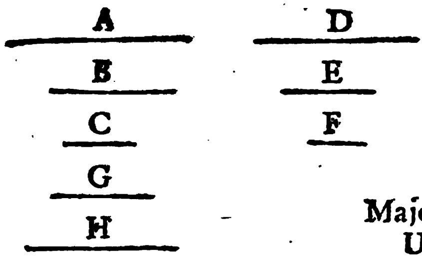
Con-

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Conſtructio.

Conſtituatur G ad C uti D ad E, atque H ad G, uti E ad F.

Demonſtratio.



QVia (*per hyp.*) ratio B ad C major quam ratio D ad E; ratio autem D ad E (*per conſtructionem.*) æqualis rationi G ad C, (*per 13. hujus*) erit
Major ratio B ad C, quam G ad C.
Unde (*per 10. hujus.*)
B major quam G:
Igitur [*per 8. hujus.*]
Major ratio A ad G, quam A ad B.
Sed (*per hyp.*)
Major ratio A ad B, quam E ad F: ergo
Multo major ratio A ad G, quam E ad F.
Quia vero (*per conſtructionem.*)
H : G :: E : F,
Major erit ratio A ad G, quam H ad G:
ideoque (*per 10. hujus.*]
A major quam H.
Quamobrem [*per 8. hujus.*]
Major ratio A ad C, quam H ad C;
Sed ex æqualitate (*per 23. hujus.*]
H : C :: D : F.
ergo ex æquo
Major ratio A ad C, quam D ad F. q. e. d.

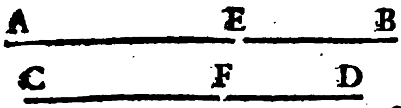
Propoſ. XXXIII. *Theor.* XXXIII.

Si fuerit major ratio totius AB ad totum CD, quam ablati AE ad ablatum CF.

Di-

Dico reliquum EB ad reliquum FD, majorem habere rationem quam totum AB ad totum CD.

Demonſtratio.



Cum (*per hyp.*) major ſit ratio AB ad CD, quam AE ad CF.

Permutando (*per 27. hujus.*) erit

Major ratio AB ad AE, quam CD ad CF:

Quare per converſionem rationis (*per 30. hujus.*)

Minor ratio AB ad EB, quam CD ad FD,

Ergo permutando (*per 27. hujus.*)

Minor ratio AB, ad CD, quam EB, ad FD:

Ideoque

Major ratio EB, ad FD, quam AB, ad CD. q. e. d.

Propoſ. XXXIV. *Theor.* XXXIV.

Si fuerint quotcumque magnitudines A. B. C. ex una parte, totidemque D. E. F. ex alia parte; ſitque major ratio primæ A ad primam D, quam ſecundæ B, ad ſecundam E, & tertiæ C, ad tertiam F, ſicque deinceps, ſi plures fuerint magnitudines.

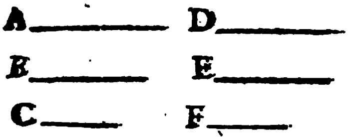
1 Dico omnes magnitudines A. B. C. ad omnes D. E F. majorem habere rationem, quam B. C relicta prima A, ad E. F relicta prima D.

2. Dico omnes magnitudines A. B. C. ad omnes D. E. F., minorem habere rationem, quam habeat prima A ad primam D.

3. Di-

3. Dico omnes magnitudines A. B. C. ad omnes D. E. F. majorem habere rationem, quam ultima C ad ultimam F.

Demonſtratio primæ partis in tribus magnitudinibus.



QVia [*per hyp.*) major eſt ratio A ad D, quam *B* ad E.

Permutando [*per 27. hujus.*]

Major ratio A ad B, quam *D* ad E.

Componendo [*per 28. hujus.*]

Major ratio *A* + *B* ad *B*, quam D + E, ad E.

Iterum permutando [*per 27. hujus.*)

Major ratio *A* + *B* ad D + E, quam *B* ad E.

Cum igitur totum *A* + *B* ad totum D + E majorem habeat rationem, quam ablatum *B* ad ablatum E:

(*per 33. hujus*] erit.

Major ratio reliquæ A ad reliquum D, quam totius A + *B* ad totum D + E.

Eodem modo demonſtrabitur,

Majorem habere ratione *B* ad *E*, quam habeat *B* + C ad *E* + F: quare multo major ratio A ad D, quam *B* + C, ad *E* + F:

Ergo permutando {*per 27. hujus.*)

Major ratio A ad *B* + C, quam D ad *E* + F.

Igitur componendo [*per 28. hujus.*].

Major ratio A + *B* + C ad *B* + C, quam D + *E* + F ad *E* + F.

Rurſus permutando [*per* 27. hujus.]

Major ratio A + *B* + C ad D + E + F, quam *B* + C ad *E* + F.

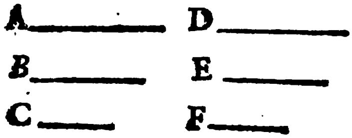
q. e. i. d.

Demonſtratio ſecundæ partis.

CUm in prima parte hujus propoſitionis fuerit demonſtratum majorem rationem habere A + *B* + C ad D + *E* + F, quam *B* + C ad *E* + F.

Si *B* + C, & *E* + F intelligantur tamquam ablata [*per 33. hujus*]

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bujus] major erit ratio reſidui A ad reſiduum D, quam A + B + C ad D + E + F. q. e. 2. d.

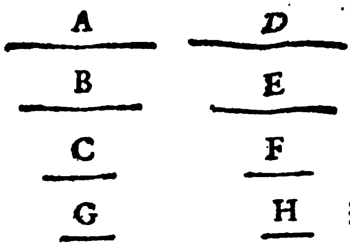
Demonſtratio tertiæ partis.

Quia [*per hyp.*] major ratio B ad E, quam C ad F.
Permutando (*per 27. hujus.*) erit
Major ratio B ad C, quam E ad F.
Componendo (*per 28. hujus.*)
Major ratio B + C ad C, quam E + F ad F,
Denuo permutando
Major ratio B + C ad E + F, quam C ad F:
Sed ex demonſtratis in prima parte
Major ratio A + B + C ad D + E + F, quam B + C ad E + F;
ergo erit etiam
Major ratio A + B + C ad D + E + F, quam C ad F. q. e. 3. d.

Demonſtratio ſi quatuor, vel plures ſint magnitudines ex utraque parte.

SI ex una parte quatuor ſint magnitudines A. B. C. G, & quatuor ex alia D. E. F. H.

Sitque major ratio { A ad D, quam B ad E; B ad E, quam C ad F; C ad F, quam G ad H. } idem demoemn-ſtrabitur



1. Cum in tribus tantum magnitudinibus B. C. G, aliiſque numero æqualibus E. F. H in ſecunda parte demonſtrata ſit,
Major ratio B ad E, quam B + C + G ad E + F + H;
Multo major erit ratio A ad D, quam B + C + G ad E + F + H,
Quare permutando (*per 27. hujus.*]
Major ratio A ad B + C + G, quam D. ad E + F + H:

Igitur

Igitur componendo [*per* 28. *hujus*.]

Major ratio A + B,C,G,ad B, C, G quam D + E,FH, ad E,FH,

ergo permutando [*per* 27. *hujus*.]

Major ratio A + B,C, G ad D + E, F H quam B, C G,ad E,FH. q. e. d.

2. Cum ex modo demonſtratis.

Major ſit ratio omnium A, B, C, G, ad omnes D, *E*, F, H, quam B, C, G ad *E*, F, H.

Erit [*per* 33. *hujus*.]

Major ratio reſidui A ad reſiduum D, quam totius A, B, C, G ad totum D, *E*, F, H. q. e.d.

3. Quia in tribus tantum magnitudinibus, non ſolum demonſtrata eſt,

Major ratio B, C, G, ad *E*, F, H, quam G, ad H, verum etiam

Major ratio A, *B*, C, G ad D,*E*, F, H, quam *B*CG, ad *E*FH.

Multo major erit ratio A*B*CG ad D,*E*,F,H,quam G ad H. q.e.d.

Datis quinque, ſex, vel ſeptem magnitudinibus colligi poterit id, quod de tribus, & quatuor magnitudinibus fuit demonſtratum.

Propoſ. XXXV. *Theor.* XXXV.

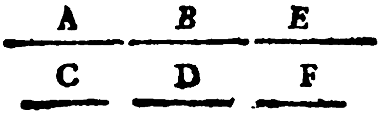
Si ratio A ad B ſit eadem ac ratio C ad D.

Dico rationem duplicatam A ad B æqualem eſſe rationi duplicatæ C ad D, & viciſſim.

Si duplicata ratio A ad B eadem eſt, ac duplicata C ad D.

Dico dimidiatam, ſeu ſubduplicatam rationem eſſe æqualem.

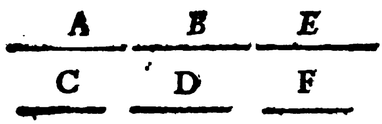
Conſtructio.



Ut A ad B ita ſit B ad E, & ut C ad D, ita D ad F.

P De-

Demonſtratio primæ Partis.



QVia (*per hyp.*) A ad *B*, ita C ad D; ſunt autem (*per conſtructionem.*) ut A ad *B*, ita *B* ad *E*, & ut C ad D, ita D ad F.

Quare [*per* 11. *hujus.*)
Ut A ad *B*, ita C ad D, & ut B ad *E*, ita D ad F;
Igitur ex æquo [*per* 22. *hujus.*]
Ut A ad *E*, ita C ad F,
Sed (*per def.* 10. *hujus.*)

Ratio A ad *E* eſt duplicata A ad *B*, & ratio C ad F duplicata C ad D; ſi ergo ratio A ad *B* ſit ratio C ad D, etiam duplicatæ A ad *E*, & C ad F eædem erunt. q. e. 1. d.

Demonſtratio ſecundæ partis.

RAtio A ad *B* bis ſumpta [*per def.* 10. *hujus.*] facit *E*, atque ratio C ad D pariter bis accepta facit F; ſed [*per hyp.*] ratio A ad *E*, eadem, ac ratio C ad F: ergo etiam ratio A ad *B* ſemel ſumpta æqualis erit rationi C ad D ſemel ſumptæ; quare ſi eædem ſunt rationes duplicatæ, etiam dimidiatæ eædem erunt. q. e. 2. d.

Elementi Quinti Finis.

LIBER

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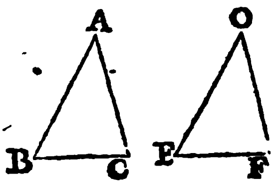
LIBER VI.

EUCLIDIS ELEMENTUM SEXTUM.

DEFINITIONES.

Definitio I.

SIMILES dicuntur illæ figuræ rectilineæ, quæ ſingulos angulos ſingulis angulis æquales habent, & latera æquales angulos comprehendentia proportionalia.



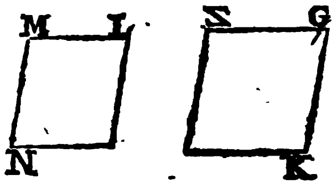
SI duo triangula ABC, OEF (idem de reliquis figuris rectilineis intelligendum) habeant angulum A = ang O: ang. B = ang. E: ang. C = ang. F: & inſuper

Si AB: BC: : OE: EF.
Si BC: CA: : EF: FO.
Si CA: AB: : FO: OE.

Dicta triangula dicuntur SIMILIA.

Def. II.

RECIPROCÆ figuræ dicuntur, cum in utraque figura antecedentes, & conſequentes rationum termini fuerint.



SI in figuris LN, ZK, latera ZG, GK, lateribus LM, MN fuerint taliter proportionalia, ut in utraque figura ſint antecedentes, & conſequentes rationum, hoc eſt.

GZ : LM : : MN : GK.

Rationis GZ ad LM antecedens eſt in figura ZK, & conſequens in figura LN: rationis verò MN ad GK antecedens eſt in figura LN, & conſequens in figura ZK; quamobrem figuræ ZK, LN RECIPROCÆ appellantur.

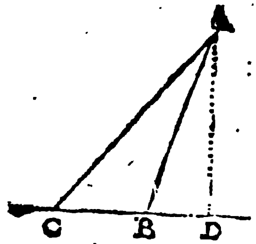
Def. III.

EXTREMA, & MEDIA ratione recta linea terminata *AB* ſecta dicitur, cum taliter ſecta fuerit, ut tota *AB* ſit ad majus ſegmentum *AC*, ut *AC* ad minus ſegmentum *CB*.



Defin. IV.

ALTITUDO cujuſcumque figuræ dicitur perpendicularis a vertice ad baſim ducta.



SI figuræ ABC vertex ſtatuatur punctum A; perpendicularis ab A ſupra baſim CB ducta, hoc eſt AD; dicitur figuræ ABC ALTITUDO.

Def. V.

RATIO ex RATIONIBUS COMPONI dicitur, cum rationum quantitates inter ſe multiplicatæ aliquam effecerint rationem.

AD habendam integram hujus Definitionis notitiam eſt animadvertendum, Denominatorem cujuſcumque rationis oſtendere quantitatem rationis cadentem inter antecedens, & conſequens, manifeſtando quanta ſit magnitudo antecedentis ad conſequens comparata; ſi enim talis denominator ſit quatuor, ſignum eſt quod antecedens quater continet conſequens; ſi vero denominator rationis ſit $\frac{1}{4}$; indicium erit, antecedens exiſtere quartam partem conſequentis.

Hoc ſtante Euclides in poſita definitione dicit, quod ratio ex rationibus compoſita dicitur, quando rationum denominatores inter ſe multiplicati, aliquam efficiunt rationem.

A. B. C. D.

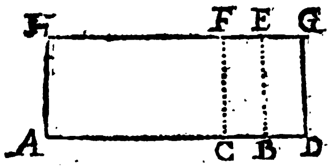
Datis igitur rationibus 12 ad 3, & 4 ad 2, ut talium rationum rationem componamus, in primis oportet accipere rationum denominatores, qui ſunt 4, & 2: ſecundo fiat multiplicatio 4 in 2, productum erit 8. denominator rationis compoſitæ ex rationibus 12 ad 3, & 4 ad 2. hoc eſt ratio 8 ad 1.

Circa hanc compoſitam rationem eſt obſervandum, quod Euclides rationes illas, ex quibus reſultat ratio compoſita, QVANTITATES appellavit, dum alij INDICES, & EXPONENTES nominant, alii vero QUOTIENTES dicunt, & hoc quia indicant, exponunt, atque manifeſtant quoties antecedens rationis contineat ſuum conſequens.

Verum dimiſſa hac terminorum varietate, circa rationem ex rationibus compoſitam, ſat erit intelligere, quod univerſaliter loquendo, quotieſcumque duæ, vel plures rationum quantitates ſimul multiplicantur productum, quod ex tali multiplicatione reſultat, vocatur ratio ex rationibus compoſita, cum ex plurium rationum quantitatibus inter ſe multiplicatis componatur.

Def. VI.

PARALLELOGRAMMUM ad aliquam rectam lineam applicatum dicitur deficere a parallelogrammo, quando non occupat totam lineam: excedere verò dicitur, quando occupat majorem lineam, ac ſit illa, cui applicatur; ita tamen, ut deficiens, aut excedens parallelogrammum eamdem habeat altitudinem cum parallelogrammo applicato, & cum eo conſtituat unum parallelogrammum.



SI parallelogrammum AF applicatum ad rectam AB non occupet totam AB, at ſolam partem AC, completo parallelogrammo AE, parallelogrammum AF dicitur deficere parallelogrammo CE; & parallelogrammum CE dicitur defectus.

Quod de defectu dictum eſt, etiam de exceſſu intelligi debet; ſi enim ſuper AB conſtitutum fueri parallelogrammum AG, hoc parallelogrammum excedere dicitur parallelogrammo BG, quod exceſſus vocatur.

Propoſ. I. Theor. I.

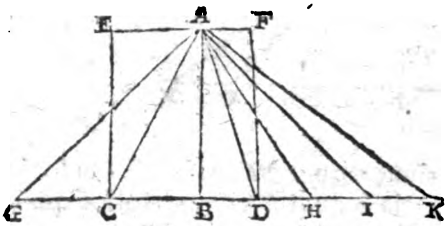
Si triangula CAB, BAD, vel parallelogramma BE, BF in eadem fuerint altitudine.

1. Dico triangulum CAB ad triangulum BAD illam habere rationem, quam habet baſis CB ad baſim BD.

2. Dico parallelogrammum BE, ad parallelo-

lelogrammum BF habere rationem baſis CB ab baſim BD.

Conſtructio primæ partis.



R Ecta CD indefinitè producatur ad utramque partem verſus G, & verſus K. Ex CG [*per* 3. *pri.*] auferantur quæcumque partes æquales ipſi CB: in præſenti verò figura una tantum pars facta fuit æqualis ipſi CB, nempe CG. Pariter in recta BK notentur DH, HI, IK ſingulæ ipſi BD æquales Notentur rectæ AG, AK, AI, AH.

Demonſtratio primæ partis.

CUm triangula BAC, CAG ſint ſuper æqualibus baſibus BC, CG, & inter eaſdem parallelas conſtituta: (*per* 38. *pri.*) erunt æqualia: ob eamdem rationem æqualia erunt etiam triangula BAD, DAH, HAI, IAK.

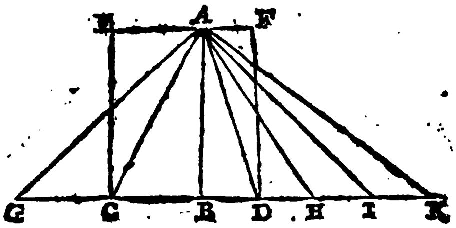
Quo ſtante quam multiplex eſt recta BG rectæ BC, tam multiplex etiam erit triangulum BAG, trianguli BAC: pariter quam multiplex eſt recta BK rectæ BD, tam multiplem etiam erit triangulum BAK trianguli BAD.

Rurſus quia dum baſis BG = BK, etiam triangulum BAG = triangulo BAK: quando BG major BK, etiam triangulum BAG majus triangulo BAK: & quando BG minor BK, etiam triangulum BAG minus triangulo BAK.

Hoc poſito ſtatuatur baſis BC prima magnitudo; BD ſecunda; triangulum BAC tertia; & triangulum BAD quarta.

Quia vero [*per demonſtrata*] primæ BC, & tertiæ BAC ſumpta fuerunt pariter multiplicia *B*G, & *B*AG; ſecundæ etiam

BD, & quartæ BAD poſita ſunt æqualiter multiplicia BK, & BAK: hæc autem æqualiter multiplicia, ex viſis, talia ſunt videlicet,



Quando BG multiplex primæ æquat BK multiplicem ſecũdæ, etiam BAG multiplex tertiæ æquat BAK multiplicẽ quartæ.
Quando BG majus BK, etiam BAG majus BAK.
Quando BG minus BK, etiam BAG minus BAK.

Secundum quamcumque multiplicationem.

Sequitur (*per def. 6. quinti.*)

ut baſis BC ad baſim BD, ita triangulum BAC, ad Triangulum BAD. q. e. 1. d.

Conſtructio ſecundæ partis.

SUper triangulorum baſibus BC, BD, & inter parallelas CD EF duo conſtituantur parallelogramma BE, BF.

Demonſtratio ſecundæ partis.

PArallelogrammum BE [*per 34. primi.*) duplum trianguli BAC, atque parallelogrammum BF duplum trianguli BAD.

Igitur (*per 15. quinti.*)

Parallelog. BE ad parallelog. BF, ut triang. BAC, ad triang. BAD.

Sed [*per demonſtrata in prima parte.*)

Ut triang. BAC ad triang. BAD, ita baſis BC ad baſim BD.

Ergo (*per 11. quinti.*) ut

Paranllelog. BE ad parallelog. BF, ita baſis BC ad baſim BD. q. e. 2. d.

Propoſ.

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Propoſ. II. Theor. II.

Si in triangulo BAC lateri BC ducta fuerit parallela DE.

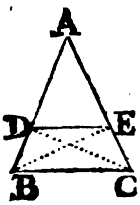
1. Dico reliqua trianguli latera AB, AC eſſe proportionaliter ſecta in D, & E.

2. Dico. Si trianguli latera AB, AC fuerint proportionaliter ſecta in D, & E, rectam DE, ipſi BC eſſe parallelam.

Conſtructio.

Notentur duæ rectæ EB, DC.

Demonſtratio primæ partis.



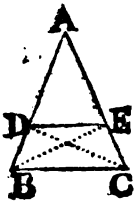
DUo triangula DBE, ECD (*per 37. pri.*) ſunt æqualia.

Quare (*per 7. quinti.*) ut
Triang. ADE ad triang. BDE, ita triang. ADE ad triang. ECD.
Sed (*per 1. hujus.*) ut
Triang. ADE ad triang. DBE, ita AD ad DB,
Atque: [*per eamdem 1. hujus.*] ut
Triang. ADE ad triang. ECD, ita AE ad EC:
Ergo (*per 11. quinti.*)
AD : DB : : AE : EC. q. e. 1. d.

Demonſtratio alterius partis.

(*per hyp.*] AD : DB : : AE : EC.
Sed (*per 1. hujus.*)
Ut AD : DB : : triang. AED : triang. DEB; atque
Ut AE : EC : : triang. ADE : triang. EDC:
Igitur (*per 11. quinti.*)

Ut



Ut triang. *AED*: triang. *DEB*: : triang. *AED*: triang. *EDC* ;

Igitur (*per 9. quinti.*

Triang. *DEB* = triang. *DCE*.

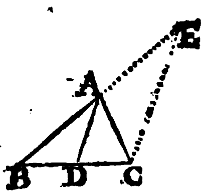
Cum autem hæc triangula æqualia ſint ſuper eadem baſi *DE* conſtituta: (*per 39. pri.*) erunt inter eaſdem parallelas: ideoque *DE* parallela *BC* : q. e. 2. d.

Propoſ. III. Theor. III.

Si trianguli BAC, angulus BAC fuerit bifariam diviſus a recta A D.

1. Dico BD: DC: : BA: AC,
   Viciſſim, ſi BD: DC: : BA: AC.
2. Dico angulum BAC eſſe bifariam diviſum a recta AD.

Conſtructio.



A Puncto C (*per 31. pri.*) notetur CE parallela AD, quæ producatur quouſque concurrat cum BA producta in E: debet enim concurrere, quia in triangulo ADB anguli ABD, ADB [*per 17. pri.*] ſunt duobus rectis minores. Cum autem (*per 29. pri.*) angulus ADB = ang. ECD ; inde ſequitur angulos ABC, ECB eſſe duobus rectis minores: quare (*per axjo. 13.*) rectæ BA, & CE debent concurrere in E.

Demonſtratio primæ partis.

Quia (*per conſtructionem.*) rectæ DA, CE ſunt parallelæ (*per 29. pri.*) ang. ECA = ang. CAD, & ang. E = DAB. Cum autem [*per hyp.*] ſit ang. CAD = ang. DAB: ſequitur ang. ECA = ang. E.

Qua-

Quare (*per 6. pri.*)
AE = AC.
Ergo (*per 7. quinti.*)
BA : AE : : BA : AC.
Cum autem in triang. BEC (*per constructionem*) ſit AD parallela EC : erit (*per 2. hujus.*)
BA : AE : : BD : DC,
Ergo (*per 11. quinti.*)
BA : AC : : BD : DC : q. e. p. d.

Demonſtratio alterius partis.

(*Per ſuppoſitum*) BD : DC : : BA : AC,
(*per 2. hujus.*)
BD : DC : : BA : AE;
Erit (*per 11. quinti.*)
BA : AC : : BA : AE;
Quare (*per 9. quinti.*)
AC = AE.

Ideoque in triang. CAE (*per 5. primi.*) ang. ACE = ang. AEC. Cum autem ob parallelas AD, EC (*per 29. primi.* ſit ang. ACE = ang. CAD, & ang. AEC = ang. DAB : erit (*per axio pri.*) ang. CAD = ang. DAB : ergo angulus BAC eſt biſariam diviſus a recta AD. q. e. 2. d.

Propoſ. IV. Theor. IV.

Si triangulum BAC habeat ſingulos angulos æquales ſingulis angulis trianguli CDE; hoc eſt

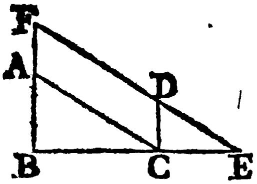
ang. ABC = DCE
ang. BCA = CED
ang. CAB = EDC

Dico circa æquales angulos, latera eſſe pro-
por-

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portionalia, & homologa eſſe illa latera, quæ æqualibus angulis ſubtenduntur, videlicet.

AB : *BC* : : *DC* : *CE*.
BC : *CA* : : *CE* : *ED*.
CA : *AB* : : *ED* : *DC*.



Conſtructio.

DAta triangula *ABC*, *DCE* taliter diſponantur, ut non ſolum homologa latera BC, CE, in recta linea BCE ſint conſtituta, verum etiam æqualium angulorum DCE, *ABC*, unus ſit externus, alter verò internus, & oppoſitus. Producantur latera BA, ED quouſque concurrant in F; debent enim concurrere, quia anguli *ABC*, *BCA* [*per* 17. *pri.*] ſunt minores duobus rectis: cum autem (*per hyp.*) angulus *BCA* = ang. *CED*; etiam anguli *ABC*, *CED* duobus rectis minores erunt; ideoque [*per axio.* 13.] rectæ *BA*, *ED* productæ ad partes *A*, & *D* debent concurrere in F.

Demonſtratio.

QVia (*per hyp.*) angulus *DCE* = ang. *ABC* (*per* 28. *pri.*] rectæ *AB*, *DC* erunt parallelæ; ob eamdem rationem etiam parallelæ erunt rectæ *CA*, *ED*: quamobrem *AD* erit parallelogrammum; ac propterea (*per* 34. *pri.*) *CD* = *AF*, *CA* = *DF*.
His poſitis, quia in triangulo *FBE* recta *AC* eſt parallela FE; erit [*per* 2. *hujus*.)
BA : *AF* : : *BC* : *CE*, ſeu *BA* : *CD* : : *BC* : *CE*,
permutando [*per* 16. *quinti*.)
BA : *BC* : : *CD* : *CE*.
Rurſus quia in eodem triangulo FBE recta DC eſt parallela BF;
erit (*per* 2. *huius*.]
BC : *CE* : : *FD* : *DE*, ſeu *BC* : *CE* : : *AC* : *DE*,
permutando [*per* 16. *quinti*.
BC : *AC* : : CE : DE.
Denuo, quia [*ex demonſtratis*.]

AB :

AB : BC : : DC : CE, atque BC : C*A* : : CE : DE,
Ex æquo (*per 22. quinti.*) erit
AB : *AC* : : *DC* : *DE*.

Quare in datis triangulis circa æquales angulos proportionalia ſunt latera, & homologa ſunt illa latera, quæ æqualibus angulis ſubtenduntur. q. e. d.

Corollarium.

EX hac propoſ. fit manifeſtum in triangulo *FBE*, rectam lineam *AC* parallelam F*E* abſcindere triangulum *ABC* ſimile toti triangulo *FBE*; cum dicta triangula [*per 29. pri.*] ſint æquiangula.

Nota. Æquiangula triangula FBE, *ABC* diceda ſunt SIMILIA juxta ſenſum Definitionis primæ hujus Libri; & hoc quia æquales habent angulos, & latera [*per demonſtrata*] circa æquales angulos proportionalia.

Propoſ. V. Theor. V.

Si duo triangula ABC, DEF latera habuerint proportionalia, nempe

AB : BC : : DE : EF,
BC : CA : : EF : FD,
CA : AB : : FD : DE,

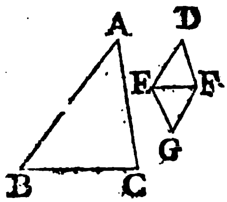
Dico eſſe æquiangula, æqualeſque habere illos angulos, ſub quibus homologa latera ſubtenduntur, videlicet

ang. *A* = D
ang. B = E
ang. C = F.

Cu-

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Conſtructio.



POnatur (*per* 23. *pri.*] angulus FEG = ang. B.

Per eandem propoſitionem fiat angulus EFG = ang. C,

Concurrantque rectæ EG, FG in G.

Demonſtratio.

QVia (*per conſtructionem*] in triangulo EGF angulus FEG = ang. B, & ang. EFG = ang. C: [*per* 32. *pri.*) & ang. G = ang. *A*: triangula B*A*C, EGF erunt æquiangula:

ergo [*per* 4. *hujus*.]

AB: BC:: GE: EF,

Sed [*per hyp.*)

AB: BC:: DE: EF,

Quare (*per* 11. *quinti*.)

GE: EF:: DE: EF,

ergo (*per* 9. *quinti*.)

GE = DE.

Eadem prorſus ratione demonſtrabitur

GF = DF.

Cum autem in triangulis EGF, EDF latera EG, GF ſint ſingula ſingulis æqualia lateribus ED, DF, atque baſis EF communis; ſequitur (*per* 8. *pri.*) ang. G = ang. D: & erunt (*per* 4. *pri.*) reliqui anguli reliquis angulis æquales, ſub quibus æqualia latera ſubtenduntur, nempè

ang. DEF = ang. FEG, &

ang. DFE = ang. EFG.

Quare triangula EDF, EGF ſunt æquiangula: ſed triangulum EGF (*per conſtructionem*) eſt æquiangulum triangulo BAC: ergo etiam triang. EDF erit æquiangulum eidem triang. BAC. q. e. d.

Propoſ.

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Propoſ. VI. Theor. VI.

Si in triangulis *BAC*, *EDF* ſit ang. B = ang. E; atque latera æquales angulos comprehendentia proportionalia, nempe

AB : BC : : DE : EF.

Dico eſſe æquiangula, æqualeſque eſſe illos angulos; ſub quibus homologa latera ſubtenduntur, nempe

ang. C = ang. F, & ang. A = ang. D.

Conſtructio.

Ad punctum E, & ad rectam EF (*per 23. pri.*) ponatur angulus FEG = ang. B, & ad punctum F conſtituatur angulus EFG = ang. C.

Demonſtratio.

In triangulis BAC, EGF (*per conſtructionem*) anguli, E, & F ſunt ſinguli ſingulis æquales angulis B, & C: quare (*per 32. pri.*) & ang. G = ang. A: unde dicta triangula BAC, EGF erunt æquiangula.

Quamobrem (*per 4. hujus.*)

AB : BC : : GE : EF.

Sed [*per hyp.*]

AB : BC : : DE : EF:

Ergo [*per 11. quinti.*]

GE : EF : : DE : EF,

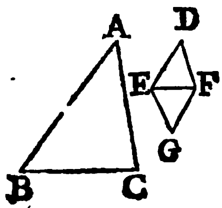
Unde (*per 9. quinti.*)

GE = DE.

Cum igitur [*per hyp.*] ang. B = ang. DEF, & (*per conſtructionem.*) idem ang. B = ang. GEF (*per axio. 1.*)

ang. DEF = ang. GEF.

Sintque latera DE, EF ſingula ſingulis lateribus GE, EF æqua-



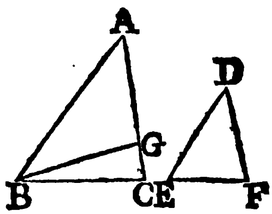
æqualia: (*per 4. pri.*) erunt reliqui anguli æquales, nempe
ang. DFE = ang. GFE, & ang. D. = ang. G.
Quare triãg. EDF æquiangulũ triang. EGF. Sed triangulum EGF [*per demonſtrata.*] æquiangulum triangulo BAC: ergo triangula BAC, EDF æquiangula. q. e. d.

Propoſ. VII. Theor. VII.

Si in triangulo ABC fuerit ang. A = ang. D. in triangulo DEF, latera verò comprehendentia angulos B, & *E* proportionalia, videlicet *AB : BC : : DE : EF*; atque reliqui anguli C, & F ambo majores, vel non majores recto.

Dico hæc triangula eſſe æquiangula, æqualeſque habere non ſolum angulos B, & *E*, verum etiam C, & F.

Conſtructio, & Demonſtratio.



SUppoſitis angulis C, & F recto minoribus, ſi anguli ABC, DEF non ſunt æquales, unus erit major, & alter minor: ponatur angulus ABC major angulo DEF. Hoc ſuppoſito ad punctum B, & ad rectam BA [*per* 23. *primi.*) ponatur angulus ABG = ang. DEF.

Quia in triangulis ABG, DEF (*per hyp.*) angulus A = ang. D, & angulus ABG [*per conſtructionem.*) = ang. E; (*per* 32. *primi.*) erit ang. BGA = ang. EFD: quare triangula ABG, DEF æquiangula.

Ideoque (*per 4. bujus.*
AB : BG : : DE : EF.

Sed

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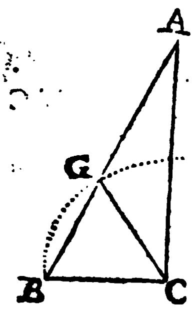
Sed [*per hyp.*)
AB : BC : : DE : EF.
Ergo (*per* 11. *quinti.*]
AB : BG : : AB : BC.
Quare (*per* 9. *quinti.*)
BG = BC,
Igitur triangulum GBC iſoſceles, unde (*per* 5. *primi.*)
ang. BCG = ang. BGC.

Cum autem, (*ut ſupponitur.*) angulus BCG ſit acutus, etiam acutus erit angulus BGC: ergo (*per* 13. *quinti.*) angulus BGA erit obtuſus: ſed angulus BGA demonſtratus eſt æqualis angulo F, qui ſuppoſitus fuit acutus: ergo angulus BGA erit acutus, & obtuſus; quod repugnat; unde angulus ABC non erit major angulo DEF.

Pariter ſi duo anguli C, & F ſinguli ſint non minores recto [*per ſuperius demonſtrata.*] erit angulus BCG = ang. BGC: cum autem angulus BCG ſupponatur non minor recto, angulus BGC erit vel rectus, vel obtuſus: ergo in triangulo BCG erunt duo anguli non minores duobus rectis, [*contra prop.* 17. *pri.*] ergo dicendum erit ang. ABC = ang. DEF: quare (*per* 32. *pri.*) triangulum ABC triangulo DEF æquiangulum. q. e. d.

SCHOLIUM.

QUærenti poſtea cur Euclides in hoc theoremate appoſuerit illa verba (utrumque vero reliquorum angulorum C, & F debere eſſe minorem, vel non minorem recto.) *Reſpondeo, hoc fuiſſe factum, quia dari poſſunt duo triangula, in quibus licet ſit unus angulus uni angulo æqualis, & circa reliquos angulos latera proportionalia, ipſa tamen non ſint æquiangula; ut patet in duobus triangulis ACB, ACG, quæ habent angulum communem A, & circa angulos ACB, ACG latera proportionalia, quia* (*per* 7. *quinti.*) *AC* : CB : : *AC* : CG: *licet hæc triangula non ſint æquiangula. Quamobrem Euclides, ut propoſitionem poneret univerſalem addidit illa verba,* reliquos angulos debere eſſe minores, vel non minores recto; *quia hac conditione ſuppoſita, triangula ſunt æquiangula.*

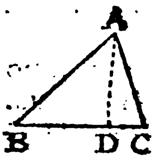


Propoſ. VIII. Theor. VIII.

In triangulo rectangulo BAC, ſi ab angulo recto A in baſim BC demiſſa fuerit perpendicularis AD.

Dico triangula BAD, DAC toti triangulo BAC, & inter ſe eſſe ſimilia.

Demonſtratio.



TRiangula *BAC*, *BDA* habent angulos BAC, BDA æquales, quia rectos: ulterius habent angulum B. communem utrique triangulo: quare [*per corol. 2. prop. 32. pri.*) & angulus C = ang. BAD: ergo triangulum BDA (*per 2. hujus*] ſimile triangulo BAC.

Eodem prorſus modo demonſtrabitur triangulum CDA æquiangulum ejdem triangulo BAC; ideoque ſimilia etiam triangula CDA, BAD.

Quod poſtea triangula BDA, CDA inter ſe ſint ſimilia, taliter demonſtratur: habent enim angulos ADB, ADC æquales, quia rectos: Rurſus [*per ſuperius demonſtrata.*] angulus BAD = C: quare (*per 32. pri.*] & angulus DAC = ang. B; ideoque triangula BDA, CDA [*per 4. huius.*) inter ſe ſimilia erunt. q. e. d.

Corrollarium primum.

EX hac propoſitione patet, in Triangulo rectangulo BAC perpendicularem ab angulo recto in baſim demiſſam, mediam eſſe proportionalem inter baſis ſegmenta BD, DC: quia ob ſimilitudinem triangulorum BDA, CDA (*per 4. hujus.*)

BD : DA :: DA : DC : q. e. d.

Corrol-

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Corrolarium ſecundum,

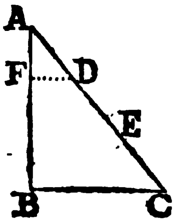
PAriter fit manifeſtum, latus BA eſſe medium proportionale inter BC, & BD, atque latus CA medium proportionale inter BC, & CD: & hoc quia ob ſimilia triangula BAC, BDA, (*per* 4. *hujus*.) CB: BA:: BA: BD.

Rurſus quia ſimilia ſunt triangula *B*AC, CDA (*per eandem* 4.) *B*C: CA:: CA: CD: q. e. d.

Propoſ. IX. Probl. I.

A data recta AB terminata quæſitam partem auferre.

Conſtructio.



A Puncto A ducatur quæcumque recta AC indefinita, quæ cum data A*B* angulum efformet *B*AC. In AC ſignetur quodcumque punctum D.

Si ex data A*B* auferenda ſit pars tertia, in AC (*per* 3. *pri.*) accipiantur tres partes æquales, & ſint AD, DE, EC.

Ducatur *B*C, & ex D (*per* 31. *pri.*) notetur DF parallela *B*C.

Dico AF tertiam eſſe partem datæ rectæ A*B*.

Demonſtratio.

QVia (*per conſtructionem.*) DF eſt parallela C*B* (*per* 2. *hujus.*)

AD: DC:: AF: F*B*;

(Componendo (*per* 18. *quinti.*)

AD: AC:: AF: AB,

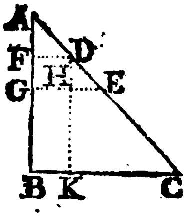
Cum autem (*per conſtructionem.*) AD ſit pars tertia ipſius AC, erit & AF tertia pars datæ rectæ A*B*. q. e. f.

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Propoſ. X. *Probl. II.*

Duabus datis rectis lineis terminatis AB, AC, quarum AB ſit inſecta, AC vero ſecta in D, & E: oportet ſimiliter ſecare AB, ac ſecta eſt recta AC.

Conſtructio.



Datæ rectæ *AB*, AC taliter diſponantur, ut conſtituant angulum BAC. A puncto C ad B recta ducatur C*B*: ex punctis D, & E [*per* 31. *primi*.) notentur rectæ DF, EG parallelæ C*B*, ſigneturquæ D*K* parallela AB.

Dico datam rectam A*B* in F, & G eſſe ſimiliter ſectam, ac ſecta eſt AC in D, & E.

Demonſtratio.

Quia [*per conſtructionem*.) DF, EG parallelæ BC; erit [*per* 30. *pri.*) DF parallela EG: quamobrem in triangulo AEG [*per* 2. *hujus*.)

AD: DE:: AF: FG.

Pariter quia in triangulo D*K*C recta E H, eſt ipſi CK parallela (*per* 2. *hujus*.)

DE: EC:: DH: HK.

Cum autem [*per* 34. *pri*.] ſit DH = FG, & H*K* = GB; erit

DE: EC:: FG. G*B*:

Quare

AF: FG:: AD: DE,

FG: G*B*:: DE: EC,

Unde data recta A*B* erit ſimiliter ſecta, ac ſecta fuit data AC. q. e. f.

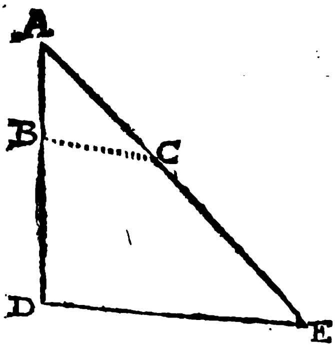
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Propoſ. XI. *Probl.* III.

Duabus datis rectis lineis AB, AC tertiam proportionalem invenire.

Conſtructio:



Datæ rectæ *AB*, AC taliter diſponantur, ut quemcumque efforment angulum BAC. In AB producta acipiatur *BD* = AC, notetur BC. Ex puncto D (*per* 31. *pri.*) notetur DE parallela BC: ellongetur AC quouſque concurrat cum DE in E.

Dico CE eſſe tertiam proportionalem duabus datis AB, AC.

Demonſtratio.

In triangulo ADE (*per conſtructionem.*) recta *BC* parallela DE: quare (*per* 2. *bujus.*)

A*B*: BD:: AC: CE,

Cum autem (*per conſtructionem*) BD = AC, erit

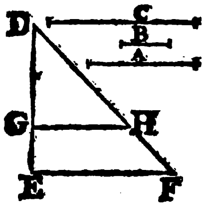
AB: AC:: AC: CE: igitur

Recta CE erit tertia proportionalis datis rectis BA, AC. q. e. f.

Propoſ. XII. *Probl.* IV.

Tribus datis rectis lineis A. B. C. quartam proportionalem invenire.

Conſtructio.



EX datis rectis lineis duæ priores A, & B ſecundum rectam DE diſponantur, ſitque DG = A, & GE = B. Notetur quæcumque recta DF, quæ angulum efficiat cum DE. In recta DF indefinitè producta (*per 3. pri.*) accipiatur DH = C: notetur HG: & ex E ducatur EF parallela GH, quæ concurrat cum DH producta in F.

Dico AF eſſe quartam proportionalem tribus datis A. B. C.

Demonſtratio.

IN triangulo EDF [*per conſtructionem.*] GH parallela EF, Quare [*per 2. hujus.*

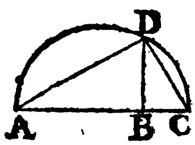
DG : GE : : DH : HF,

Ergo HF eſt quarta proportionalis tribus datis A.B.C. q. e. f.

Propoſ. XIII. Probl. V.

Duabus datis rectis lineis AB, BC mediam proportionalem invenire.

Conſtructio.



DAtæ rectæ, inter quas invenienda eſt media proportionalis, diſponantur ſecundum rectam ABC. Recta AC bifariam dividatur, & ſupra ipſam notetur ſemicirculus ADC. Ex puncto B erigatur BD perpendicularis ipſi AC.

Dico BD eſſe mediam proportionalem inter *AB*, & *BC*.

Notentur rectæ *DA*, DC.

Demonſtratio.

ANgulus *A*DC in ſemicirculo (*per 31. tertij.*] rectus eſt: quare in triangulo rectangulo *A*DC ab angulo recto D in baſim

baſim AC (*per conſtructionem.*) ducta eſt perpendicularis DB: quare (*per corol. 1. prop. 8. hujus*) erit DB media proportionalis inter datas rectas *AB*, *BC*. q. e. f.

Propoſ. XIV. Theor. IX.

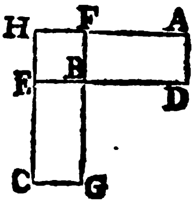
Si parallelogrammum AB æquale fuerit parallelogrammo BC, habueritque angulum FBD = ang. EBG.

Dico latera circa æquales angulos eſſe reciprocè proportionalia: & viciſſim.

Si latera circa æquales angulos fuerint reciprocè proportionalia.

Dico parallelogramma eſſe æqualia.

Conſtructio.



DAta parallelogramma *AB*, EG taliter diſponantur, ut æqualium angulorum latera DB, BE componant rectam lineam DBE: producantur latera AF, CE donec concurrant in H.

Demonſtratio primæ partis.

CUm anguli FBD, EBG (*per hyp.*) ſint æquales, atque latera E *B*, *BD* [*per conſtructionem.*) in rectum conſtituta, (*per corol. 3. prop. 15. pri.*] etiam F*B*, *B*G, erunt in rectum diſpoſita.

His ſtantibus quia (*per hyp.*] parallelogrammum *AB* = parallelogrammo EG erit,

[*per 7. quinti.*

Parallelog. *AB* ad parallelog. FE, ut parallelog. EG ad parallelog. FE:

Sed (*per 1. hujus.*)

Parallelog. *AB* ad parallelog: FE, ut DB, ad *BE*,
pariterque
Parallelog. *EG* ad parallelog. F*E*, ut G*B*, ad *BF*:
Quare [*per* 11. *quinti.*]
D*B*: *BE*: : G*B*: *BF*. q. e. 1. d.

Demonſtratio ſecundæ partis.

[*per hyp.*] D*B*: *BE*: : G*B*: *BF*;
Sed (*per* 1. *hujus.*)
Ut D*B* ad *BE*, ita parallelog. *AB* ad parallelog. F*E*, atque
Ut G*B* ad *BF*, ita parallelog. G*E* ad parallelog. F*E*:
Quare (*per* 11. *quinti.*]
Ut parallelog. *AB* ad parallelog. F*E*, ita parallelog. *EG* ad parallelog. F*E*,
ergo [*per* 7. *quinti.*)
Parallelog. *AB* = parallelog. EG. q. e. 2. d.

Propoſ. XV. Theor. X.

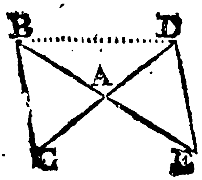
Si triangulum BAC fuerit æquale triangulo DAE, habueritque angulum BAC æqualem angulo DAE.

Dico latera circa æquales angulos eſſe reciprocè proportionalia: & viciſsim

Si circa æquales angulos latera fuerint reciprocè proportionalia.

Dico triangula fore æqualia.

Conſtructio.



DAta triangula *BAC*, *DAE* taliter ſecundum angulos æquales *BAC*, *DAE* diſponantur, ut latera *BA*, *AE* componant rectam *BAE*; quo facto (*per corol.* 3. *prop.* 15.)

15. *primi*.) etiam latera *CA*, *AD* erunt in rectum conſtituta. Ducatur *BD*.

Demonſtratio primæ partis.

QVoniam (*per hyp.*] triang. *BAC* = triang. *DAE*.
[*per 7. quinti*.)
Triang. *BAC* ad triang. *BAD*, ut triang. *DAE* ad triang. *BAD*;
Sed *per* 1. *hujus*.]
Triang. *BAC* : triang. *BAD* : : *CA* : *AD* : atque
Triang. *DAE* : triang. *BAD* : *EA* : *AB*,
Ergo (*per* 11. *quinti*.)
CA : *AD* : : *EA* : *AB*. q. e. 1. d.

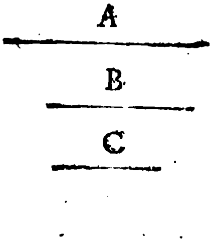
Demonſtratio ſecundæ partis.

IN triangulis *BAC*, *DAE* cum angulis æqualibus *BAC*, *DAE*.
[*per hyp.*]
CA : *AD* : : *EA* : *AB*.
Sed [*per* 1. *hujus*.)
CA : *AD* : : triang. *CAB* : triang. *BAD*; pariterque
EA : *AB* : : triang. *EAD* : triang. *DAB*.
Ergo (*per* 11. *quinti*.]
Triang. *CAB* : triang. *BAD* : : triang. *EAD* : triang. *DAB*.
ergo [*per* 9. *quinti*.]
triang. *CAB* = triang. *EAD*. q. e. 2. d.

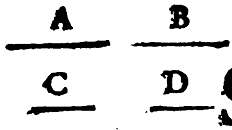
Propoſ. XVI. Theor. XI.

Si quatuor rectæ lineæ A, B, C, D, fuerint proportionales.

Dico A × D = B × C: & viciſſim,
Si A × B = B × C.
Dico quatuor lineas A, B, C, D eſſe proportionales.



De-



Demonſtratio primæ partis.

SI unum rectangulum ex prima A,& quarta D fuerit efformatum, aliud vero rectangulum a ſecunda B, & tertia C comprehenſum; hæc duo rectangula erunt duo parallelogramma æquiangula, in quibus circa æquales angulos (*per hyp.*) latera erunt reciprocè proportionalia quare [*per 14. bujus.*] dicta rectangula æqualia erunt. q. e. 1. d.

Demonſtratio ſecundæ partis.

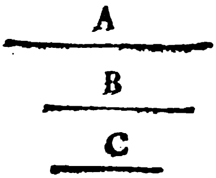
QVia (*per hyp.*) rectangulum ex A,& D efformatum æquat rectangulum a rectis B & C comprehenſum: duo erunt parallelogramma æquiangula, & æqualia; quare (*per 14. huius.*) latera circa æquales angulos erunt reciprocè proportionalia; nempe A ad B, uti C ad D. q. e. 2. d.

Propoſ. XVII. Theor. XII.

Si tres rectæ A, B, C fuerint proportionales.
Dico rectangulum a prima A, & tertia C contentum æquale eſſe quadrato a media B deſcripto; & viciſſim.

Si rectangulum a prima A & tertia C efformatum æquale fuerit quadrato mediæ B.
Dico tres rectas A, B, C eſſe proportionales.

Demonſtratio primæ partis.



REctangulum ex prima A, & tertia C efformatum, atque quadratum Mediæ B ſunt parallelogramma æquiangula, quæ (*per hyp.*) circa æquales augulos habent latera reciprocè proportionalia: ergo

go [*per* 14. *hujus.*) erunt æqualia. q.e.i.d.

Demonſtratio ſecundæ partis.

Rectangulum comprehenſum a rectis A, & C [*per hyp.*] eſt æquale quadrato rectæ *B*: cum autem hæc rectangula ſint æquiangula, & æqualia parallelogramma [*per* 14. *hujus.*] circa æquales angulos erunt latera reciprocè proportionalia: quare A ad B, uti B ad C. q.e.i.d.

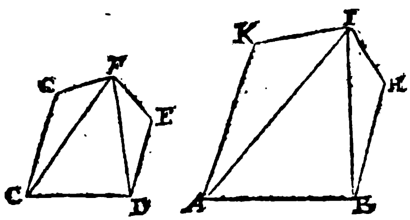
SCHOLIUM.

Circa has demonſtrationes eſt obſervandum quodlibet Rectangulum eſſe parallelogrammum: & ideo de parallelogrammo demonſtrata, valent etiam de rectangulo.

Propoſ. XVIII. Probl. VI.

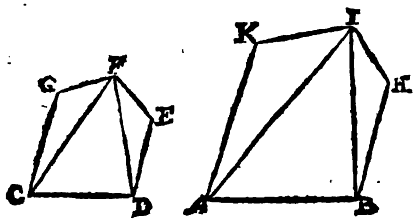
Datis rectilineo CDEFG, rectaque AB, ad ipſam AB applicandum eſt rectilineum ſimile, ſimiliterque poſitum. Dato rectilineo CDEFG.

Conſtructio.



Datum rectilineum CDEFG rectis FC, FD reſolvatur in triangula FGC, FCD, FED numero minora quantum fieri poteſt.

Ad punctum A, & ad rectam AB [*per* 23. *pri.*) conſtituatur an-



angulus BAI = ang. DCF: pariter ad punctum B, & ad rectam AB applicetur angulus ABI = ang. CDF: hoc facto producantur rectæ AI, BI donec concurrant in I; debent enim concurrere, quia diſcedunt a duobus angulis minoribus duobus rectis, qui (*per conſtructionem*) æquales ſunt angulis DCF, CDF, qui [*per* 17. *pri.*] ſunt duobus rectis minores.

Eodem prorſus modo ad punctum A, & ad rectam AI ponatur angulus IAK = ang. FCG: pariter ad punctum I efformetur ang. AIK = ang. CFG, rectæque AK, IK concurrant in K.

Tandem ad B, & I extrema rectæ BI applicentur anguli IBH, BIH ſinguli ſingulis æquales angulis FDE, DFE, concurrantque rectæ BH, IH in H.

Dico rectilineum ABHIK ad rectam AB datam applicatum ſimile eſſe, & ſimiliter poſitum dato rectilineo CDEFG.

Nota. Figuræ ſimiles dicuntur SIMILITER POSITÆ, quæ ad eamdem partem angulos habent æquales, & latera proportionalia.

Demonſtratio.

IN triangulo AKI (*per conſtructionem*) anguli IAK, AIK ſunt æquales angulis GCF. GFC in triangulo FCG: quare [*per* 32. *pri.*] ang. K = ang. G. Eadem ratione in triangulis IBH, FDE demonſtrabitur ang. H = ang. E.

Rurſus cum [*per conſtructionem.*] ang. KAI = ang. GCF, & ang. IAB = ang. FCD; erit (*per axio.* 2.) totus angulus KAB = toti ang. GCD: eodem modo demonſtrabitur ang. ABH = ang. CDE, & ang. HIK = ang. EFG: quare poſita rectilinea erunt æquiangula.

Quod poſtea latera circa æquales angulos ſint proportionalia, ita demonſtrabitur. Trian-

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Triangula CGF, AKI [*per conſtructionem*, & *per demonſtrata*.] ſunt æquiangula; ideoque (*per 4. hujus*.)

GC : CF :: KA : *A*I.

Inſuper quia triangula CFD, *A* I *B* & ipſa (*per conſtructionem*.] ſunt æquiangula (*per eamdem* 4.)

CF : CD :: *A*I : *A*B:

Ergo ex æqualitate (*per* 22. *quinti*.)

GC : CD :: K*A* : *A*B:

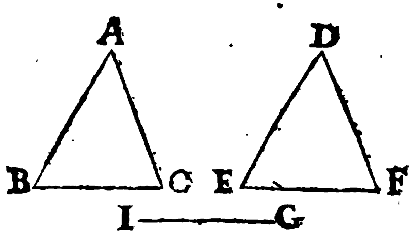
igitur circa æquales angulos C, & *A* latera ſunt proportionalia.

Simili prorſus modo demonſtrabitur circa reliquos rectilineorum æquales angulos latera eſſe proportionalia; ideoque rectilineum ABHIK ad datam rectam *A*B applicatum, ſimile, & ſimiliter poſitum dato rectilineo CDEFG. q. e. f.

Propoſ. XIX. Theor. XIII.

Si triangula BAC, EDF fuerint ſimilia. Dico inter ſe habere duplicatam rationem laterum homologorum.

Conſtructio, & Demonſtratio.



In datis triangulis ſimilibus B*A*C, EDF ſit

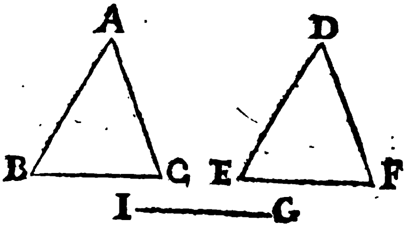
ang. *A* = ang. D
ang. B = ang. E
ang. C = ang. F.

Homologa vero latera ſint BC, & EF. Oportet demonſtrare, triang. B*A*C, ad triang. EDF rationem habere duplicatam lateris *B*C ad latus EF.

Quia vero hæc homologa latera *B*C, *E*F poſſunt eſſe inter ſe æqualia, & inæqualia: ſi fuerint æqualia, ut in prima figura, veritas hujus theorematis ita demonſtratur.

Æqualium laterum homologorum BC, & EF [*per* 11. *hujus*.] tertia proportionalis inveniatur IG, quæ, & ipſa lateribus

BC



BC,EF æqualis erit; ideoque ratio BC ad IG, quæ *juxta def.* 10. *quinti.*) dicitur duplicata illius rationis, quam habet BC ad EF, erit ratio æqualitatis.

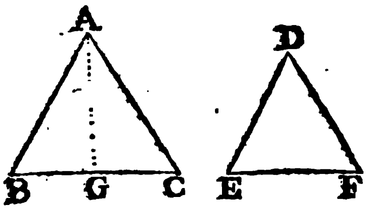
Cum autem (*per hyp.*) in triangulis *BAC*, EDF ſit

CB: *BA*: : FE: ED, atque

BC: *CA*: . EF: FD,

Poſita antecedentium *BC*, & *EF* æqualitate [*per* 14. *quinti.*) etiam conſequentia *BA*, & *ED* æqualia erunt: ideoque *BA* = *ED*, & C*A* = DF: quare [*per* 8. *pri.*) triangulum *BAC* = triang. EDF: ex hoc ſequitur quod ratio trianguli *BAC* ad triangulum EDF, ſit ratio æqualitatis; ſed ex dictis eſt etiam ratio æqualitatis duplicata homologorum laterum æqualium *BC*, *EF*: quare trian. *BAC* ad triang. E*DF* rationem habebit duplicatam homologorum laterum *BC*, *EF*. q. e. i. d.

Si vero homologa latera *BC*, *EF* fuerint inæqualia *BG* ſcilicet majus, & *EF* minus, ut in præſenti figura, eadem veritas taliter demonſtrabitur.



Laterum homologórum *BC*, *EF* (*per* 11. *hujus.*) tertia proportionalis inveniatur *B*G, quæ minor erit quam *BC*, quia *BC* major ponitur quam EF. Ducatur *AG*.

Hac conſtructione poſita, quia (*per hyp.*] in triangulis ſimilibus *BAC*, *EDF*.

AB: BC: : *DE*: *EF*,

alternando [*per* 16. *quinti.*)

AB: DE: : *BC*: EF.

Sed (*per conſtructionem*)

BC: EF: : EF: *BG*,

Ergo [*per* 11. *quinti.*)

AB: DE: : EF: *BG*.

Duo

Duo igitur triangula ABG,DEF [*per hyp.*]angulos habent*B*,& E æquales,atque latera circa æquales angulos reciprocè proportionalia: ergo [*per* 15.*hujus*.] triang. ABG = triang. DEF, unde [*per* 7. *quinti* .]

Triang. A*B*C : triang. A*B*G : : triang. A*BC* : triang. DEF.

Sed [*per* 1. *hujus*.]

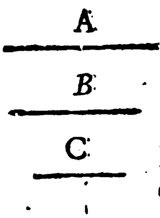
Triang. *ABC* : triang. *ABG* : : *B*C : *B*G.

Ergo [*per* 11. *quinti*.]

Triang. A*B*C : triang. DEF : : *B*C : BG :

Cum autem (*per conſtructionem*) ratio BC ad BG ſit duplicata illius, quam habet homologum latus BC, ad homologum latus EF; etiam triangulum A*B*C, ad triangulum DEF habebit rationem duplicatam laterum homologorum BC, E*F*. q. e. d.

Corollarium.



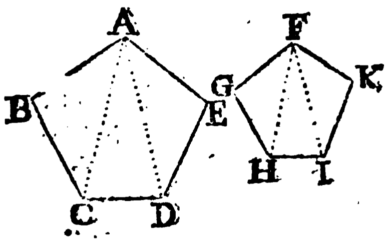
EX hoc theoremate fit manifeſtum; quod ſi tres rectæ lineæ A. *B*. C. fuerint proportionales, triangulum ſupra primam A ad ſimile triangulum ſupra ſecundam B, illam habere rationem, quam habet prima A ad tertiam C.

Propoſ. XX. *Theor.* XIV.

Si fuerint ſimilia Polygona ABCDE,FGHIK.

1. Dico hæc Poligona partiri in triangula ſimilia numero æqualia, & homologa totis.

2. Dico Polygonum ad Polygonum habere rationem duplicatam laterum homologorum.



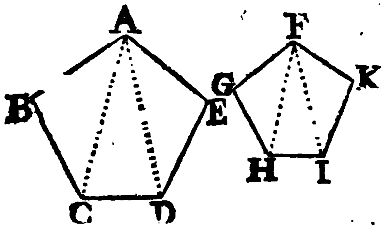
Conſtructio.

AB angulis æqualibus A, & F ad oppoſitos angulos C, D, H, I, rectæ ducantur AC, AD, FH, FI.

De-

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Demonſtratio.



CUm [*per hyp.*) ſimilia ſint data polygona, tot erunt anguli in uno polygono, quot in alio. quare triangula numero æqualia erunt in dictis polygonis.

Quoniam vero ang. *B* [*per hyp.*] = ang. G, & latera circa iſtos æquales angulos proportionalia ſunt; (*per 6. hujus.*) triangula A*B*C, FGH erunt æquiangula; ideoque ang. BCA = GH*F*, pariterque ang. *B*AC = ang. G*F*H, quibus homologa latera ſubtenduntur: unde *per 4. hujus.*] hujuſmodi triangula habebunt latera circa æquales angulos proportionalia.

Eadem prorſus ratione æquiangula erunt triangula *AED*, *F*KI: quare (*per 4. hujus.*) ſimilia.

Quantum poſtea ad triangula CAD, HFI, quia [*per hyp.*] ang. BCD = ang. GHI, demptis æqualibus angulis BCA, GHF, [*per axio. 3.*) ang. ACD = ang. FHI. Eodem modo demonſtrabitur ang. ADC = ang. FIH: unde (*per 32. pri.*) & ang. CAD = ang. HFI: ideoque etiam triangula ACD, FHI (*per 4. hujus*] erunt ſimilia: Idem de aliis triangulis dicendum, ſi plura fuerint.

Quod vero hæc triangula ſint homologa totis, videlicet, quod ita ſint ſingula triangula in uno polygono ad ſingula triangula ſimilia in altero polygono, ut polygonum ad polygonum, ita demonſtratur.

Triangulum ABC ad triangulum FGH (*per 19. hujus.*) habet duplicatam rationem homologorum laterum AC, FH; & triangulum CAD ad triang. HFI rationem habet duplicatam eorumdem laterum homologorum AC, FH.

Quare (*per 11. quinti.*)

Triang. ABC: triang. *F*GH: triang. ACD: triang. *F*HI.

Cum autem [*per eamdem 9.*] triang. ACD ad triang. *FHI* ſit in duplicata ratione laterum homologorum AD, *F*I, in qua ratione eſt etiam triang. ADE ad triang. *F*IK; ex hoc ſequitur.

Triang. ACD: triang. *F*HI:: triang. ADE: triang. FIK.

Igi-

Igitur triangula unius polygoni, alterius polygoni triangulis proportionalia ſunt, & hujus proportionalitatis antecedentia ſunt in uno polygono, & conſequentia in altero polygono.

Sed (*per* 12. *quinti*.)

ut unum antecedens ad unum conſequens, ita omnia antecedentia ad omnia conſequentia: ergo

triang. *ABC* : trang. *FGH* : : omnia triang. { *ABC*, *ACD*, *ADE* } :

ad triang. { *FGH*, *FHI*, *FIK* } ;

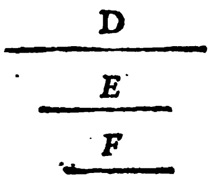
ſed triang. { *ABC*, *ACD*, *ADE* } = (*per axio.* 9.) poligono ABCDE, &

triang. { *FGH*, *FHI*, *FIK* } = polygono *FGHIK* : ergo

ut triang. *ABC* ad triang. *FGH*,
ita polygonum ABCDE, ad polygonum FGHIK.

Cum autem ratio triang. *ABC* ad triang. *FGH* [*per* 19 *hujus*.] ſit ratio duplicata homologorum laterum *AB*, FG; etiam ratio polygoni *ABCDE*, ad polygonum *FGHIK* erit duplicata laterum homologorum *AB*, *FG*, q. e. d.

Corollarium.



EX hac propoſitione colligitur, quod ſi tres rectæ lineæ *D. E. F.* fuerint proportionales, polygonum ſupra primam D deſcriptum ad ſimile polygonum factum ſupra ſecundam E, illam habere rationem, quam habet prima linea D ad tertiam *F*: & hoc quia ratio D ad *F* eſt duplicata rationis D ad E, uti ſupra in def. 10. libri quinti dictum fuit.

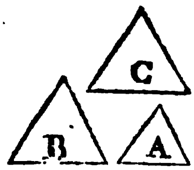
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Propoſ. XXI. Theor. XV.

Si rectilinea A, & B ſimilia fuerint rectilineo C.

Dico inter ſe eſſe ſimilia.

Demonſtratio.



Cum [*per hyp.*) rectilineum A ſimile ſit rectilineo C, non ſolum anguli rectilinei A, æquales erunt angulis rectilinei C, verum etiam latera circa æquales angulos erunt proportionalia; idem pariter dicendum de rectilineo *B*: quare (*per axio primum*] ſinguli anguli rectilinei *A* æquales ſingulis angulis rectilinei B: atque [*per* 11. *quinti.*) latera rectilinei *A* proportionalia lateribus rectilinei B: ergo [*per def.* 1. *hujus.*) rectilineum *A* ſimile rectilineo B. q. e. d.

Propoſ. XXII. Theor. XVI.

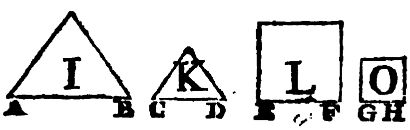
Si quatuor rectæ lineæ AB, CD, EF, GH fuerint proportionales.

Dico ab hujuſmodi lineis rectilinea ſimilia, ſimiliterque deſcripta, eſſe proportionalia: & viciſſim.

Si a quatuor rectis lineis rectilinea ſimilia deſcripta, fuerint proportionalia.

Dico quatuor lineas eſſe proportionales.

Conſtructio.



Super primam AB, & ſecundam CD (*per* 18. *hujus.*) ſimilia deſcriban-

bantur rectilinea I, & K. Pari modo ſuper tertiam EF, & quartam GH ſignentur alia ſimilia rectilinea L, & O; de iſtis rectilineis primo eſt demonſtrandum rectil. I: rectil. K:: rectil. L: rectil. O.

Demonſtratio primæ partis.

QVia (*per hyp.*) rectilineum I ſimile eſt rectilineo K: (*per 20. hujus.*) I ad K habebit rationem duplicatam homologorum laterum AB, CD: ſimiliter rectilineum L ad rectilineum O eſt in duplicata ratione homologorum laterum EF, GH; ſed (*per 35. quinti.*) duplicata AB ad CD eadem eſt, ac duplicata EF ad GH:

Quare [*per 11. quinti.*] ut
Rectil. I ad rectil. K, ita rectil. L ad rectil. O. q. e. d.

Demonſtratio ſecundæ partis.

RAtio I ad K (*per corol. 20. hujus.*] eſt duplicata AB ad CD; & ratio L ad O duplicata EF ad GH; ſed [*per hyp.*] ratio I ad K eadem, ac ratio L ad O:

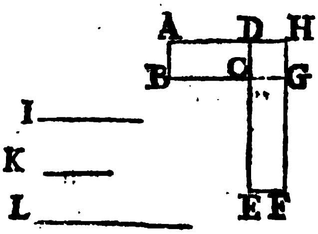
Ergo (*per 11. quinti.*)
Duplicata ratio AB ad CD eadem ac duplicata ratio EF ad GH: cum autem [*per 35. quinti.*] duplicatæ rationes inter ſe æquales, etiam ſubduplicatas æquales habeant: ſequitur dimidiatam AB ad CD æqualem eſſe dimidiatæ EF, GH: igitur AB ad CD, uti EF ad GH. q. e. d.

Propoſ. XXIII. *Theor.* XVII.

Si parallelogramma AC, EG fuerint æquiangula.

Dico inter ſe habere illam rationem, quæ ex lateribus componitur.

Conſtructio.



DAta parallelogramma AC, EG uniantur ſecuudum angulos æquales BCD, ECG taliter, ut BC, & CG ſint in directum conſtituta, efformentque rectam BCG: quo ſtante [*per corollar. 3. prop. 15. primi.*) etiam latera DC. CE erunt in rectum poſita: unde DCE erit recta linea.

Producantur latera AD, FG donec concurrant in H; debent enim concurrere, quia FG ſecat parallelam BG; unde (*per axio. 23.*) ſecabit aliam parallelam AH.

Ulterius (*per 12. bujus.*) fiat

ut BC: CG:: I: K; &

ut DC: CE:: K: L.

Demonſtratio.

(*per 1. bujus*] ut BC ad CG, ita parallelog. BD ad parallelog. CH,

Sed (*per conſtructionem.*]

ut BC ad CG, ita I ad K:

Ergo [*per 11. quinti.*]

ut parallelog. BD ad parallelog. CH, ita I ad K;

Pariter (*per eamdem 1. bujus.*)

ut DC ad CE, ita parallelog. CH ad parallelog. CF,

Sed (*per conſtructionem.*)

ut DC ad CE, ita K ad L:

Ergo (*per 11. quinti.*]

ut parallelog. CH ad parallelog. CF, ita K ad L,

unde ex æquo (*per 22. quinti.*)

ut parallelog. BD ad parallelog. CF, ita I ad L,

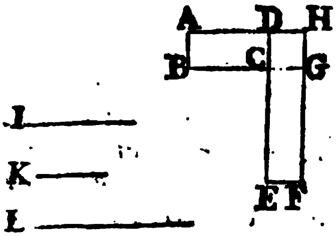
Sed [*ut infra demonſtrabitur.*]

ratio I ad L eſt compoſita ex rationibus I ad K, & K ad L;

Ergo

ratio parallelog. BD ad parallelog. CF erit compoſita ex rationibus I ad K, & K ad L,

Sed



Sed [*per conſtructionem.*] ratio compoſita ex rationibus I ad K, & K ad L eadem eſt, ac compoſita ex rationibus BC ad CG, & DC ad CE.

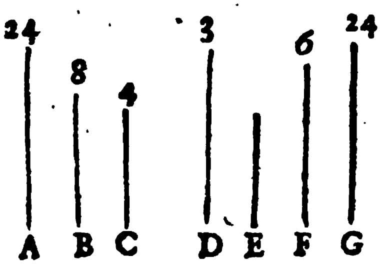
ergo [*per 11. quinti.*] ratio parallelog. BD ad parallelog. CF erit compoſita ex rationibus laterũ BC ad CG, & DC ad CE. q.e.d.

Nota. Componentium rationum antecedentia debent eſſe in uno parallelogrammo, & conſequentia in alio.

SCHOLIUM.

DUm Euclides in hac Propoſ. XXIII. *dicit æquiangula parallelogramma inter ſe habere compoſitam ex duabus rationibus, quas habent latera circa unum angulum in uno parallelogrammo, ad duo latera circa alium angulum æqualem in alio parallelogrammo, hoc ſolum vult intelligere, quod ſi duæ laterum rationes diſponantur in tribus quantitatibus; quemadmodum prima quantitas ad tertiam, ita parallelog. ad parallelog. Inſuper cum dicat idem Author, primam magnitudinem ad tertiam, rationem habere compoſitam ex rationibus primæ ad ſecundam, & ſecundæ ad tertiam: circa hujuſmodi aſſertum neceſſarium putavi in hoc loco addere ſequentem demonſtrationem ex Eutochio antiquiſſimo Euclidis interprete deſumptam.*

Theorema demonſtrandum.



SI fuerint tres quæcumque magnitudines A.B.C.

Dico rationem A ad C eſſe compoſitam ex rationibus A ad B, & ad C iuxtà Def. V. hujus Libri.

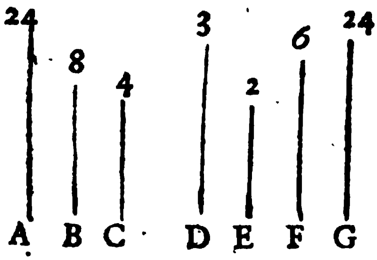
Ad hoc Theorema demonſtrandum cum eodem Eutochio aſſume ſequens axioma.

AXIOMA.

DENOMINATOR, ſivè QUANTITAS illius rationis, quam habet antecedens A ad conſequens B, ducta in conſequens B Producit antecedens A.

Veritas hujus axiomatis ſatis conſtat per ſolam terminorum explicationem, quia omnis rationis quantitas, ſiuè denominator oſtendit habitudinem illam, quam habet antecedens A ad conſequens B: quare ſi conſequens B multiplicetur per illum DENOMINATOREM, *productum erit idem antecedens A.*

Conſtructio.



Denominator rationis A ad B ſit D, & Denominator rationis B ad C, ſit E.

Productum ex his denominatoribus D, & E ſit F.

Dico productum ex F in C eſſe A. Productum ex C in F ſit G: ſi ergo G æqualis fuerit ipſi A; erit verum dicere A eſſe productum ex F in C.

Demonſtratio,

Quia D eſt denominator rationis A *ad* B *[per axio. ſuperius poſitum.] productum ex D in* B *erit* A,

Eadem ratione, cum E ſit denominator rationis B ad C, productum ex E in C, erit B.

Quoniam igitur C multiplicando E facit B, & C multiplicando F facit G; (per 15. quinti.)

F : E :: G : B,

Rurſus quia D multiplicando E facit F, & D multiplicando B facit A (per 15. quinti.)

E : B :: F : A,

Igitur permutando [per 16. quinti.)

E : F :: B : A.

Con-

Convertendo [per corol. 4. quinti.]

F : E :: A : B:

ut autem F ad E (per demonſtrata.] ita G ad B,

ergo (per 11. *quinti.)*

A : B :: G : B,

Quare *(per 9. quinti.)*

A = G.

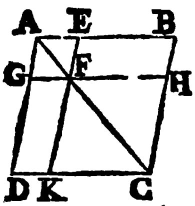
Quia vero (per conſtructionem.) G eſt productum ex F in C, etiam A erit productum ex F in C: quare F erit quantitas, ſiuè denominator rationis primæ magnitudinis A ad tertiam C; ſed F eſt productum ex D denominatore rationis A ad B, & ex E denominatore rationis B ad C: ergo (per def. 5. hujus.) ratio A ad C erit compoſita ex rationibus A ad B, & B ad C. q. e. d.

Propoſ. XXIV. *Theor.* XVIII.

In parallelogrammo DB.

Dico parallelogramma GE, KH circa diametrum AC conſiſtentia, eſſe ſimilia toti parallelogrammo DB, & inter ſe.

Conſtructio.



IN parallelogrammo DK ducatur diameter AC: in dicta diametro notetur punctum F, per quod ducatur recta GH parallela AB, vel DC. Pariter per idem punctum F notetur EK parallela AD, vel BC.

Demonſtratio.

ANgulus DAB communis eſt utrique parallelogrammo DB, GE; ergo æqualis in ambobus parallelogrammis.

Angulus AGF (*per 29. pri.*] æqualis ang. D; & angulus AEF (*per eamdem 29.*) æqualis ang. B.

Pariter angulus GFE (*per 15. pri.*) æqualis angulo KFH; angulus KFH [*per 34. pri.*] æqualis ang. DCB: quare paral-

lellog.GE æquiangulum parallelogrammo DB.

Cum auté (*per conſtruct.*)GF ſit parallela *DC*, (*per corol*: 4. *bujus*.) erit

AG : GF : : AD : DC.

GF : FA : : DC : CA.

AF : FE : : AC : CB.

Quare ex æquo (*per* 22. *quinti*.)

GF : FE : : DC : CB.

Eadem ratione demonſtrabitur

FE : EA : : CB : BA : atque

EA : AG : : BA : AD.

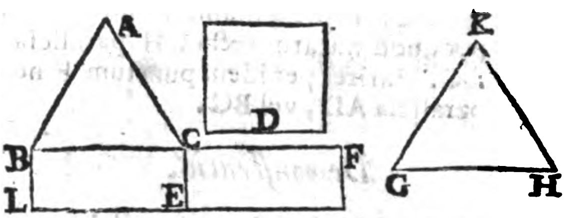
Quamobrem parallelogrammum GE (*per def.* 1. *bujus*.) ſimile erit parallelogrammo DB.

Eodem prorſus modo demonſtrabitur parallelogrammum KH ſimile eidem parallelogrammo DB : quare (*per* 21. *bujus*.] etiam paralleiogamma GE, & KH inter ſe ſimilia. q. e. d.

Propoſ. XXV. Probl. VII.

Duobus datis rectilineis BAC, & D: conſtituere aliud rectilineum GKH ſimile rectilineo BAC, & æquale rectilineo D.

Conſtructio.



AD unum latus illius rectilinei, cui ſimile conſtitui debet, hoc eſt ad latus BC (*per* 44. *vel* 45: *pri.*] in quovis angulo applicetur parallelogrammum BE, æquale rectilineo BAC. Super rectam CE in angulo ECF, qui æqualis ſit angulo CBL (*per*

(*per* 45. *pri.*) conſtituatur paralellog. *EF* æquale rectilineo D.

Inter rectas BC, CF [*per* 13. *bujus.*] inveniatur media proportionalis GH: ad GH (*per* 18. *bujus.*] applicetur rectilineum GKH ſimile, ſimiliterque poſitum rectilineo BAC.

Dico, rectilineum GKH eſſe ſimile dato rectilineo BAC, & æquale rectilineo D.

Demonſtratio.

QVia (*per conſtructionem*] angulus FCE = ang. CBL; (*per* 14. *pri.*) BF erit una linea recta: idem verificatur de linea LI.

Rurſus, quia (*per conſtructionem.*)eſt rectilineum BE ad rectil. C, ita rectil. BAC ad rectil. D.

Ac (*per* 1. *bujus.*]
Rectil. BE: rectil. CI :: BC: CF,
Eſt enim (*per* 11. *quinti.*)
Rectil. BAC: rectil. D :: BC: CF,
Sed [*per corol. prop.* 19. & 20. *bujus.*)
BC: CF: rectil. BAC: rectil. GKH,
Ergo [*per* 11. *quinti.*]
Rectil. BAC: rectil. D :: rectil. BAC: rectil. GKH.
Quare (*per* 9. *quinti.*)
Rectil. GKH = rectil. D. q. e. f.

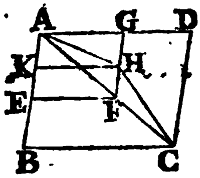
Propoſ. XXVI. Theor. XIX.

Si à parallelogrammo BD ablatum fuerit parallelogrammum EG ſimile, ſimiliterque poſitum toti parallelogrammo BD, communemque cum eo habens angulum A.

Dico parallelogrammum EG circa eamdem diametrum AC conſiſtere.

Con-

Conſtructio.



DUcantur rectæ A F, FC; hoc facto, ſi AFC eſt una linea recta, evidens eſt veritas præſentis propoſitionis.

Si verò AF, FC non componunt rectam lineam, ab A ad C recta ducatur AHC, quæ ſeccabit FG, vel FE: ſecet FG in H; per H notetur HK parallela AG.

Demonſtratio.

QVia (*per conſtructionem.*) AHC eſt recta linea, & diameter parallelogrammi BD: [*per 24. bujus.*] parallelogrammum BD erit ſimile parallelogrammo KG: ſed (*per hyp.*] parallelogrammum EG eſt ſimile parallelogrammo BD: ergo (*per 21. bujus.*) parallelog. KG ſimile parallelog. EG, & ſimiliter poſitum:

Quare (*per def. 1. bujus.*]
GA : AE : : GA : AK,
Ideoque (*per 9. quinti.*)
AE = AK;

Quod cum repugnet veritati axio. 9. AHC non erit linea recta. Cum autem hoc de quacumq, alia linea ab A ad C ducta idem demonſtrari poſſit, non autem de recta AFC; hæc ſola erit dicenda recta linea ab A ad C ducta: ideoque parallelogrammum EG erit circa eamdem diametrum cum toto parallelogrammo BD. q. e. d.

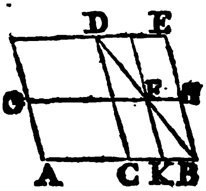
Propoſ. XXVII. Theor. XX.

Si recta AB diviſa ſit bifariam in C, & non bifariam in K, & parti AC applicetur parallelogramma AD, cujus defectus erit CE, & parti AK applicetur parallelogrammum AF,

cujus

cujus defectus erit KH, & hi duo defectus CE, KH ſint duo parallelogramma ſimilia.

Dico AD eſſe majus quam AF.



Conſtructio.

IN parallelogrammo CE, quod eſt defectus parallelogrammi AD, ducatur diameter DB. In hac diametro erit punctum F [*per 26. bujus.*] cum KH, & CE ſint ſimilia.

Demonſtratio.

QVia [*per 43. primi.*) complementum CF = complemento FE.

Communi addito KH erit

CF + KH = FE + KH,

Sed [*per 36. pri.*]

CF + KH = CG,

Quare [*per axio. 1.*)

CG = FE + KH,

Communi addito CF erit

CG + CF, hoc eſt AF = FE + KH + CF.

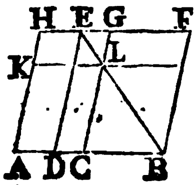
Cum autem [*per axio. 9.*] parallelogrammum CE majus ſit FE + KH + CF, ſitq; parallelogrammum CE (*per 36. pri.*) æquale parallelogrammo AD; erit etiam

Parallelogrammum AD majus parallelogrammo AF. q. e. d.

Idem valebit, ſi diviſa AB bifariam in C, & non bifariam in D parti AC applicetur AL, cujus defectus erit LB, & parti AD applicetur AE, cujus defectus eit EB, & hi defectus LB, EB ſint ſimiles, nam ducta diametro BE, hæc tranſibit per L [*per 26. bujus.*] Et erit AL majus quam AE, ut demonſtro.

De-

Demonſtratio.



QVia (*per hyp.*] AC = CB: ipſi vero AC (*per* 34. *pri.*] æqualis eſt HG, & ipſi CB æqualis GF [*per axio.* 1.] erit HG = GF. Quare [*per* 36. *primi*.) parallelog. GK = parallelog. FL.

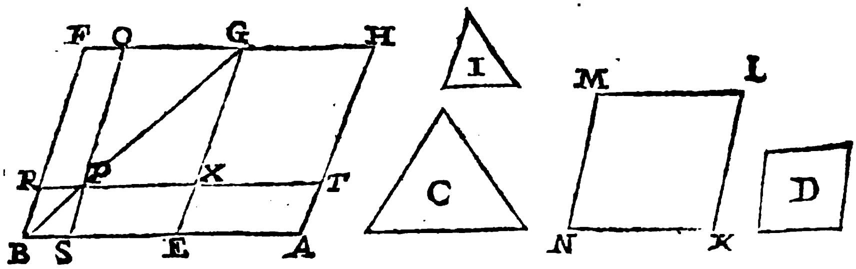
Sed (*per* 43. *primi*.)
Parallelog. FL = parallelog. LD,
ergo (*per axio.* 1.)
KG = LD,
Sed (*per axio.* 9.)
KG majus quam KE:
Ergo, & LD majus quam KE,
Addito communi KD:

erit LD + KD, hoc eſt parallelog. AL ad dimidiam AC applicatum, majus quam KE + KD, hoc eſt AE, applicatum ad AB cum defectu DF ſimili defectui BL. q. e. d.

Propoſ. XXVIII. Probl. VIII.

Ad datam rectam AB applicare parallelogrammum æquale dato rectilineo C, quod deficiat parallelogrammo ſimili alteri dato parallelogrammo D. Oportet autem rectilineum C, cui æquale eſt applicandum parallelogrammum, non eſſe majus illo quod ad dimidiam BE applicari poteſt, & eſt ſimile dato parallelogrammo D.

Con-



Conſtructio.

DAta recta AB (*per* 10. *pri.*) biſariam dividatur in E, ſuper dimidiam BE [*per* 18. *hujus.*) conſtituatur parallelogramum BG ſimile,& ſimiliter poſitum parallelog. dato D. Compleatur parallelog. BH: ducatur EG parallela AH.

Hoc facto parallelog. EH, vel eſt æquale dato rectilineo C, vel majus (minus enim eſſe nequit) (*per ſuppoſitionem.*)

Si igitur parallelog. EH adæquat C, cum defectus ſit parallelog. EF [*per conſtructionem.*] ſimile ipſi D, factum erit. q. e. f.

Si poſtea parallelog. EH majus eſt dato rectilineo C [*per* 45. *pri.*) inveniatur exceſſus parallelog. EH ſupra rectilineum C quod ſit I.

(*Per prop.* 25. *hujus.*] conſtituatur parallelog. KM ſimile parallelog. D, vel EF, & æquale rectilineo I, quod minus erit parallelog. EF, cum I ſit exceſſus parallelogrammi EF ſupra rectilineum C.

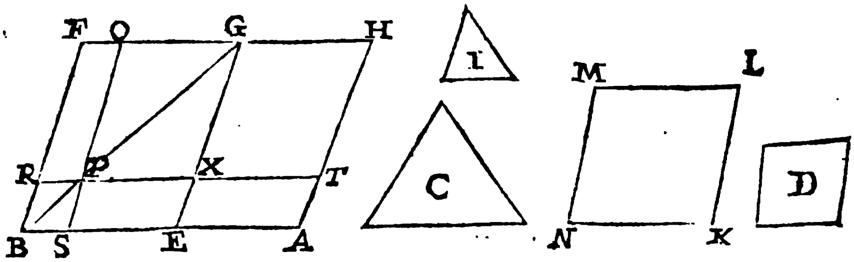
Cum autem (*per conſtructionem.*] parallelogramma EF, & KM ſint ambo ſimilia rectilineo D; (*per* 21. *hujus.*) inter ſe ſimilia erunt: quare [*per def.* 1. *hujus.*] habebunt latera circa æquales angulos proportionalia, videlicet

KL : LM : : EG : GF,

Quia verò (*ex demonſtratis.*) parallelog. EF majus eſt parallelogrammo KM, etiam recta EG major quam KL, & recta GF major quam LM.

Tan-

Tandem fiat GX = KL, atque GO = LM. Per O notetur OS parallela *F*B. Per X ducatur recta RT parallela *BA*.



Dico parallelogrammum AP ad AR applicatum, æquale eſſe rectilineo *C*, ejuſque defectum RS eſſe ſimile dato parallelogrammo D.

Ducatur GB, quæ (*per 26. hujus.*) tranſibit per P. nam cum ſit

OG : GX :: FG : GE,

erunt parallelogramma OX, FE ſimilia.

Demonſtratio.

Cum (*per conſtructionem*] KM ſit exceſſus parallelogrammi EF ſupra rectilineum C: erunt *K*M, & C = EF.

Sed XO factum fuit æquale *K*M: ergo XO eſt exceſſus parallelogrammi EF ſupra C.

Quare parallelogramma OR, RE æqualia rectilineo C: ſed (*per 36. pri.*) ipſi RE = ET: ergo OR + ET = rectilineo C:

Quia vero (*per 43. primi.*) O*R* = EP; erit ET + EP, videlicet parallel. AP = dato rectilineo C.

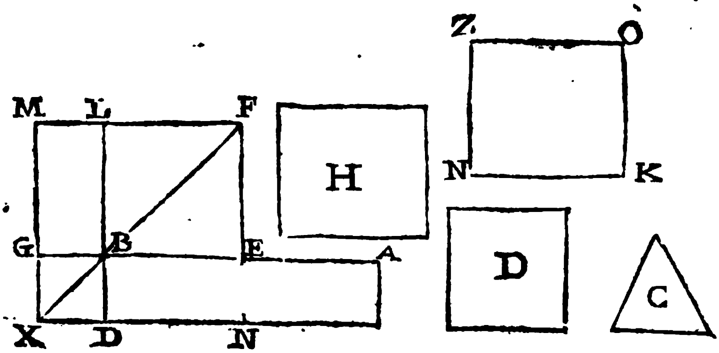
Demum quia defectus RS (*per 24. bujus.*) ſimilis eſt EF, & EF

(*per*

(*per conſtructionem.*) ſimile ipſi D: [*per* 21. *bujus.*] RS ſimile ipſi D : quare parallelogrammum AP ad AB rectam applicatum, erit æquale dato rectilineo C, ejuſque defectus RS erit parallelogrammum ſimile dato parallelogrammo D. q. e. f.

Propoſ. XXIX. Probl. IX.

Ad datam rectam AB applicare parallelogrammum æquale dato rectilineo C, excedens parallelogrammo ſimili alteri dato parallelogrammo D.



Conſtructio.

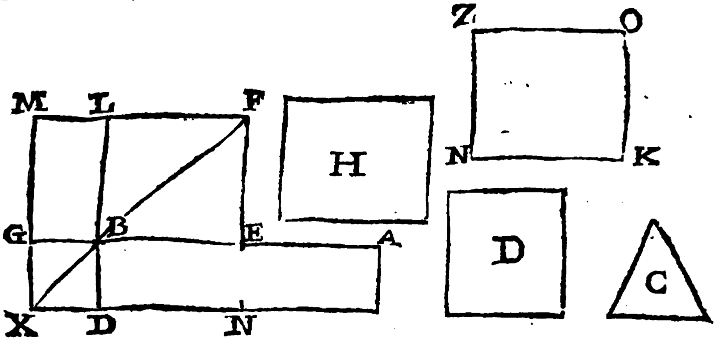
DAta recta AB (*per* 10. *pri.*) biſariam dividatur in E. Supra dimidiam BE [*per* 18. *bujus.*] conſtituatur parallelogrammum EL ſimile, & ſimiliter poſitum dato paralellogrammo D.

Pariter (*per* 14. *ſecundi*) rectilineo C, & parallelogrammo EL æquale conſtituatur quadratum H, cui quadrato H (*per* 25. *buius.*) æquale fiat parallelog *KZ*, ſimile, atque ſimiliter poſitum parallelog. EL.

Hoc facto erit *Z*K majus quam EL. Cum autem K*Z*, & EL facta ſint ſimilia, ſimiliterque poſita; erunt latera *K*O, O*Z* majora lateribus EF, FL: productis minoribus lateribus EF, FL, fiat FN = OK, & FM = O*Z*. Com-

Compleatur parallelogrammum MN, quod erit ſimile, ſimiliterque poſitum parallelogrammo EL: quare [*per 26. hujus.*] parallelogramma MN, EL erunt circa eamdem diametrum FX, atque MN = KZ ob ſimilitudinem, quam habent MN. & ZK, & laterum æqualitatem.

Producatur LB in D, compleaturque parallelogrammum DA. Dico parallelog. AD applicatum ad rectam AB æquale eſſe dato rectilineo C, & exceſſum DG ſimile eſſe dato parallelogrammo D.



Demonſtratio.

QVia [*per 36. pri.*] parallelog. AN = parallelog. BN atque (*per 43. pri.*) parallelog. BN = parallelogrammo BM: (*per axio. 1.*) erit parallelog. AN = parallelog. BM, Communi addito NG erit AN + NG, hoc eſt AX = BM + NG.

Cum autem (*per conſtructionem*) MN = KZ, & KZ = EL + C, Erit BM + NG = C: ſed AX = BM + NG: ergo, & AX = C.

Demum quia parallelog. GD ſimile eſt parallelogrammo D: ſequitur ad datam AB fuiſſe applicatum parallelog. AX æquale dato rectilineo C, & excedere parallelogrammo GD ſimili dato parallelogrammo D. q. e. f.

Pro-

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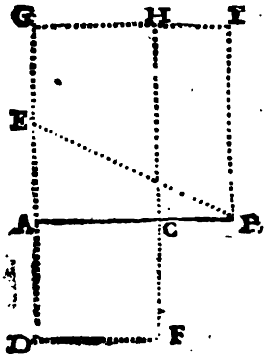
Propoſ. XXX. Probl. X.

Datam rectam AB terminatam taliter ſecare, ut ſit tota ad majus ſegmentum, uti majus ſegmentum ad minus.

Conſtructio.

Conſtructio hujus problematis illa eadem eſt, quæ in propoſ. 11. lib. 2. propoſita fuit pro conſtituendo rectangulo CI = quadrato AF. Tali igitur conſtructione ſuppoſita:

Dico rectam AB in C eſſe taliter ſectam, ut ſit AB ad AC, uti AC ad CB: ideoque datam rectam extrema, & media ratione ſectam dicunt.



Demonſtratio.

Quia (*per* 11. *ſecundi.*) rectangulum CI = quad. AF. (*per* 14. *hujus.*)
HC : CF : : AC : CB;
Sed [*per def. quadrati.*)
AC = CF, & HC = AB,
Quare mutatis HC in AB, & F in AC, erit
AB : AC : : AC : CB. q.e.f.

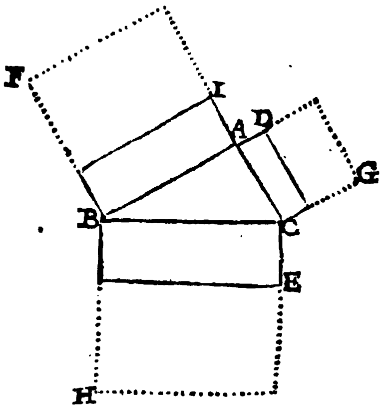
Propoſ. XXXI. Theor. XXI.

In triangulo rectangulo BAC cum angulo recto A.

Dico figuram ſupra BC deſcriptam æqualem eſſe figuris ſibi ſimilibus a lateribus BA, AC deſcriptis.

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Conſtructio.



AD dati trianguli latera AB, BC, CA [*per 18. bujus.*) applicentur ſimiles figure BI, CD, BE.

Dico figuram BE deſcriptam a latere ſubtendente angulum rectum A, æqualem eſſe figuris ſimilibus BI, CD lateribus BA, AC angulum rectum comprehendentibus, applicatis.

Conſtructio.

Super latera trianguli rectanguli [*per 46. primi.*) tria deſcribantur quadrata CH, AF, AG.

Demonſtratio.

QVia tam deſcriptæ figuræ, quam quadrata [*per conſtructionem.*) ſunt inter ſe ſimilia.

[*per 20. bujus.*]

quad. AF eſt ad quad. CH in duplicata ratione AB ad BC; pariterq;
fig. BI eſt ad fig. BE in duplicata ratione AB ad BC:
Quare (*per 11. quinti.*)
Ut quad. AF ad quad. CH, ita fig. BI ad fig. BE;
Ob eamdem rationem erit etiam
Ut quad. AG ad quad. CH, ita fig. CD ad fig. BE.
His demonſtratis: erit
Ut prima magnitudo quad. AF ad ſecundam quad. CH,
Ita tertia fig. BI ad quartam figuram BE.
Pariter ut quinta quad. AG, ad ſecundam quad. CH,
Ita ſexta fig. CD ad quartam fig. BE:
Ergo (*per 24. quinti.*)
Ut prima quad. AF cum quinta quad. AG ad ſecundam quad. CH,
Ita tertia fig. BI cum ſexta fig. CD ad quartam fig. BE:
Sed [*per 47. pri.*] Qua-

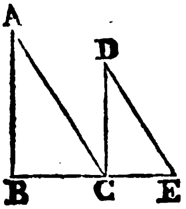
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Quad. AF,& AG ad quad. CH rationem habent æqualitatis;
Quare etiam
Ratio figurarum BI, & CD ad fig. BE erit ratio æqualitatis,
Ideoque figuræ BI, CD a lateribus BA, AC deſcriptæ = fig. BE.
q. e. d.

Propoſ. XXXII. *Theor.* XXII.

Si duo triangula BAC, CDE habeant latera BA, AC proportionalia lateribus CD, DE, atque ſecundum unum angulum ACD taliter compoſita fuerint, ut homologa latera AB, DC, atque AC, DE ſint parallela.

Dico reliqua horum triangulorum latera BC, CE eſſe in directum poſita.



Demonſtratio.

Quia (*per hyp.*) latus AB parallelum DC [*per 29. pri.*) ang. BAC = ang. ACD. Pariter quia ſupponitur latus AC parallelum lateri DE [*per eamdem 29.*) ang. ACD = ang. CDE: quare [*per axio. 1.*) ang. A = ang. D.

Cum autem triangula BAC, CDE [*per demonſtrata*] æquales habeant angulos A, & D, atque (*per hyp.*) circa iſtos æquales angulos latera proportionalia: [*per 6. hujus.*) dicta triangula erunt æquiangula; ideoque ang. ABC = ang. DCE. additis æqualibus angulis BAC, ACD: erunt

anguli $\left.\begin{matrix}A\\B\end{matrix}\right\}$ = angulis $\left.\begin{matrix}ACD\\DCE\end{matrix}\right\}$ hoc eſt angulo ACE.

Denuo addito communi angulo ACB: erunt

anguli $\left.\begin{matrix}A\\B\\ACB\end{matrix}\right\}$ = angulis $\left.\begin{matrix}ACE\\ACB\end{matrix}\right\}$:

 Sed

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Sed [per 32. pri.)

anguli A, B, ACB } = duobus rectis: quare etiam

anguli ACE, ACB } = duobus rectis:

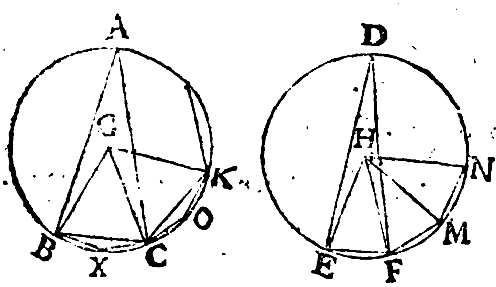
Igitur (per 14. primi.]
latus BC erit in rectum cum latere CE. q. e. d.

Propoſ. XXXIII. *Theor.* XXIII.

In æqualibus circulis, ABC, DEF, ſi ad centra conſtituti ſint anguli BGC, EHF, vel ad peripherias BAC, EDF.

Dico angulum BGC eſſe ad angulum EHF, ut peripheria BC ad peripheriam EF: Sectoremque BGC ad ſectorem EHF, ut peripheria BC, ad peripheriam EF.

Conſtructio prioris partis.



DUcantur rectæ BC, EF, atque (per 1. quarti.)in circulo ABC accomodetur CK = BC: pariter in circulo EDF adaptentur rectæ FM, MN = EF, notenturque rectæ KG, MH, NH.

Demonſtratio primæ partis.

QVoniam (*per conſtructionem.*) BC = CK (*per* 28. *tertij.*) arcus BC = arcui CK: unde (*per* 27. *tertij.*) ang. BGC = ang.

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ang. CGK. Eadem ratione in circulo EDF æquales erunt arcus EF, FM, MN, & anguli EHF, FHM, MHN: ergo

Quam multiplex eſt arcus BCK arcus BC,
tam multiplex etiam erit ang. BGK anguli BGC.
Pariter quam multiplex eſt arcus EFMN arcus EF,
tam multiplex etiam ang. EHN anguli EHF.
Ulterius ſi arcus BCK æqualis ſit arcui EFMN;
[*per* 27. *tertij*] & ang. BGK = ang. EHN.
Si arcus BCK minor fuerit arcu EFMN,
& ang. BGK minor ang. EHN.
Si denique arcus BCK major fuerit arcu EFMN,
& angulus BGK major erit angulo EHN.

Si ergo hæc pariter multiplicia tam arcuum BC, EF, quam angulorum BGC, EHF ſervant ordinem æqualitatis ſuperioritatis, & inferioritatis præſcritum in def. 6. libri quinti, fatendum erit,

Arcum BC eſſe ad arcum EF, uti ang. BGC ad ang. EHF. q. e. 1. d.

Hoc idem demonſtrabitur de angulis A, & D ad circumferentiam conſtitutis; quia (*per* 28. *tertij*.) dicti anguli A, & D ſunt medietates angulorum ad centra BGC, EHF; quare ut ang. A ad ang. D, ita arcus *BC* ad arcum EF. q. e. 2. d.

Conſtructio alterius partis.

In ſegmentis *BC*, *CK* conſtituantur anguli *BXC*, *COK*.

Demonſtratio.

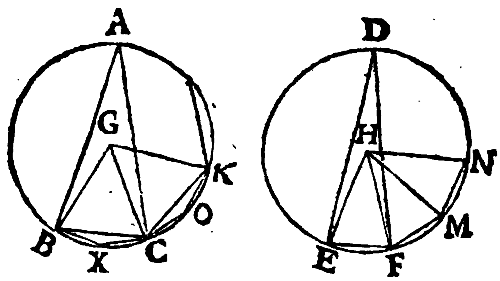
Cum anguli *BXC*, COK inſiſtant æqualibus arcubus *BAC*, *CBAK* (*per* 27. *tertij*.) erunt æquales; quare (*per def.* 10. *tertij*.) ſegmentum *BXC* ſimile ſegmento COK, ideoque [*per* 24. *tertij*.) dicta ſegmenta inter ſe æqualia.

Quia vero (*per* 4. *primi*.) etiam æqualia ſunt triangula *BGC*, CGK, ſi æqualibus ſegmentis addantur æqualia triangula, reſultabunt ſectores *BGC*, *CGK* inter ſe æquales: Quamobrem quam multiplex eſt ſector BG*K* ſectoris *BGC*, tam multiplex etiam arcus *BC*K, arcus *BC*.

Ulterius ſi arcus BCK ſit æqualis arcui EFMN,

etiam

etiam ſector *B*GK erit æqualis ſectori EHN.
Si arcus major arcu, etiam ſector major ſectore, &
Si arcus minor arcu, etiam ſector minor ſectore.
Hoc ſtante ſequitur
Quando ſector *B*G*K* major eſt ſectore EHN,
etiam arcus *B*CK major arcu EFMN.
Quando ſector = ſectori, etiam arcus = arcui; &
Quando ſector minor ſectore, etiam arcus minor arcu.



His poſitis ſi ſector *B*G*C* ſtatuatur prima magnitudo; ſector EHF ſecunda; arcus *B**C* tertia; & arcus EF quarta.

Quia (*per conſtructionem.*) primæ, & tertiæ ſumpta ſunt pariter multiplicia, videlicet ſector *B*GK, & arcus *B*CK: pariter ſecundæ, & quartæ accepta ſunt æqualiter multiplicia, nempe ſector EHN, & arcus EFMN: de iſtis autem æqualiter multiplicibus demonſtratum fuit, quod

Quando multiplex primæ magnitudinis ſuperat multiplicem ſecundæ, etiam multiplex tertiæ ſuperat multiplicem quartæ.

Quando æquale, æquale; & quando minus, minus: ſequitur

[*per def.* 6. *quinti*.]

Sectorem *B*G*C* eſſe ad ſectorem EHF, ut arcum *B**C* ad arcum EF. q. e. d.

Corollarium I.

EX hoc Theoremate fit manifeſtum, ut ſector ad ſectorem, ita eſſe angulum unius ſectoris ad angulum alterius ſectoris;

quia

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quia [*per ſuperius demonſtrata*.) tam anguli, quam ſectores inter ſe ſunt ut arcus ad arcum: quare

(*per* 11. *quinti*.)

Sector ad ſectorem ut angulus ſectoris ad angulum ſectoris.

Corollarium II.

RUrſus perſpicuum, eſt ita eſſe angulum in centro *BGC* ad quatuor rectos, uti eſt arcus *BC* illi angulo ſubtenſus, ad totam circumferentiam: quia

Ut ang. in centro *BGC* ad ang. rectum in centro,
Ita circumferentia *BC* ad circumferentiam ſubtenſam angulo recto: quare
Ut angulus in centro, ad quatuor rectos (*per* 15. *quinti*.)
Ita arcus illi angulo ſubtenſus ad quadruplum quadrantis,
hoc eſt ad totam circumferentiam:
Convertendo igitur,
Ut quatuor recti ad angulum in centro, ita tota circumferentia ad arcum angulo in centro ſubtenſum. q. e. d.

Elementi ſexti finis.

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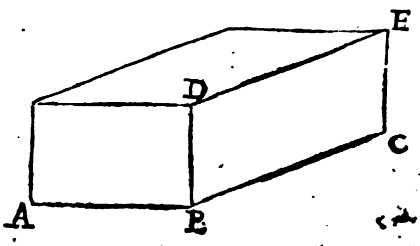
EUCLIDIS
ELEMENTUM UNDECIMUM, ET SOLIDORUM ELEMENTUM PRIMUM.

DEFINITIONES.

Def. I.

SOLIDUM, ſiue CORPUS dicitur illud, quod longitudinem, latitudinem, atque craſſitiem habet.

IN explicatione defin. 1. primi Elementi dictum fuit in magnitudinibus, triplicem dari partium componentium ordinationem, nempè in longum, in latum, & in craſſum. In def. 2. ejuſdem primi Elementi illa magnitudo, quæ ſolam habet longitudinem, dicta fuit LINEA. In defin. 5. illa magnitudo, quæ longitudinem, & latitudinem obtinet, appellata fuit SUPERFICIES. In hac verò defin. illæ magnitudines, quarum componentes partes in longum, latum, atque craſſum ordinantur, a Mathematicis SOLIDA, ſiue CORPORA appellantur, illaque doctrina, quæ ſolida, ſiue corpora contemplatur, peculiari vocabulo STEREOMETRIA dicitur: hoc cernere licet in appoſito ſolido A E, in quo BC eſt longitudo, BA latitudo, BD vero craſſities.



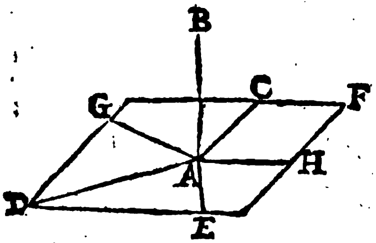
Def. II.

TERMINI ſolidorum dicuntur ſuperficies.

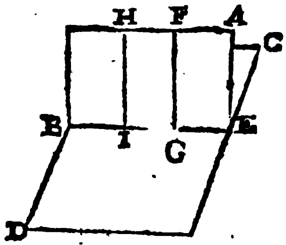
QVemadmodum Euclides in defin. 3. Lib. primi Elementorum lineæ terminos puncta nominavit, atque in defin. 6. ejuſdem Elementi ſuperficiei terminos lineas vocavit, ita in hoc loco ſolidorum, ſiuè corporum terminos ſuperficies dicit.

Def. III.

RECTA LINEA BA dicitur ad PLANUM FD recta, quando angulos efficit rectos cum omnibus lineis CA, HA, EA &c. in propoſito plano ductis, illamque tangentibus in A.



DUm recta linea AB taliter inſiſtit plano FD, ut ſolum punctum A ſit in plano FD, reliqua vero extra, anguloſque facit rectos cum omnibus lineis CA, HA, EA &c. in plano FD ductis, & concurrentibus in A; tunc recta linea AB dicitur RECTA ſubiecto plano FD.

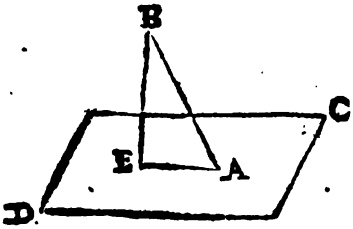


Defi. IV.

Planum AB ad planum DC dicitur RECTUM, cum rectæ lineæ FG, HI &c. in plano AB ductæ, communique planorum ſectioni BE perpendiculares, alteri plano CD rectæ ſunt.

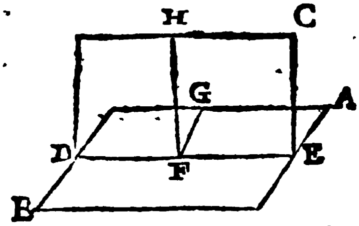
Def.

Def. V.

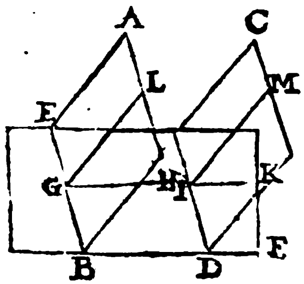


INCLINATIO lineæ rectæ BA ad planum CD dicitur angulus acutus B AE efformatus a linea BA inſiſtente ſubiecto plano CD, & a recta AE in eodem plano CD ducta a puncto A ad punctum E, in quod cadit BE recta plano CD.

Def. VI.



INCLINATIO plani CD ad planum AB vocatur angulus acutus HFG comprehenſus a recta linea HF in plano CD conſtituta, & perpendiculari communi planorum ſectioni DE, & a recta linea FG ducta in plano AB, & perpendiculari eidem communi planorum ſectioni DE.



Def. VII.

SIMILITER inclinata dicuntur plana AB, CD ad planum EF, quando anguli inclinationis planorum LGH, MIK ſunt æquales.

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Def. VIII.

PARALLELA Plana vocantur AB, CD, quando in quamcumque partem producta, numquam concurrunt.

Def. IX.

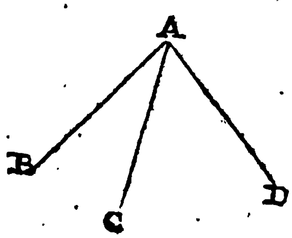
SIMILIA Solida dicuntur illa, quæ a planis ſimilibus, & numero æqualibus continentur.

Def. X.

ÆQUALIA, & SIMILIA appellantur illa ſolida, quæ a ſimilibus planis numero, ac magnitudine æqualibus comprehenduntur.

Def. XI.

SOLIDUS angulus A ille dicitur, qui a pluribus quam duobus planis angulis BAD, BAC, CAD ad idem punctum A conſtitutis, neque in eodem plano exiſtentibus, continetur.



SI concipiamus tres planos angulos BAD, BAC, CAD, ad idem punctum A eſſe conſtitutos, neque in eodem exiſtere plano, quia linea AC eſt intelligẽda extra planum anguli BAD; angulus A a tribus angulis planis BAD, BAC, CAD comprehenſus, vocatur angulus ſolidus.

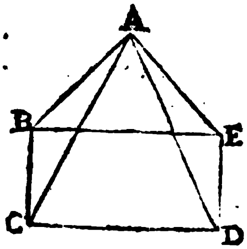
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Notandum. Duo tantum plani anguli nequeunt ſolidum angulum componere, cum ad minus tres requirantur.

Def. XII.

PYRAMIS dicitur figura ſolida BCDEA contenta à planis BAC, CAD, DAE, EAB a plano BD ad punctum A ductis.

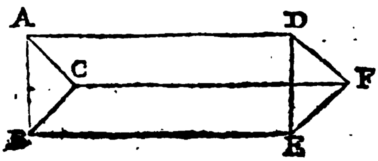


Notandum. Punctum A, ad quod concurrunt plana BAC, CAD &c. Pyramidis VERTEX nuncupatur: Planum BD dicitur BASIS pyramidis. Diſtantia puncti A a plano BD vocatur ALTITUDO pyramidis.

Si pyramidis baſis ſit triangulum, etiam pyramis triangularis dicitur: ſi baſis ſit quadrangulum, vocatur quadrangularis; ſi pentagonum pentagonalis &c.

Def. XIII.

PRISMA vocatur figura ſolida ABEDCF a planis contenta, quorum duo oppoſita ACB, DFE ſunt æqualia, ſimilia, & parallela; reliqua vero plana AE, AF, BF ſunt parallelogramma.



Notandum. Quando priſmatum oppoſita plana ſunt triangula, tunc priſma dicitur triangulare; quando ſunt quadrangula, priſma vocatur quadrangulare &c.

Def. XIV.

SPHÆRA dicitur, quando ſemicirculi manente diametro circum ductus ſemicirculus in ſe ipſum revolvitur, unde moveri ceperat, circumaſſumpta figura.

HÆc eſt Sphæræ Defin. ab Euclide tradita in hoc loco. Theodoſius Lib. I. Sphæricorum Defin. I. aliter ſphæram definivit, dicendo

Spæra dicitur figura ſolida unica ſuperficie comprehenſa, ad quam ab uno puncto eorum, quæ intra figuram ſunt poſita, omnes cadentes rectæ lineæ, ſunt inter ſe æquales.

Def. XV.

CENTRUM Sphæræ dicitur illud punctum, a quo omnes rectæ lineæ ad ſphæræ ſuperficiem ductæ, ſunt æquales.

Def. XVI.

AXIS ſphæræ appellatur quæcumque recta linea per ſphæræ centrum ducta, & utrimque a ſuperficiæ ſphæræ terminata, ſupra quam immobilem revolvitur ſphæra.

Def. XVII.

DIAMETER ſphæræ vocatur quælibet recta linea per ſphæræ centrum ducta, & utrimque a ſphæræ ſuperficie terminata.

Def.

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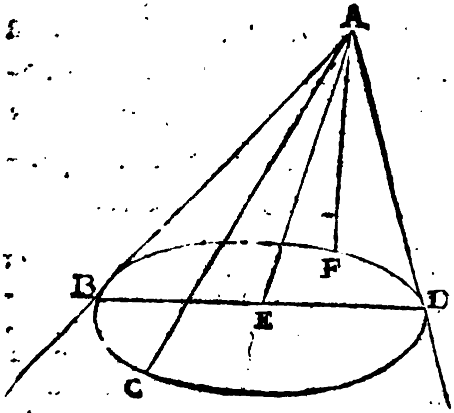
Def. XVIII.

POLI ſphæræ appellantur duo puncta axem terminantia.

Def. XIX.

CONICA ſuperficies dicitur illa, quæ deſcribitur, dum ab aliquo puncto immobili extra circuli planum accepto ad circuli circumferentiam ducta recta linea in utramque partem producta, manente puncto convertitur circa circuli peripheriam, quouſque circuli revolutionem abſolvat.

Fig I.

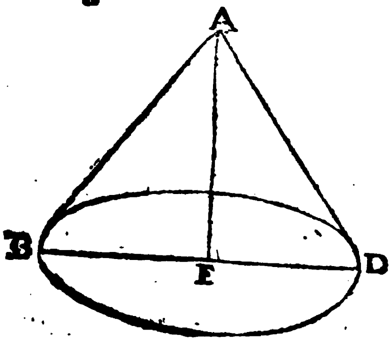


IN appoſita figura ſi extra planum circuli BCD acceptum fuerit punctum A, a quo ad dati circuli circumferentiam ducta ſit recta AB, & manente puncto A, recta AB intelligatur moveri per totam circumferentiam BCD, per talem motum genita ſuperficies dicitur CONICA ſuperficies.

Def. XX.

CONUS appellatur illa figura ſolida, quæ circulo, & conica ſuperficie comprehenditur, uti eſt ſolidum BAD.

Fig. II.

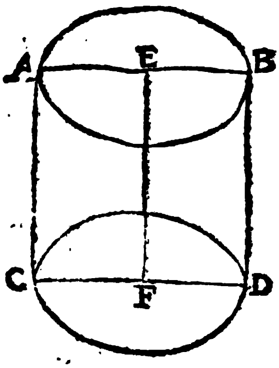


1. VERTEX coni dicitur punctum A.
2. BASIS coni vocatur circulus BCD.
3. AXIS coni appellatur recta AE a vertice ad baſis centrum ducta.
4. CONUS RECTUS illæ dicitur, qui habet AXEM AE, *fig.* 2. baſi BC rectum.
5. CONUS SCALENUS vocatur illæ, qui AXEM AE habet baſi BCD inclinatum. *fig.* I.

Def. XXI.

CYLINDRICA ſuperficies dicitur illa, quæ deſignatur, dum duorum circulorum æqualium, & parallelorum diametri parallelæ, una cum recta linea diametrorum terminos ab eadem parte coniungente, in circulorum planis circa manentia centra circumferuntur quouſque integram compleant revolutionem.

Fig.I.



SI fuerint duo circuli AB, CD inter ſe æquales, & paralleli; pariterque horum circulorum diametri AEB, CFD æquediſtantes, coniungatque ad eaſdem partes A, & C diametrorum extremitates recta AC. Dum dictæ diametri circa manentia circulorum centra E, F, una cum recta AC revolvuntur, completa revolutione ſuperficies ſignata a recta linea AC, CYLINDRICA ſuperficies appellatur.

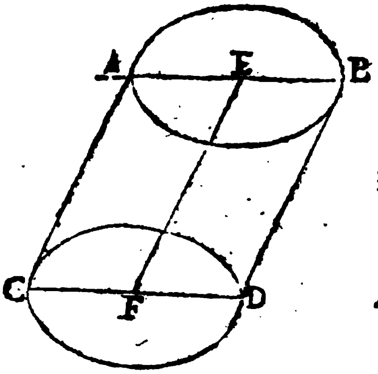
Def.

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Def. XXII.

CYLINDRUS dicitur illa figura ſolida, quæ a duobus æqualibus, & parallelis circulis, ac cylindrica ſuperficie comprehenditur.

Fig. II.



1. BASES Cylindri vocantur duo circuli AB, CD æquales, & paralleli.
2. AXIS Cylindri dicitur recta EF coniungens circulorum centra E, & F.
3. LATUS Cylindri appellatur recta AC ad eaſdem partes coniungens circulorum diametros AB, CD.
4. RECTUS Cylindrus nuncupatur, qui axem habet ad baſes rectum, ut in *Fig. I.*
5. SCALENUS Cylindrus dicitur, qui Axem habet ad baſes inclinatum, ut in *Fig. II.*

Def. XXIII.

SIMILES recti Coni, & Cylindri vocantur illi, qui axes, & baſium diametros habent proportionales.

Def. XXIV.

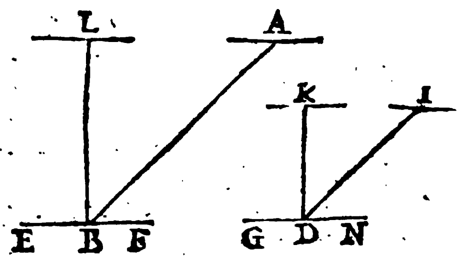
SIMILES Scaleni Coni, & Cylindri appellantur illi, qui AXES habent baſibus ſimiliter inclinatos, atque baſium diametris proportionales.

SCHO.

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SCHOLIUM.

QUia Euclides tantummodo conos, atque Cylindros rectos definivit, non autem Scalenos, cum ſatis manifeſtum ſit ad habendam ſimilitudinem in Conis, & in Cylindris Scalenis, haud ſatis eſſe AXES, & baſium diametros eſſe proportionales; Ut ea, quæ ab eodem Euclide de ſolis Conis, & Cylindris rectis ſimilibus fuerunt demonſtrata, etiam Scalenis ſimilibus accomodari poſſint, neceſſarium fuit definire etiam Conos, atque Cylindros Scalenos ſimiles: quemadmodum in ſuperiore definitione factum fuit. Hac enim poſita definitione, quæ de Conis, ac Cylindris rectis ſimilibus demonſtrantur, etiam de ſcalenis pariter ſimilibus verificantur: hocque taliter ſuadetur.



Sint in Cylindris Scalenis ſimilibus diametri baſium EF, GN, *& Axes BA, DI.* Super *baſes* EF, GN *in eadem altitudine cum* Cylindris Scalenis *intelligantur* Cylindri *recti, quorum Axes ſint BL, DK.*

Dico Cylindros rectos eſſe ſimiles: quia cum ſupponantur ſimiles Cylindri Scaleni (per def. 24. hujus.) angulus BAL = angulo DIK: *pariter quia (per hyp.) æquales etiam ſunt anguli BLA,* DKI, *quia recti; triangula ALB,* IKD: *(per 32. pri.) erunt æquiangula: quare (per 4. ſexti.) etiam ſimilia; ideoque*

LB: BA :: KD: DI

Cum autem (*per hyp.*)

BA: DI :: EF : GN:

Erit (*per 11. quinti.*)

LB: EF: KD :: GN,

Et alternando (*per 16. quinti.*)

LB: KD :: EF : GN.

Igitur in cylindris rectis axes LB, KD *ſunt baſium diametris* EF, GN *proportionales: ideoq; cylindri ſunt ſimiles: quia verò cylindrus rectus, cujus Axis BL = Cylindro Scaleno cum Axe BA, atq; Cylin-*

lindrus rectus, cum Axe DK = Cylindro Scaleno in Axe DL (inferius demonſtrabitur) cum eamdem habeant baſim, & altitudinem; ea, quæ de Cylindris rectis ſimilibus dicuntur, etiam ſcalenis ſimilibus ſuperius definitis accommodari poterunt.

Quod de Cylindris dictum eſt, eſt de Conis intelligendum. Quare &c.

Def. XXV.

CUBUS dicitur figura ſolida ſub ſex æqualibus quadratis contenta.

Def. XXVI.

TETRAEDRUM vocatur illa ſolida figura, quæ ſub quatuor triangulis æqualibus, & æquilateris continetur.

Def. XXVII.

OCTAEDRUM appellatur figura Solida ſub octo triangulis æqualibus, & æquilateris comprehenſa.

Def. XXVIII.

DODECAEDRUM nominatur illa figura ſolida, quæ ſub duodecim Pentagonis æqualibus, æquilateris, & æquiangulis continetur.

Def. XXIX.

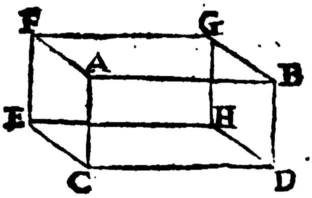
ICOSAEDRUM dicitur illa figura ſolida, quæ ſub viginti triangulis æqualibus, & æquilateris comprehenditur.

Def.

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Def, XXX.

PARALLELEPIPEDUM nuncupatur illa figura ſolida, a ſex planis figuris quadrilateris contenta, quarum oppoſitæ ſunt parallelæ.



SI ſolida figura FD contineatur a ſex figuris planis quadrilateris CF, DG, CB, EG, CH, AG, quarum oppoſitæ CF, DG: CB, EG: CH, AG parallelæ ſint; talis figura dicenda erit PARALLELEPIPEDUM.

Def. XXXI.

Solida figura in ſolida figura INSCRIBI dicitur, quando omnes anguli figuræ inſcriptæ conſtituuntur, vel in angulis, vel in lateribus, vel denique in planis illius figuræ, cui inſcribitur.

Def. XXXII.

Solida figura ſolidæ figuræ CIRCUMSCRIBI dicitur, quando vel anguli, vel latera, vel denique plana figuræ circumſcriptæ, tangunt omnes angulos illius figuræ, circum quam deſcribitur.

Poſtulatum I.

Per duas rectas lineas ad punctum concurrentes planum ducere.

Poſtulatum II.

Per tria puncta planum ducere.

Pro-

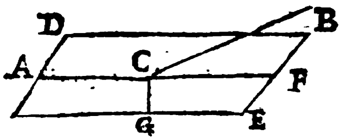
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Propoſ. I. Theo. I.

Si rectæ lineæ ACB pars AC fuerit in plano DE.

Dico totam lineam ACB eſſe in eodem plano DE.

Conſtructio.



SI pars AC poteſt eſſe in plano DE, pars vero CB extra planum DE; a puncto C in plano DE [*per 11. primi.*) ducatur CG perpendicularis ipſi AC. Vlterius in eodem plano DE a puncto C ipſi CG notetur perpendicularis CF.

Demonſtratio.

TRes rectæ lineæ CA, CG, CF [*per conſtructionem.*) ſunt in eodem plano DE. Anguli ACG, FCG, quia facti ſunt recti, erunt duobus rectis æquales: quare [*per 14. pri.*) ACF erit linea recta: cum autem [*per hyp.*] recta ſit linea ACB, duæ rectæ lineæ ACF, ACB, idem ſegmentum AC commune habebunt: quod eſt contra Axio. 10. Si ergo pars AC eſt in plano DE, etiam tota ACB in eodem erit plano DE. q.e.d.

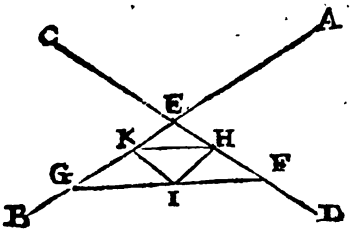
Propoſ. II. Theor. II.

Si duæ rectæ lineæ AB, CD ſe mutuo ſecent in E.

Dico eas eſſe in eodem plano, atque triangulum omne in uno exiſtere plano.

Con-

Conſtructio.



IN rectis EB, ED, utcumque notentur duo puncta G, & F, unianturque recta GF. Pariter in lateribus trianguli E G F ſignentur puncta H, I, *K*, addanturque rectæ HI, I*K*, KH.

Demonſtratio ſecundæ partis.

SI trianguli GEF aliqua pars EKIF poteſt eſſe in uno plano; altera, pars vero GKI in alio plano: etiam rectarum EG, FG partes E*K*, FI erunt in uno plano, partes verò KG, IG in alio plano: quod cum fieri nequeat [*per* 1. *hujus.*) in illo plano, in quo eſt pars E*K*IF, eſt etiam alia pars KGI. Cum autem hoc valeat de quibuſcumque alijs partibus trianguli EGF: ſequitur quod totum triangulum EGF ſit in uno plano. q. e. 1. d.

Demonſtratio primæ partis.

TOtum triangulum GEF [*per demonſtrata.*] eſt in uno plano; ſed in illo plano, in quo eſt triangulum G E F, ſunt etiam rectæ EG, EF: ergo rectæ EG, EF in eodem ſunt plano. Quia vero (*per* 1. *hujus.*) in illo plano, in quo eſt GE, reperitur etiam EA; & in plano, in quo poſita eſt FE, eſt etiam EC: ergo rectæ BA, DC ſe mutuo ſecantes in E, in uno erunt plano. q. e. 2. d.

Propoſ. III. Theor. III.

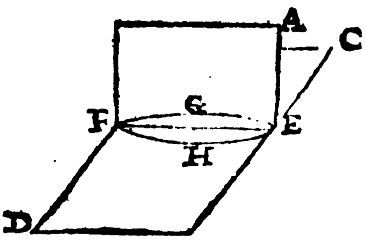
Si planum AF ſecet planum CB.

Dico lineam EF, communem ſectionem planorum AF, CB eſſe lineam rectam.

Con-

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Conſtructio, & Demonſtratio.



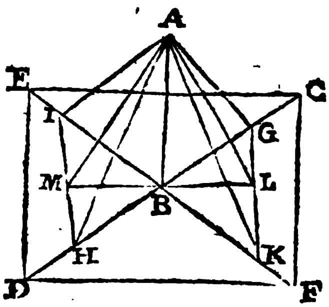
Quia in quovis plano a puncto ad punctum ſemper duci poteſt linea recta, ſi linea FE in utroque plano AF,CB conſtituta, non eſt recta linea; in plano AF a puncto F ad E recta ducatur EGF; pariter in plano CB ab E ad F notetur recta EHF: quo facto duæ rectæ lineæ claudent ſpatium: contra Axio. 14: ergo ſola linea EF communis ſectio planorum AF, CB, erit linea recta. q. e. d.

Propoſ. IV. *Theor.* IV.

Si recta linea AB perpendiculariter inſiſtat duabus rectis lineis CD, EF ſe mutuo ſecantibus in B.

Dico AB rectam eſſe plano CD, per rectas CD, EF ducto.

Conſtructio.



Fiat BH = BG, atque BK = BI: notentur rectæ IH,& GK. Per B ducatur recta MBL, addanturque rectæ IA, MA, HA, KA, LA, GA.

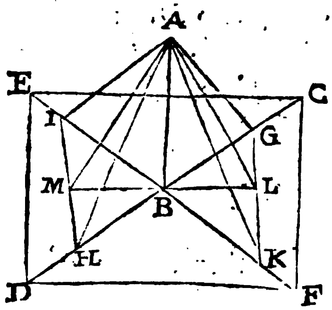
Demonſtratio.

In triangulis $\left.\begin{matrix}ABG\\ABH\end{matrix}\right)$ (*per conſtructionem.*] latera AB, BG ſunt ſingula ſingulis æqualia lateribus AB, BH, & [*per hyp.*] angulus ABG = ang. ABH: quare (*per 4. primi.*) AG = AH.

Ob

Ob eamdem rationem in triangulis ABI ABK } AI = AK.

In triangulis IBH GBK } latera IB, BH (per conſtructionem:) ſingula ſingulis æqualia lateribus KB, BG, & ang. IBH (per 15. pri.) = ang. KBG: ergo (per. 4. pri.) IH = KG: atque ang. BIH = ang. BKG.



In triangulis BIM BKL } (per demonſtrata.) ang. BIM = ang. BKL, & ang. IBM (per 15. pri.) = ang. KBL, latuſque IB (per conſtructionem.) = lateri BK: quare (per 26. pri.) IM = KL, & MB = BL.

In triangulis IAH KAG latera IA, AH (per demonſtrata) ſingula ſingulis æqualia lateribus KA, AG, baſiſque IH = baſi KG: igitur (per 8. primi.) angulus HIA = ang. GKA:

In triangulis AIM AKL } (per demonſtrata) latera AI, IM ſingula ſingulis æqualia lateribus AK, KL, & angulus AIM = ang. AKL: igitur (per 4. pri.) AM = AL.

In triangulis ABM ABL } latera AB, BM (per demonſtata) ſingula ſingulis æqualia lateribus AB, BL, atque baſis AM = baſi AL: ergo (per 8. pri.) angulus ABM = ang. ABL. quare (per def. 10. pri.) dicti anguli ABM, ABL erunt ambo recti.

Quod de recta AB in ordine ad rectam MBL demonſtratum eſt, de quacumq; alia recta linea in plano EF ducta, & tranſeunte per punctum B oſtendi poterit: quare (per def. 3. hujus) linea AB erit recta plano EF per rectas CD, EF ducto. q. e. d.

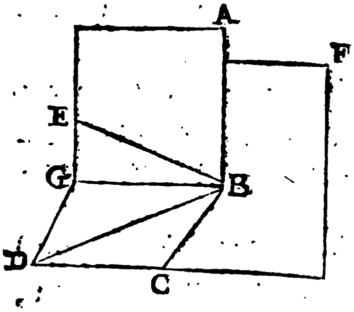
Propoſ. V. Theor. V.

Si recta AB ad rectos angulos inſiſtat tribus

rectis lineis BC, BD, BE ſeſe mutuo tangentibus in B.

Dico tres rectas BC, BD, BE in eodem exiſtere plano.

Conſtructio; & Demonſtratio.



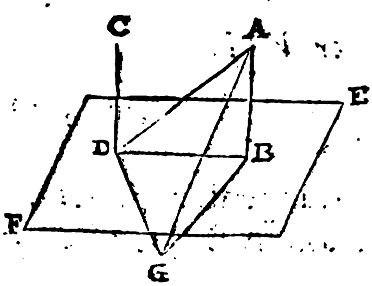
QVia (*per 2. hujus.*) rectæ BC, BD ſunt in uno plano, ſit hoc planum FD: ſi recta BE non eſt in plano FD (*per eamdem 2.*) erit in eodem plano cum AB: ſit planum rectarum AB, EB planum AG.

Quoniam verò planum AG occurrit plano FD (*per 3. hujus.*) BG communis ſectio talium planorum erit linea recta.

Hoc ſtante, quia (*per hyp.*) AB eſt perpendicularis lineis BC, BD (*per 4. hujus.*) AB recta erit plano FD: ſed BG eſt in plano FD: ergo angulus *ABG* erit rectus: cum autem (*per hyp.*) ſit etiam rectus angulus *ABE*; erit ang. *ABG* = ang. *ABE*: quod repugnat, cum angulus *ABG* ſit totum, & ang. *ABE* pars: quare recta BE erit & ipſa in plano rectarum *BC*, *BD*. q. e. d.

Propoſ. VI. Theor. VI.

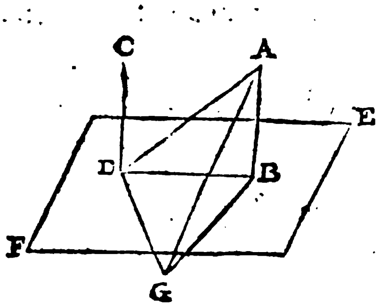
Si duæ rectæ lineæ AB, CD rectæ ſint ad planum EF.



Dico AB, & CD eſſe parallelas.

Conſtructio.

DUcatur recta *BD*. A puncto D (*per 11. pri.*) noteV tur



tur DG perpendicularis ipſi D*B*, in plano EF,& æqualis *B*A: ad-addanturq; rectæ *B*G, AG, AD.

Demonſtratio.

QVia rectæ lineæ A*B*, CD (*per hyp.*) rectæ ſunt plano EF [*per def.3.bujus*] anguli A*B*D, CD*B* erunt recti.

In triangulis $\left.\begin{matrix} ABD \\ GDB \end{matrix}\right\}$ (*per conſtructionem*] latera A*B*, *B*D ſunt ſingula ſingulis æqualia lateribus GD, D*B*, & anguli A*B*D, GD*B* pariter æquales, quia recti: ergo [*per 4.pri.*) AD = G*B*.

In triangulis $\left.\begin{matrix} ABG \\ GDA \end{matrix}\right\}$ (*per conſtructionem, & per demonſtrata.*) latera A*B*, *B*G ſingula ſingulis ſunt æqualia lateribus GD, DA, baſiſq; AG cõmunis ergo (*per 8.pri.*] angulus A*B*G = ang. GDA: ſed ang. A*B*G (*per def. 3. bujus.*] eſt rectus: quare etiam ang. GDA rectus erit.

Cum autem (*per hyp.*] ſit etiam rectus angulus GDC: ſequitur quod recta GD perpendicularis ſit tribus lineis D*B*,DA, DC: quare (*per 5. bujus*,] rectæ D*B*, DA, DC in eodem erunt plano: ſed [*per 2. bujus*] in illo plano, in quo ſunt D*B*, DA eſt etiam A*B*: ergo in eodem plano erunt A*B*, & CD.

Ulterius quia duo interni, & ad eaſdem partes anguli A*B*D, CD*B* *per hyp.*) ſunt ambo recti, erunt duobus rectis æquales: ideoque (*per 28. pri.*) rectæ A*B*, CD erunt parallelæ. q. e. d.

Propoſ. VII. Theor. VII.

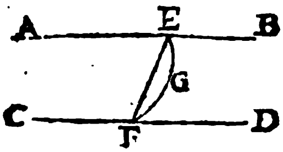
Si in duabus rectis lineis parallelis AB, CD accepta ſint puncta E, & F.

Dico lineam rectam coniungentem puncta E, & F in plano parallelarum AB, CD exiſtere.

Con-

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Conſtructio, & Demonſtratio.



SI recta *EF* non eſt in plano parallelarum *AB*, CD, erit in alio plano. Hoc aliud planum, ſi producatur neceſſario ſecabit parallelarum planum, quia puncta E, & F ſunt in plano parallelarum. Sit igitur communis horum planorum ſectio EGF, quæ [*per 3. hujus*] erit linea recta: quamobrem duæ rectæ EF, & EGF claudent ſpatium, quod repugnat [*per axio. 14.*]: ergo recta EF eſt in plano parallelarum *AB*, CD. q. e. d.

Propoſ. VIII. Theor. VIII.

Si duæ rectæ lineæ AB, CD ſint parallelæ, fueritque earum una, nempe AB, recta ad planum EF.

Dico & alteram CD eidem plano EF rectam eſſe.

Conſtructio.

DUcatur recta *BD*, quæ erit in plano EF. A puncto D in eodem plano EF ducatur DG ipſi *DB* perpendicularis, & æqualis *BA*, adiunganturque rectæ *GB*, GA, DA.

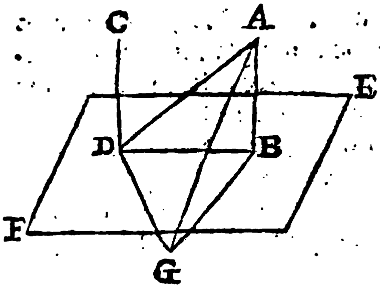
Demonſtratio.

QVia *AB* [*per hyp.*] eſt recta plano EF (*per def. 3. hujus.*) erit angulus *ABD* rectus; ſunt autem (*per 29. primi.*) anguli *ABD*, *CDB* duobus rectis æquales: quare angulus *CDB* rectus erit.

In triangulis *GDB*, *ABD* latera G D, D *B* [*per conſtruct.*] ſunt ſingula ſingulis æqualia lateribus *AB*, *BD*, & angulus GD*B* = ang. *ABD*, quia ambo recti; ergo [*per 4. pri.*] *B*G = *AD*.

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In triangulis $\left.\begin{matrix} GDA \\ GBA \end{matrix}\right\}$ latera GD, DA (*per conſtruct.* & *per demonſtrata.*] ſunt ſingula ſingulis æqualia lateribus *AB*, *BG*, baſiſque *AG* communis: ergo (*per* 8. *pri.*] ang. GD*A*, = ang. *ABG*; ſed ang. *A*BG (*per hyp.* eſt rectus: ergo etiam angulus GD*A* rectus erit: ex quo ſequitur, rectam GD perpendicularem eſſe duabus rectis D*B*, D*A* concurrentibus in D: quare [*per* 4. *hujus.*] GD recta erit plano per *B*D, & *A*D ducto; ſed (*per* 2. *hujus.*] in illo plano, in quo ſunt *B*D, & *AD*, eſt etiam *AB*; & in plano, in quo eſt *AB* (*per hyp.*) eſt & *CD*: ergo tres rectæ *B*D, *AD*, CD concurrentes in D in uno erunt plano: (quare [*per* 4. *hujus.*] G D perpendicularis etiam erit ipſi CD.



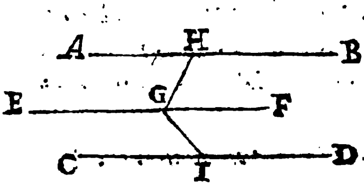
Cum igitur CD ſit perpendicularis rectis D*B*, *D*G concurrentibus in D (*per* 4. *hujus.*) CD recta erit plano EF per rectas *BD*, GD ducto q. e. d.

Propoſ. IX. Theor. IX.

Si duæ rectæ lineæ AB, CD ſint ipſi EF parallelæ, quamvis EF in alio ſit plano.

Dico AB, & CD fore inter ſe parallelas.

Conſtructio.



In recta E*F* aſſumatur quodcumque punctum G, a quo [*per* 12. *primi.* ad rectas *AB*, CD notentur perpendiculares GH, GI.

Demonſtratio.

QVia (*per hyp.*] AB eſt parallela ipſi EF, & angulus GHA (*per conſtructionem.*) rectus: [*per* 29. *pri.*] & ang. EGH rectus erit. Eadem prorſus ratione rectus erit angulus EGI.

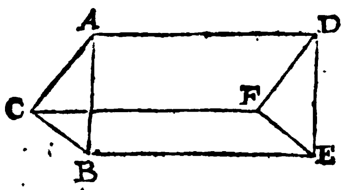
Quoniam vero EG perpendicularis eſt duabus rectis GH, GI concurrentibus in G: [*per* 4. *hujus.*] EG recta erit plano per GH, & GI ducto. Cum autem (*per hyp.*] CI ſit parallela ipſi EG (*per* 8. *hujus.*) etiam CI recta erit eidem plano per GH, & GI ſignato. Eadem ratione plano per GH, & HI ducto recta eſt AH: quare [*per* 6. *hujus.*] AH parallela CI. q. e. d.

Propoſ. X. *Theor.* X.

Si duæ rectæ lineæ AC, BC, concurrentes ad punctum C, ſingulæ ſingulis parallelæ ſint duabus rectis DF, EF concurrentibus in F, & in alio plano exiſtentibus.

Dico angulos ACB, DFE ab illis lineis comprehenſos eſſe æquales.

Conſtructio.

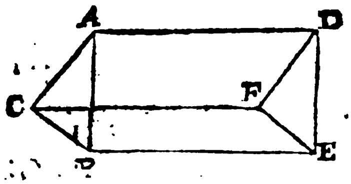


FIat CA = FD, atque CB = FE. Ducantur AB, DE, adducaturque CF.

Demonſtratio.

QVoniam rectæ AC, DF [*per hyp. & per conſtructionem.*) parallelæ ſunt, & æquales (*per* 33. *pri.*] AD, & CF ſunt parallelæ, & æquales. Eadem ratione CF parallela, & æqualis ipſi BE: quare [*per* 9. *hujus.*] AD parallela, & æqualis BE: ergo (*per* 33. *primi.*) AB = DE:

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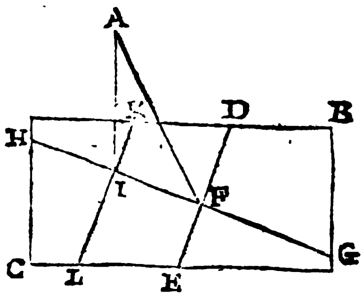


In triangulis ACB / DFE } latera AC, CB [*per conſtructionem.*) ſunt ſingula ſingulis æqualia lateribus DF, FE, baſiſque AB [*per demonſtrata.*] æqualis baſi DE: igitur (*per* 8. *pri.*) angulus ACB = ang. DFE. q. e. d.

Propoſ. XI. *Probl.* I.

Dato quovis puncto A extra planum BC; ad idem planum rectam ducere.

Conſtructio.



IN plano BC utcumque recta notetur DE; ad DE ex A [*per* 12. *primi.*) perpendicularis ducatur AF: a puncto F [*per* 11. *primi.*] in plano BC notetur FH perpendicularis ipſi DE: a puncto A dato [*per* 12. *primi.*) notetur AI perpendicularis rectæ HG.

Dico AI rectam eſſe plano BC.

Per I ducatur KIL parallela ED.

Demonſtratio.

QVia DF (*per conſtructionem.*) perpendicularis eſt rectis FA, FH: (*per* 4. *bujus.*] DF recta erit plano per FA, & FH ducto: quare (*per* 8. *bujus.*) etiam KI eidem plano per FA, FH ducto, recta erit.

Quia vero AI (*per* 2. *bujus.*) eſt in plano rectarum FA, FH tangitque KL in I: (*per def.* 3. *bujus.*) angulus AIK erit rectus: ergo AI perpendicularis erit rectis IK, IF: unde (*per* 4. *bujus.*] AI recta erit plano BC per rectas IK, IF ducto. q. e. f.

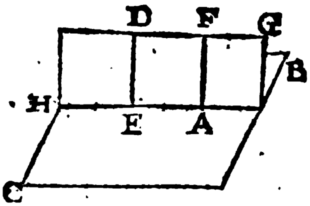
Pro-

Propoſ. XII. Probl. II.

Puncto A in plano BC dato.

Ab eodem puncto A eidem plano BC rectam lineam excitare.

Conſtructio.



Ex puncto D utcumque extra planum BC ſignato [*per 11. hujus.*] ad planum BC recta ducatur DE, quæ ſi cadat in A factum eſt, quod erat faciendum, ſi cadat extra A, ut in E, ab E ad A ducatur EA. A puncto A in plano rectarum DE, EA, hoc eſt in plano GH (*per 11. pri.*) ipſi EA perpendicularis ducatur AF.

Dico AF rectam eſſe plano BC.

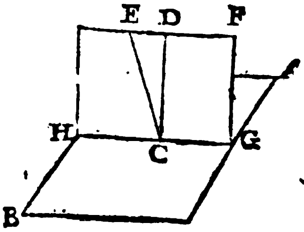
Demonſtratio.

Quia (*per conſtructionem.*] AF parallela ED, ED vero facta eſt recta plano BC [*per 8. hujus.*) erit etiam AF eidem plano BC recta. q. e. f.

Propoſ. XIII. Theor. XI.

Ab eodem puncto C in plano AB accepto, duæ rectæ lineæ CD, CE plano AB rectæ ad eaſdem partes duci nequeunt.

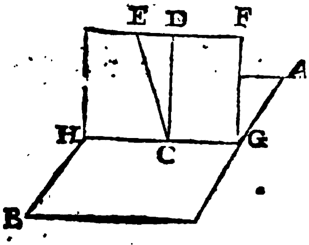
Conſtructio.



Per duas rectas lineas CD, CE ducatur planum FH, communiſque ſectio plani FH cum plano AB ſit recta HCG.

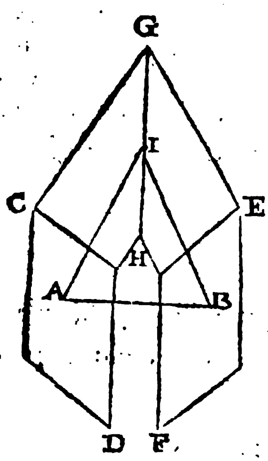
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Demonſtratio.



SI lineæ DC, EC ambæ rectæ ſunt plano AB: (*per def.3.hujus.*) erunt anguli DCH, ECH recti, & ideo æquales; hoc eſt impoſſibile, quia DCH eſt totum, & ECH eſt pars, ergo eſt imdoſſibile, DC, EC ambas eſſe rectas plano *AB*.

Propoſ. XIV. *Theor.* XII.



Si linea AB ſit recta planis CD, EF.

Dico illa plana eſſe parallela.

Conſtructio, & Demonſtratio.

SI plana CD, EF non concedantur parallela, producta concurrent: concurrant igitur ad partes C, & E in recta GH, in qua accipiatur punctum I, ad quod in planis DCGH, FEGH ducantur rectæ lineæ AI, BI; quæ conſtituant triangulum AIB.

Cum autem [*per hyp.*] AB recta ſit planis DCGH, FEGH: (*per def.3. hujus.*) anguli BAI, ABI erunt recti: ideoque duobus rectis æquales, quod cum repugnet *Prop.*17.*pri.*, plana CD, EF non poterunt concurrere: unde erunt parallela. q. e. d.

Propoſ. XV. *Theor.* XIII.

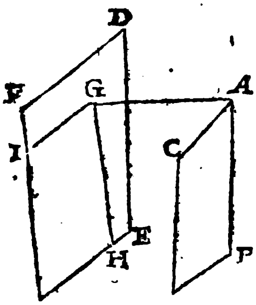
Si duæ rectæ lineæ AC, AB concurrentes in A, ſint ſingulæ ſingulis duabus rectis lineis DF,

DE

DE concurrentibus in D, & in alio plano exiſtentibus, parallelæ.

Dico planum per *AB*, & *AC* ductum parallelum eſſe plano per DF, DE deſcripto.

Conſtructio.



EX puncto A (*per* 11. *bujus.*] recta ad planum FE ducatur GA. Ex puncto G in plano FE (*per* 31. *pri.*) notentur GH parallela DE, atque GI parallela *DF*.

Demonſtratio.

QVia (*per hyp. & per conſtruct.*) rectæ AB, GH parallelæ ſunt ipſi DE: (*per* 9. *bujus.*] erit AB parallela *GH*: eadem ratione AC parallela GI: quare (*per* 29. *pri.*] erunt ang. BAG, HGA duobus rectis æquales; ſed ang. HGA (*per conſtruct.* eſt rectus: ergo & angulus BA*G* rectus erit: eadem prorſus ratione demonſtrabitur rectus angulus CAG.

His manentibus, cum linea AG perpendicularis ſit duabus lineis AB, AC concurrentibus in A: (*per* 4. *bujus.*) AG recta erit plano BC per lineas AB, AC ducto: quare (*per* 14. *bujus.*) plana BC, EF erunt parallela. q. e. d.

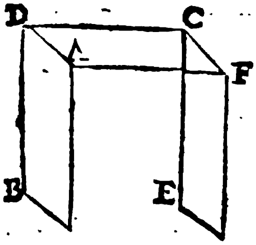
SCHOLIUM.

QVia ad ſequentes propoſitiones demonſtrandas opus eſt per datum punctum extra planum acceptum, ducere planum alteri plano parallelum; ideo opportunum duxi in hoc loco ſequens probl. demonſtrare.

Dato

Dato plano *AB*, & puncto C extra planum *AB* poſito: per punctum C ducere planum plano *AB* parallelum.

Conſtructio.



IN dato plano AB ducantur duæ quæcumque lineæ DA, DB efformantes angulum ADB. A puncto D ad C notetur recta DC.

In plano rectarum DC, DB, hoc eſt in plano CB a puncto C (per 31. pri.) ducatur CE parallela DB, & in plano rectarum CD, DA, nempe in plano CA notetur CF parallela DA.

Dico planum per CE, & CF ductum dato plano AB eſſe parallelum.

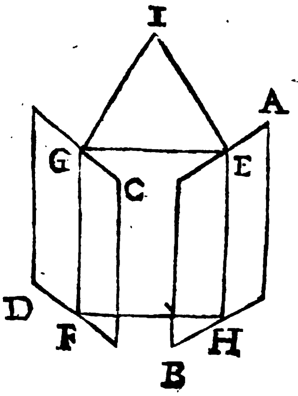
Demonſtratio.

QUia rectæ CE, CF concurrentes in C (per conſtruct.) parallelæ ſunt rectis DB, DA concurrentibus in D (per 15. hujus.) plana AB, FE per dictas lineas ducta erunt parallela. q. e. f.

Propoſ. XVI. Theor. XIV.

Si duo plana parallela AB, CD ſecentur a plano EF.

Dico communes ſectiones EH, GF eſſe parallelas.



Conſtructio, & Demonſtratio.

QUia communes ſectiones plani EF cum planis AB, CD, quæ ſunt EH, GF ambæ reperiuntur in plano ſecante EF, ſi non ſunt parallelæ poterunt concurrere; ſi fieri poteſt concurrant in I productæ ad partes E, & G.

Hoc

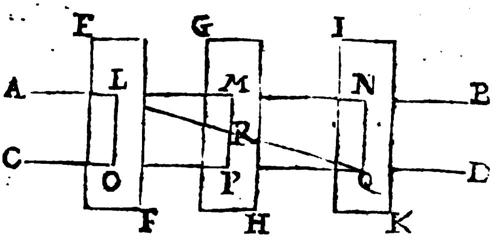
Hoc poſito:] *per* 1. *hujus*.) recta linea FGI tota erit in plano CD, & linea EH tota erit in plano AB: ſi ergo rectæ FGI, HEI concurrunt in I, etiam plana AB, CD debent concurrere, cum ſit impoſſibile concurrere lineas, non autem illa plana, in quibus concurrentes lineæ reperiuntur: ſed plana (*per def.* 8. *hujus*.] nequeunt concurrere: ergo neque communes ſectiones GF, EH poterunt concurrere: quare erunt parallelæ. q. e. d.

Propoſ. XVII. Theor. XV.

Si rectæ lineæ AB, CD quomodocumque diſpoſitæ ſecentur a planis parallelis EF, GH, IK.

Dico lineas AB, CD ſecari in partes proportionales, videlicet LM : MN :: OP : PQ.

Conſtructio.



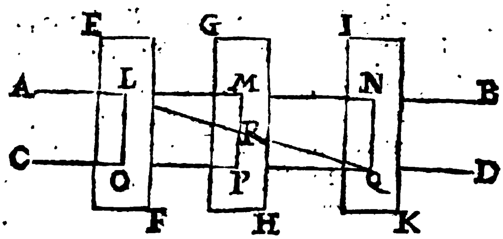
In plano EF ab L ad O ducatur recta LO: pariter in plano IK ab N ad Q notetur recta NQ. A puncto L ad Q ducatur recta LQ, quæ plano GH occurrat in R. A puncto R ad puncta M, & P in plano GH notentur rectæ RM, RP.

Demonſtratio.

Triangulum LNQ [*per* 2. *hujus*] eſt in uno plano: quod pariter de triangulo LOQ eſt intelligendum.

Quia vero parallela plana GH, IK ſecantur a plano trianguli LNQ; atque parallela plana EF, GH ſecantur a plano trianguli LOQ: [*per* 16. *hujus*. (communes talium planorum ſectiones erunt parallelæ: quare MR parallela NQ; & RP parallela LO.

His

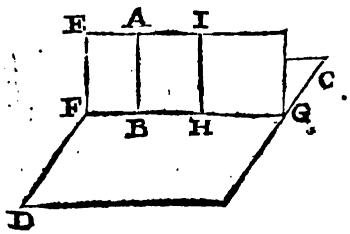


His demonſtratis in triangulo LNQ, [*per* 2. *ſexti*.]
LM : MN : : LR : RQ.
Ob eamdem cauſam in triangulo LOQ.
LR : RQ : : OP : PQ.
Quare (*per* 11. *quinti*.)
LM : MN : : OP : PQ. q. e. d.

Propoſ. XVIII. *Theor.* XVI.

Si linea AB plano CD recta fuerit.
Dico omnia plana per AB ducta eidem plano CD eſſe recta.

Conſtructio.



PEr *A*B ducatur quodcumque planum EG, cujus communis ſectio cum plano CD ſit FG in communi ſectione FG accipiatur quodvis punctum H, a quo in plano E G (*per* 31. *pri.*] notetur HI ipſi B*A* parallela.

Demonſtratio.

QVoniam [*per conſtructionem.*] lineæ B*A*, HI ſunt parallelæ, atque (*per hyp.*) *AB* recta eſt plano C*D*: (*per* 8. *huius*.]. & HI plano CD recta erit: ergo (*per def.* 3. *hujus*.] & HI erit perpendicularis communi ſectioni FG. Quod autem de recta HI demonſtratum eſt, valet etiam de quacumque alia linea

in

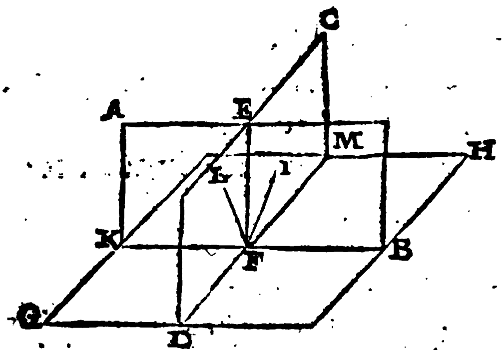
in plano EG ducta parallela ipſi *BA*: quare (*per def. 4. hujus.*) planum EG ad planum CD rectum erit: ſed demonſtrata de plano EG, etiam cuivis plano per BA ducto applicari poſſunt: ergo omnia plana per AB ducta, plano CD recta erunt. q. e. d.

Propoſ. XIX. Theor. XVII.

Si duo plana ſe ad invicem ſecantia in communi ſectione EF, plano HG recta fuerint.

Dico EF communem talium planorum ſectionem, plano HG rectam eſſe.

Conſtructio, & Demonſtratio.



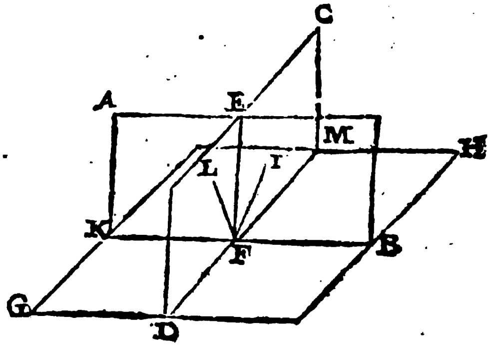
Communis ſectio plani AB cum plano GH ſit *K*B: atque communis ſectio plani CD cum eodem plano GH ſit DM. Hoc poſito EF communis ſectio planorum AB, CD vel erit perpendicularis ad utramque communem ſectionem KB, DM, vel ad unam tantum, vel ad neutram.

Si EF perpendicularis eſt ad ambas communes ſectiones KB, DM (*per 4. hujus.*) recta erit plano GH per illas communes ſectiones ducto.

Si EF eſt uni tantum rectarum KB, DM, ver. gr. ipſi KB perpendicularis; quia (*per hyp.*) planum AB rectum eſt ad planum GH, & EF perpendicularis ad KB, ut ſupponimus: EF (*per def. 4. hujus.*) recta erit plano GH.

Si denique EF non ſit perpendicularis ad KB, neque ad DM; ex puncto F in plano AB ipſi KB perpendicularis ducatur FI; pariter.

riterque in plano CD notetur FL perpendicularis ipſi DM.



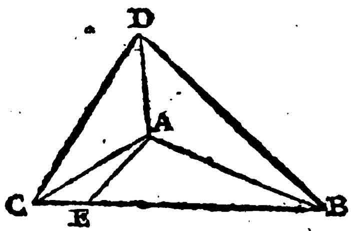
His poſitis, quoniam planum *AB* (*per hyp.*) rectum eſt ad planum GH; erit IF [*per def.* 4. *hujus.*] recta ad planum GH: ob eamdem cauſam LF ad idem planum GH recta erit: quare ab eodem puncto F in plano GH accepto ad eaſdem partes ductæ erunt duæ lineæ FI, FL eidem plano GH rectæ: quod cum ſit contra demonſtrata in 13. hujus, FE communis ſectio planorum *AB*, DC recta erit ad planum GH. q. e. d.

Propoſ. XX. Theor. XVIII.

Si ſolidus angulus A a tribus angulis planis B*A*C, B*A*D, C*A*D fuerit contentus.

Dico ex his tribus angulis duos ſimul acceptos reliquo eſſe majores.

Conſtructio, & Demonſtratio.



QVando tres anguli plani ſolidum angulum comprehendentes ſunt æquales, per ſe patet duos exiſtere tertio majores.

Quando duo tantum ſunt æquales, tertius vero inæqualis perſpicuum eſt, unum æqualium cum inæquali alterum æqualium ſuperare.

Tota igitur dubietas habetur, dum tres anguli plani conſtituentes angulum ſolidum ſunt inæquales; unde circa hanc ſolam planorum angulorum conſtitutionem oportet demonſtrare, quod medius, atque minor angulus

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lus ſimul accepti ſuperant maximum angulum.

Sit angulus BAC omnium maximus, CAD verò minimus. Ad punctum A, & ad rectam BA (*per* 23. *pri.*) conſtituatur angulus BAE = ang. BAD, fiatque recta AE = rectæ AD. Per puncta B, & E notetur BE, & producatur donec ſecet AC in C. Addantur CD, & BD.

His taliter peractis in triangulis $\left.\begin{matrix}BAD\\BAE\end{matrix}\right\}$ (*per conſtructionem.*) latera BA, AD ſingula ſingulis ſunt æqualia lateribus BA, AE, atque ang. BAD = ang. BAE: quare (*per* 4. *pri.*) BD = BE. Quia vero (*per* 20. *pri.*) in triangulo *BDC* latera *BD*, *DC* majora ſunt quam BC, demptis æqualibus BD, BE: (*per axio.* 5.) DC majus quam EC.

In triangulis $\left.\begin{matrix}DAC\\EAC\end{matrix}\right\}$ (*per conſtructionem,*) latera DA, AC ſingula ſingulis æqualia lateribus EA, AC, baſis vero DC (*per demonſtrata.*) major quam EC: ergo (*per* 25. *primi.*) angulus DAC major angulo EAC.

Cum autem (*per conſtructionem.*) ang. BAD = ang. BAE, ſi ang. BAD addatur ang. DAC, & ang. BAE adiungatur ang. EAC: (*per axio.* 5.) erunt

duo ang. $\left.\begin{matrix}BAD\\DAC\end{matrix}\right\}$ majores duobus ang. $\left.\begin{matrix}BAE\\EAC\end{matrix}\right\}$; ſed

anguli $\left.\begin{matrix}BAE\\EAC\end{matrix}\right\}$ æquant angulum BAC: ergo

anguli $\left.\begin{matrix}BAD\\DAC\end{matrix}\right\}$ majores angulo BAC. q. e. d.

Propoſ. XXI. Theor. XIX.

Si ſolidus angulus *A* a tribus angulis planis *BAC*, *CAD*, *DAB*, fuerit comprehenſus.

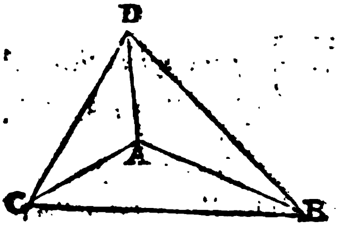
Dico iſtos tres angulos quatuor rectis minores eſſe.

Con-

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Conſtructio.

IN rectis AB, AC, AD utcumque notentur puncta B, C, D, addanturque rectæ BC, CD, DB.



Demonſtratio.

SI intelligatur planum per rectas BC, CD, DB ductum, conſtituti erunt tres anguli ſolidi B, C, D ſinguli a tribus angulis planis contenti:

Quare (*per 20. hujus.*)

In ang. ſolido B duo ang. ABC, ABD } majores ang. CBD.

In ang. ſolido C duo ang. ACB, ACD } majores ang. BCD.

In ang. ſolido D duo ang. ADC, ADB } majores ang. BDC.

Ergo ſex anguli plani ABC, ABD, &c. majores erunt tribus angulis planis CBD, BCD, BDC.

Quia vero (*per 32. pri.*) tres anguli CBD, BCD, BDC ſunt duobus rectis æquales; ſex anguli ABC, ABD &c. erunt majores duobus rectis; ſed iſti ſex anguli ABC, ABD &c. additis tribus angulis ad punctum A conſtitutis (*per 32. pri.*) æquant ſex angulos rectos: ſi ab hac ſumma demantur ſex anguli ABC, ABD &c. qui (*per demonſtrata.*) ſunt majores duobus rectis; remanebunt ad punctum A tres anguli Plani *BAD*, *DAC*, *CAB*, comprehendentes angulum ſolidum *A*, quatuor rectis minores. q. e. d.

SCHOLIUM.

QUamvis ab Euclide hoc Theorema fuerit tantumodo propoſitum de angulo ſolido a tribus angulis planis efformato, adhuc tamen de quocumque ſolido angulo, a pluribus angulis planis compoſito verificatur, uti a Clavio, aliiſque Scriptoribus in hoc loco fuit demonſtratum.

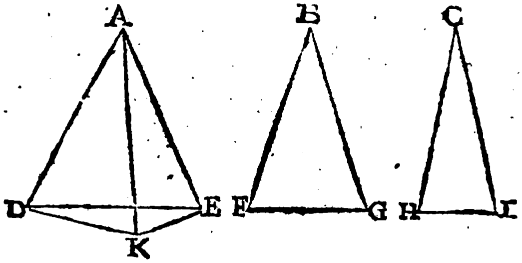
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Propoſ. XXII. Theor. XX.

Si fuerint tres anguli plani A,B,C ab æqualibus rectis lineis AD, AE : BF, BG: CH, CI: comprehenſi, quorum duo quomodocumque ſumpti, reliquo ſint majores.

Dico ex baſibus DE, FG, HI poſſe triangulum efformari.



Conſtructio, & Demonſtratio.

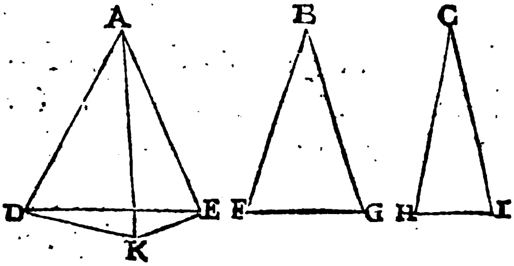
DAti tres anguli *A*, *B*, *C* poſſunt eſſe omnes inter ſe æqualles, vel duo tantum, vel omnes inæquales.

Si dati anguli A, B, C fuerint æquales, cum [*per hyp.*) æquales etiam ſint rectæ AD, *A*E &c; erunt (*per* 4. *pri.*) baſes DE, FG, HI æquales: quare ex his baſibus duæ, quomodocumque ſimul acceptæ, erunt majores tertia.

Si ex datis angulis duo tantum fuerint æquales (*per* 4.*primi.*] duæ etiam baſes erunt æquales: ideoque una æqualium baſium cum tertia inæquali, aliam æqualem ſuperabit.

Si denium tres anguli dati A, B, C fuerint inæquales, & angulus A maximus, reliqui vero B, & C æquales, vel inæquales inter ſe: erit (*per* 24. *pri.*) DE baſium maxima; reliquæ verò baſes FG, HI, inter ſe æquales, vel inæquales: quare DE + FG ſuperabit HI, pariterque DE + HI major erit quam FG.

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His manentibus oportet demonſtrare, minores baſes FG, HI ſimul ſumptas ſuperare maximam DE.

Fiat [per 23. pri.) angulus DAK = ang. B, ſitque AK = AD. Quia vero (per hyp. ang. A major eſt ang. B, recta AK cadet intra rectas AD; AE. Notentur rectæ DK, KE.

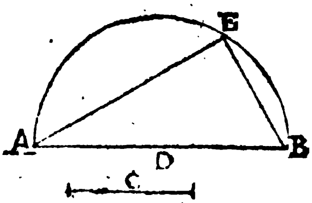
Quoniam verò duo anguli B, & C majores ſunt ang. DAE, & ang. B [per conſtruct.) æqualis ang. DAK: erit ang. C major ang. EAK.

Cum autem in triangulis DAK / FBG } latera AD, AK (per hyp.) ſingula ſingulis ſint æqualia lateribus BF, BG; & ang. DAK [per conſtruct.] = ang. B: (per 4. pri.) DK = FG.

In triangulis HCI / AE } [per hyp. & per conſtruct.) latera CH, CI ſingula ſingulis æqualia ſunt lateribus AK, AE; angulus autem C [per demonſtrata) major ang. EAK: igitur (per 24. pri.] erit HI major EK.

Cum igitur DK (per demonſtrata] ſit æqualis FG: [per axio. 4.) FG + HI major erit quam DK + KE: quare [per 22. pri.] ex tribus baſibus DE, FG, HI poterit triangulum efformari. q.e.d.

LEMMA.



Duabus datis rectis lineis inæqualibus, nempe AB majori, & C minori, potentiam majoris lineæ AB ſupra potentiam minoris C invenire.

Con-

Conſtructio.

SUper *AB* deſcribatur ſemicirculus AE*B*: in hoc ſemicirculo (*per* 1. *quarti.*) accommodetur *BE* = C: iungatur AE. Dico quadratum rectæ AE eſſe potentiam majoris lineæ A*B* ſupra potentiam minoris lineæ C.

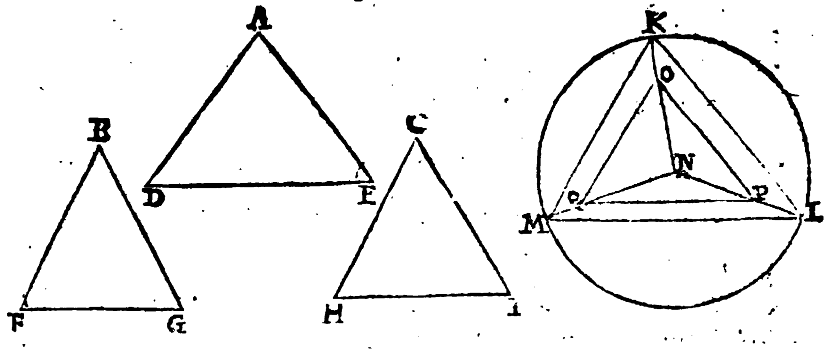
Demonſtratio.

IN ſemicirculo AE*B* (*per* 31. *tertij.* angulus *AEB* eſt rectus: quare (*per* 47. *pri.*) quad. *AB* = quad. $\left.\begin{matrix} BE \\ AE \end{matrix}\right\}$: ſed quad *BE* (*per conſtructionem.*) = quad. C: ergo quad. *AB* = $\left.\begin{matrix} \text{quad. } AE \\ \text{quad. } C \end{matrix}\right\}$ ideoque recta *AB* plus poteſt quam C quad. *AE*. q. e. f.

Propoſ. XXIII. *Prob.* III.

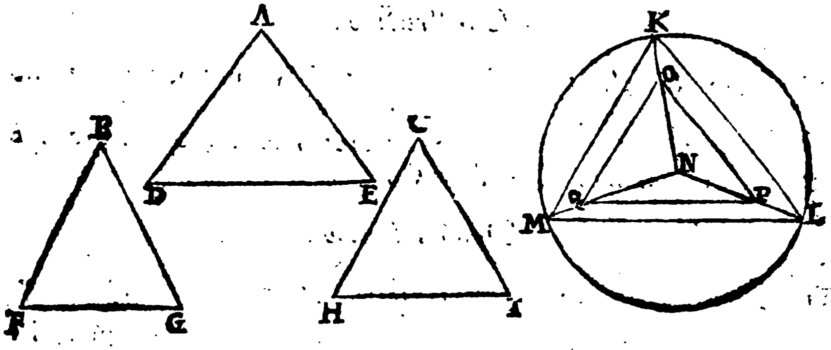
Ex datis tribus angulis planis A, B, C quatuor rectis minoribus, quorum duo quomodocumque accepti reliquo ſint majores; ſolidum angulum conſtituere.

Conſtructio.



IN una linearum comprehendentium datos angulos *A*, *B*, *C*, ut in AD, ſignetur punctum D. Fiant omnes reliquæ *AE*. BF,

 BG,



BG, *CH*, *CI* ipſi *AD* æquales, addanturque rectæ *DE*, *FG*, *HI*, ex quibus (*per 22. bujus*) poterit triangulum efformari: Conſtituatur ergo [*per 22. pri.*] ex tribus lineis DE, FG, HI triangulum KLM, ſitque

KL = DE:
KM = FG:
ML = HI:

Circa triangulum MKL [*per 5. quarti*) deſcribatur circulus, cujus centrum ſit N, a quo ad angulos trianguli MKL ducantur rectæ NM, NK, NL

Dico omnes rectas *AD*, *AE*, cæteraſque datos angulos *A*, *B*, *C* comprehendentes, ſuperare rectas NK, NM, NL a circuli centro N ad trianguli angulos M, K, L ductas.

Quia verò tres dati anguli *A*, *B*, *C* tales eſſe poſſunt, ut centrum circuli circa triangulum MKL deſcripti, aliquando ſit intra triangulum, aliquando in uno trianguli latere, & aliquando extra triangulum: in omni caſu opus eſt demonſtrare, rectas *AD*, *AE* &c. ſuperare NK, NM, NL.

Quando centrum N cadit intra triangulum. Conſtructio, & Demonſtratio.

SI rectæ *AD*, *AE* &c. non concedantur majores quam NK, NM, NL, erunt vel æquales, vel minores: ſint primo, ſi eſſe poſſunt, æquales. Hoc ſuppoſito,

In triangulis $\left.\begin{matrix} DAE \\ KNL \end{matrix}\right\}$ latera *DA*, *AE* erunt ſingula ſingulis

æqua-

æqualia lateribus KN, NL, atque (*per conſtructionem*) baſis DE = baſi KL, ergo (*per* 8. *primi*.] ang. A = ang. KNL. Eadem ratione erit ang. B = ang. KNM, & ang. C = ang. MNL; ſed tres anguli ad punctum N conſtituti. [*per corol.* 1. *prop.* 15. *primi.*) ſunt quatuor rectis æquales: ergo & tres anguli A, B, C, quatuor rectis æquales erunt, quod eſt contra ſuppoſitum: quare rectæ AD, AE &c. nequeunt eſſe æquales rectis NK, NM, NL.

Secundo ſint Minores, ſi eſſe poſſunt. Fiant NO, NP, NS æquales rectis AD, AE &c. ducatur recta OP: Hoc facto

In triangulo KNL [*per def. circuli*.] NK = NL, [*per conſtructionem*.) NO = NP: quare (*per* 19. *quinti*.) OK = PL: ergo

NO: OK:: NP: PL,

unde (*per* 2. *ſexti*) OP parallela ipſi KL: igitur (*per corol. prop.* 4. *ſexti*.] triangula KNL, ONP ſimilia: ideoque

NL: LK:: NP: PO,

ergo (*per* 14. *quinti*.) KL major, quam OP.

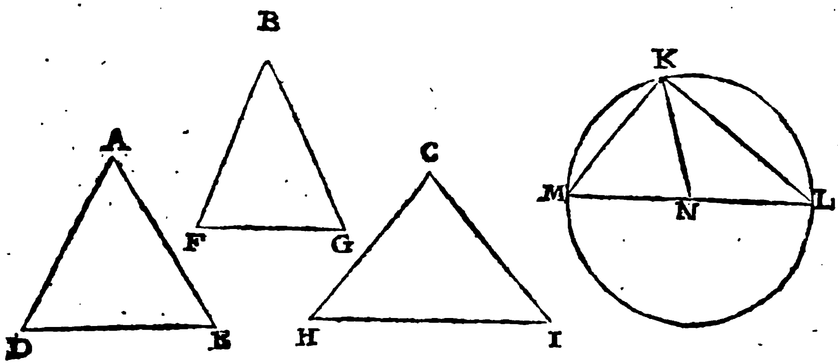
Cum autem KL (*per conſtructionem*.) ſit æqualis DE; erit OP minor DE.

In triangulis $\left.\begin{matrix}DAE\\ONP\end{matrix}\right\}$ latera AD, AE erunt ſingula ſingulis æqualia lateribus NO, NP, baſiſque DE (*per demonſtrata*) major quam OP: ergo [*per* 25. *pri.*] ang. A major ang. ONP.

Eodem argumento ducta OS, demonſtrabitur ang. B major ang. ONS, & ang. C major ang. SNP.

Cum autem tres anguli circa punctum N [*per corol.* 1. *prop.* 15. *pri.*] ſint quatuor rectis æquales; tres anguli dati A, B, C quatuor rectis majores erunt, cõtra hypoteſim, quia ſupponuntur quatuor rectis minores: ex quo ſequitur, lineas NK, NL, NM non eſſe majores rectis AD, AE &c. Vlterius cum [*per demonſtrata*.] neque æquales eſſe poſſint, rectæ AD, AE, &c. ſuperabunt NK, NL, NM, q. e. i. d.

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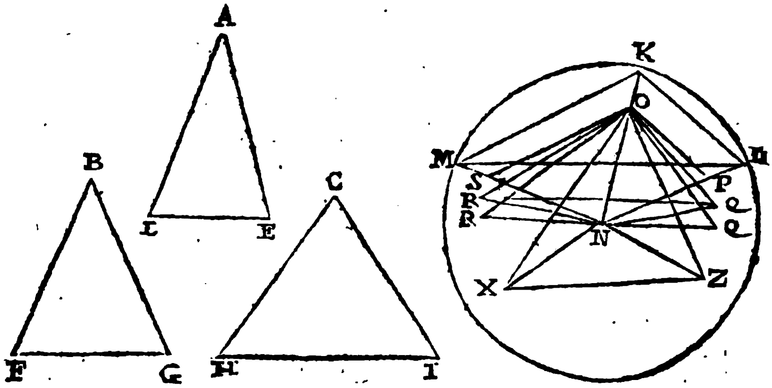
Quando centrum N cadit in unum latus trianguli. Conſtructio, & Demonſtratio.

ML = HI
KL = DE
KM = FG

QVando centrum N cadit in latus M L, ducatur NK: in hoc caſu, ſi dicamus CH, CI æquales eſſe rectis N L, NM, NK, rectæ CH, CI æquabunt OL, NM: ſed (*per conſtructionem.*) NL, NM, hoc eſt ML æqualis eſt ipſi HI: quare in triangulo CHI latera CH, CI non erunt majora, quam HI contra demonſtrata (*in* 20. *primi.*): ergo rectæ CH, CI &c. non erunt rectis NL, NM, NK. Cum autem neque poſſint eſſe minores, quia, ſi hoc eſſet, latus HI ſuperaret latera CH, CI; quod magis eſt contra eamdem 20. primi; quando igitur centrum N eſt in latere trianguli, rectæ AD, AE &c. majores ſunt rectis NK, NM, NL. q. e. 2. d.

Quan-

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Quando centrum N cadit extra triangulum. Conſtructio, & Demonſtratio.

EXiſtente centro N extra triangulum MKL, ſi primo dicamus rectas AD, AE &c. æquales eſſe rectis NM, NK, NL, in triangulis $\left.\begin{matrix}\text{KNL}\\\text{DAE}\end{matrix}\right\}$ ſingula latera erunt ſingulis lateribus æqualia, baſiſque LK [*per conſtruct.*) æqualis baſi DE [*per 8. pri.*): quare erit ang. A = ang. KNL. Eadem ratione ang. C = ang. KNM: quare totus angulus MNL æqualis duobus angulis A, & C. Cum autem [*per hyp.*) anguli A, & C ſimul accepti ſint majores angulo B, etiam angulus MNL major erit angulo B.

Quia vero in triangulis $\left.\begin{matrix}\text{MNL}\\\text{FBG}\end{matrix}\right\}$ latera MN, NL ſupponuntur æqualia lateribus FB, BG, atque baſis ML [*per conſtruct.*) æqualis baſi FG: (*per 8. pri.*) erit ang. MNL = ang. B; ergo ang. MNL major, & æqualis erit angulo B. Quamobrem rectæ AD, AE &c. non ſunt æquales rectis NM, NK, NL.

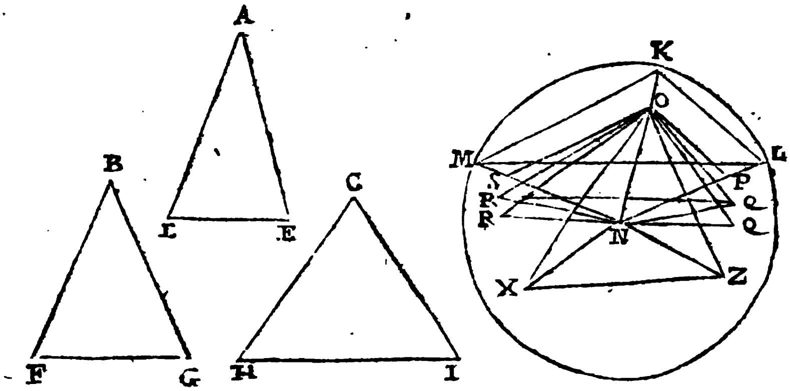
Si poſtea dicamus rectas AD, AE &c. minores eſſe quam NM, NK, NL; (*per 3. pri.*] fiant NS, NO, NP æquales ipſi AD, AE &c. ducanturque OS, OP.

Hoc facto [ut in prima parte) non ſolum demonſtrabitur angulus A major angulo ONP, verum etiam ang. C major ang. ONS.

Ad punctum N, & ad rectam NO (*per 23. pri.*) conſtituatur angulus ONQ = ang. A, atq; ang. ONR = ang. C, ſintq; rectæ NQ, NR æquales ipſis AD, AE &c; addantur rectæ OR, OQ, quæ neceſſario intra OS, & OP cadere debent, quia circumferentia circuli ex centro N, intervallo NO deſcripta debet tranſire per puncta QPOSR.

Quia vero efformato angulo ONR = ang. A, & ang. ONQ = ang. C poteſt contingere, quod lineæ NR, NQ dictos angulos efformantes, angulum conſtituant ad partes trianguli MKL, quod ſint in directum poſitæ, quodque ad oppoſitas partes trianguli MKL angulum conſtituant, ut in fig. 1.

Si contingat quod NR, NQ angulum efforment ad partes trianguli MKL, ducatur RQ.



Rectæ OS, OP [*per demonſtrata in prima parte.*] parallelæ ſunt rectis KM, KL: quare (*per 29. pri.*) ang. MKN = ang. SON, & ang. LKN = ang. PON, atque totus ang. MKL = toti ang. SOP: ſed ang. ROQ [*per axio. 9.*) minor ang. SOP: ergo, & ang. ROQ minor ang. MKL. His manentibus.

In triangulis DAE / ONQ } latera AD, AE ſingula ſingulis æqualia lateribus NO, NQ, & ang. A (*per conſtructionem.*) æqualis ang. ONQ: quare (*per 4. pri.*) baſis DE = baſi OQ. Eadem ratione in triangulis HCI / ONR } baſis HI = baſi OR. Cum autem

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(*per conſtructionem*) ſit DE = KL, & HI = KM; erit OQ = KL, & OR = KM.

In triangulis LKM QOR } [*per demonſtrata.*] latera KL, KM ſingula ſingulis æqualia lateribus OQ, OR, angulus verò LKM major angulo QOR: quare (*per 24. primi.*) baſis ML major, quam baſis RQ; ſed [*per conſtructionem.*] ML = FG: quare FG major quam RQ.

In triangulis FBG RNQ (*per conſtructionem.*) latera FB, BG ſingula ſingulis æqualia lateribus NR, NQ, & (*per demonſtrata.*) baſis FG major baſi RQ; (*per 25. pri.*) quare ang. B major angulo RNQ. Quia vero (*per conſtructionem.*) ang. RNQ eſt æqualis angulis A, & C: igitur ang. B major angulis A, & C, contra ſuppoſitum: ergo rectæ AD, AE &c. nequeunt eſſe minores rectis NL, NK, NM. Cum autem [*per demonſtrata.*) neque poſſint eſſe æquales, reſtat quod ſint majores. q. e. 3. d.

2. Si eveniat quod NR, NQ ſint in directum poſitæ ducantur rectæ OR, OQ, quarum OR, uti ſupra, demonſtrabitur æqualis ipſi KM, & OQ æqualis KL, quare

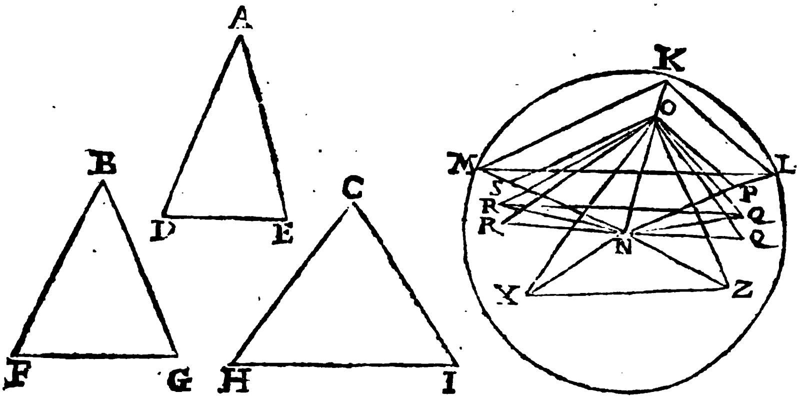
In triangulis MKL ROQ } latera MK, KL ſunt ſingula ſingulis æqualia lateribus RO. OQ, & ang. MKL major ang. ROQ: ergo (*per 24. pri.*, ML major quam RQ: ſed ML *per conſtruct.*) æqualis HI: ergo RQ minor quam HI: cum autem [*per conſtructionem.*] ſit NR = CH, & NQ = CI: erit HI major quam CH, cum CI: quod repugnat (*per 20. pri.*]

3. Si contingat quod rectæ NR, NQ infra centrum N conſtituant angulum XNZ ductis lineis OX, OZ, uti ſupra demonſtrabitur OX = KM, OZ = KL: quare

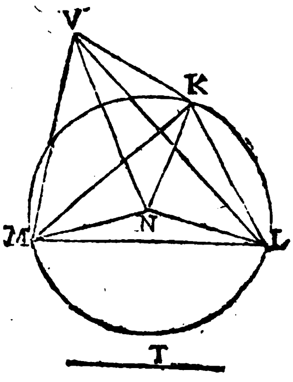
In triangulis MKL XOZ } latera KM, KL ſingula ſingulis æqualia lateribus XO, OZ, angulus verò MKL major angulo XOZ: quare [*per 24. pri.*) ML major quam XZ; ſed (*per conſtruct.*) ML = HI, ergo HI major quam XZ.

In triangulis HCI XNZ } latera CH, CI (*per conſtructionem.*) ſingula ſingulis æqualia lateribus NX, NZ, & (*per demonſtrata.*]

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tá.] baſis HI major quam baſis XZ: ergo (*per* 25.*pri.*) angulus C major angulo XNZ; ſed angulus XNZ cum duobus angulis KNX, KNZ complet quatuor rectos: ergo angulus C cum angulis A, & B ſuperabit quatuor rectos: quod eſt contra ſuppoſitum, quia tres anguli A, B, C ſupponuntur quatuor rectis minores: quamobrem non erit verum rectas AD, AE &c. minores eſſe rectis NK, NM, NL, at neque æquales eſſe poſſunt [*per demonſtrata*): ergo ſolum reſtat quod ſint majores. q. e. 3. d.



Demonſtratis lineis AD, AE &c. maoribus quam ſint lineæ NK, NM, NL a circuli centro ad trianguli angulos ductis (per Lemma antecedens) inveniatur exceſſus potentiæ unius rectarum AD, AE &c. ſupra potentiam rectæ NK, vel NM, vel NL, taliſque exceſſus ſit quadratum rectæ T.

Ex centro N plano circuli MKL [*per* 12.*hujus.*) recta excitetur NV = ipſi T, ductiſque rectis MV, KV, LV.

Dico ad punctum V conſtitutum eſſe angulum ſolidum a tribus datis angulis planis A, B, C comprehenſum.

De-

Demonſtratio.

QVia NV (*per conſtructionem*) recta eſt plano circuli LKM, (*per def. 3. hujus.*] anguli VNL, VNK, VNM erunt recti: quare [*per 47. pri.*) quad. VL = quad. VN) quad. NL). Cum autem] *per conſtruct.*) quad. AD æquale ſit ijſdem quad. VN quad. NL } erunt quad. quad. VN quad. NL } = quad. AD: unde (*per ſcol. prop. 46. pri.*) recta VL = rectæ AD.

Denuo in triangulis VNL VNK } latera VN, NL ſingula ſingulis æqualia lateribus VN, NK, æqualeſque anguli VNL, VNK, quia recti: igitur (*per 4. pri.*] VL = VK; ſed VL fuit demonſtrata æqualis ipſi AD: ergo, & VK eidem *AD* æqualis erit. Eadem ratione poterit demonſtrari VM = *AD*: quare tres rectæ VL, VK, VM inter ſe, & rectis *AD*, *AE* &c. æquales erunt.

In triangulis DAE KVL } latera *DA*, *AE* [*per demonſtrata.*) ſingula ſingulis æqualia lateribus VL, VK, baſiſque *DE* (*per conſtruct.*) æqualis baſi LK: quare [*per 8. pri.*] ang. *DAE* = ang. LVK. Iiſdem fundamentis poterit demonſtrari angulus B = ang. LVM, & ang. C = ang. MVK: quare angulus ſolidus V efformatus erit ex tribus angulis planis LVK, LVM, MVK æqualibus tribus angulis datis *A*, *B*, C. q. e. f.

Propoſ. XXIV. Theor. XXI.

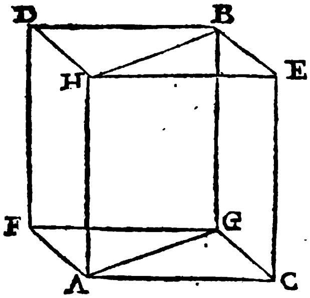
Si Parallelepipedum AB a ſex planis AE, FB, AG, HB, AD, CB fuerit comprehenſum. Dico plana oppoſita AE, FB &c. eſſe parallelogramma ſimilia, & æqualia.

Conſtructio.

In oppoſitis planis AG, HB notentur rectæ *A*G, HB.

De-

Demonſtratio primæ partis.



QVia plana parallela *AG*, *HB* ſecantur a plano BC. [*per* 16. *hujus*.) rectæ EB, & CG erunt parallelæ. Pariter quia parallela plana *AE*, FB ſecantur a plano BC [*per eamdem* 16.] rectæ BG, EC, & ipſæ erunt parallelæ: ergo (*per def.* 40. *pri.*) quadrilatera figura BC erit parallelogrammum. Eadem ratione demonſtrari poterunt parallelogramma reliqua plana parallelepipedum comprehendentia. q. e. d.

Demonſtratio ſecundæ partis.

REctæ lineæ *FA*, *AC* (*per demonſtrata in prima parte*) parallelæ ſunt rectis D*H*, HE: quare [*per* 10. *bujus*.) ang. *FAC* = ang. DHE, eamdem ob cauſam reliqui anguli parallelogrammi *AG* = ang. parallelogrammi *DE*.

Quoniam vero (*per* 34. *pri.*) eſt F*A* = DH, & *AC* = HE: erit F*A*: *AC*:: *D*H : HE.

Cum autem eadem analogia habeatur in reliquis lateribus oppoſitorum parallelogrammorum *AG*, HB: (*per def.* 1. 6. [parallelogramma *AG*, HB (idem valet de reliquis) erunt ſimilia, q. e. 2. d.

Demonſtratio tertiæ partis.

IN triangulis $\left.\begin{matrix} ACG \\ HEB \end{matrix}\right\}$ [*per* 34. *primi*, & *per demonſtrata*] latera *AC*, *CG* ſunt ſingula ſingulis æqualia lateribus HE, EB, & ang. *ACG* = ang. HEB: ergo [*per* 4. *pri.*] triangulum *ACG* = triangulo HEB: eadem ratione triang. *AFG* = triang. HDB: quare parallelogrammum FC = oppoſito DE. Cum autem idem de alijs parallelogrammis oppoſitis verificetur in parallelepipedo plana oppoſita erunt parallelogramma ſimilia, & æqualia. q. e. d.

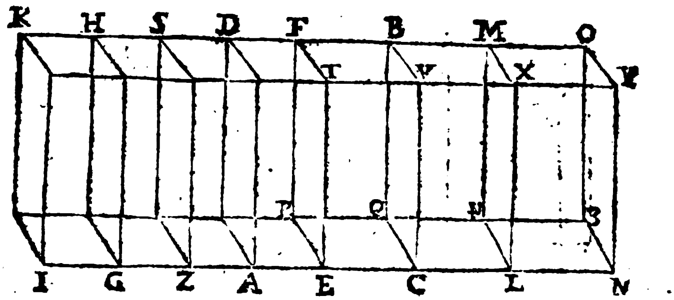
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Propoſ. XXV. Theor. XXII.

Si parallelepipedum AB ſectum fuerit plano EF oppoſitis planis AD, CB parallelo.

Dico ut baſis EV ad baſim AT, ita ſolidum EB ad ſolidum AF.



Conſtructio.

PArallelepipedo AB in oppoſitas partes IV, NO indefinitè producto, in recta AI accipiantur quæcumque partes AZ, ZG, GI ſingulæ æquales ipſi AE. Pariter in recta CN notentur partes CL, LN ſingulæ æquales ipſi EC: per puncta Z, G, I, L, N (*per Lemma ſuperius poſitum*) notentur plana ZS, GH, IK, LM, NO parallela plano AD, vel CB.

Demonſtratio.

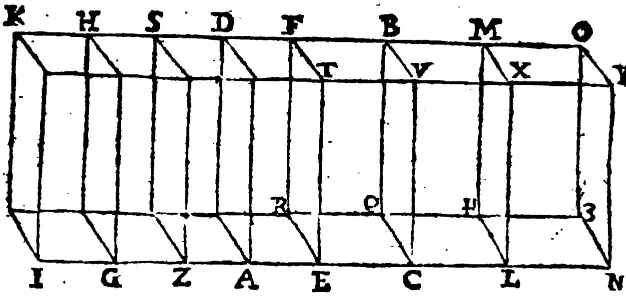
QVia ſolidum EB (*per hyp.*) a parallelis planis continetur:) [*per def. 29. hujus.*) eſt parallepipedum, in quo [*per 24. hujus.*] plana oppoſita EF, CB &c. ſunt parallelogramma ſimilia, & æqualia. Eadem ratione ſolida ED, AS, ZH, GK, CM, LO erunt parallelepipeda, in quibus (*per 24. hujus.*) oppoſita plana ſunt parallelogramma ſimilia, & æqualia.

Ulterius quia parallelogramma EV, CX, LY, [*per conſtru-*

ctio-

ctionem.) baſes habent æquales EC, CL, LN, & in ijſdem parallelis continentur: (*per* 36.*pri.*) erunt æqualia.

Pariter quia anguli unius parallelogrammi [*per* 29.*pri.*) ſunt aliorum parallelogrammorum angulis æquales, atque circa æquales angulos latera proportionalia, cum antecedentia ſint antecedentibus, & conſequentia conſequentibus æqualia: [*per def.*10 *hujus*.] parallelepipeda EB, CM, LO, erunt æqualia. Eadem ratione æqualia erunt parallelepipeda ED, AS, ZH, GK.



His manentibus quam multiplex baſis ND ipſius baſis CT, tam multiplex etiam erit parallelepipedum EO parallelepipedi EB, & quam multiplex eſt baſis IT ipſius baſis AT, tam multiplex etiam erit parallelepipedum IF parallelepipedi AF.

Ulterius quando baſis EY eſt æqualis baſi IT, cætera etiam plana parallelepipedi EO ſunt æqualia, & ſimilia planis parallelepipedi EK (*per def.* 10. *hujus*) parallelepipedum EO = parallelepipedo IF: quando baſis EY minor baſe IT, etiam ſolidum EO minus ſolido IF; & quando baſis EY major baſe IF, etiam parallelepipedum EO majus parallelepipedo IF.

Ponatur baſis EV prima magnitudo; baſis AT ſecunda; ſolidum EB tertia; ſolidum AF quarta: Primæ, atque tertiæ magnitudinis (*per demonſtata*) æqualiter multiplicia ſunt EY, & ſolidum EO: ſecundæ vero, & quartæ pariter multiplicia ſunt IT, & ſolidum IF. Cum autem hæc pariter multiplicia obtineant conditiones Def. 6. Libri quinti, (*per eamdem def.*) ut baſis EV ad baſim AT, ita parallelepipedum EB ad parallelepipedum AF. q. e. d.

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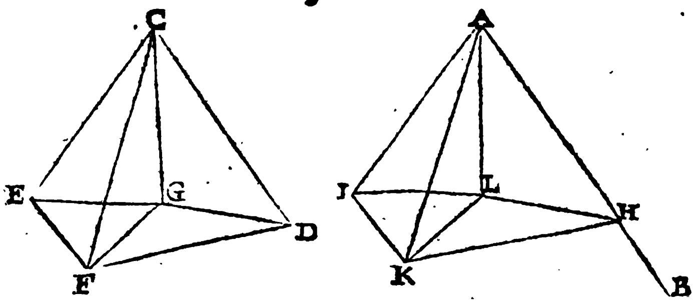
SCHOLIUM.

QVod de ſolis parallelepipedis ſuperius demonſtratum eſt, valet etiam de quibuſcumque priſmatis, quia portiones priſmatum ſectarum plano oppoſitis planis parallelo, ad inuicem ſunt ut baſes.

Propoſ. XXVI. Probl. IV.

Ad punctum datum A, & ad rectam AB conſtituere angulum ſolidum æqualem dato angulo ſolido C a tribus angulis planis DCE, DCF, FCE comprehenſo.

Conſtructio.



IN recta CF accipiatur quodcumque punctum F, a quo (*per* 11. *hujus*] ad planum rectarum CD, CE perpendicularis ducatur FG, ſignentur rectæ GC, GD, GE, FE, FD.

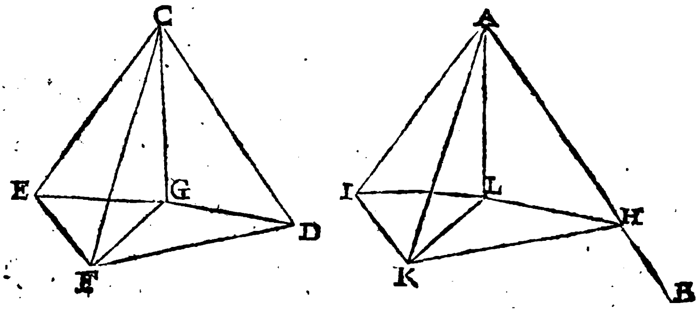
In data recta AB [*per* 3. *pri.*] fiat AH = CD. Ad punctum A, & ad rectam AB (*per* 23. *pri.*) conſtituatur angulus HAI = angulo DCE, ſitque recta AI = CE.

In plano rectarum AH, AI ad punctum A, & ad rectam AB fiat ang. HAL = ang. DCG, ſitque AL = CG; notentur rectæ LI, LH.

Ex puncto L [*per* 12. *hujus*) ad planum rectarum BA, AI excitetur perpendicularis LK = ipſi GF: addanturque rectæ lineæ KA, KH, KI.

Dico ſolidum angulum A = dato ſolido angulo C.

De-



Demonſtratio.

IN triangulis $\left.\begin{matrix}HAL\\DCG\end{matrix}\right\}$ [*per conſtructionem.*] latera HA, AL ſingula ſingulis æqualia lateribus DC, CG, & angulus HAL = angulo DCG: ergo [*per 4. pri.*] HL = DG.

Quia (*per conſtructionem.*) ang. HAI = ang. DCE, & ang. HAL = ang. DCG: erit ang. LAI = ang. GCE.

In rriangulis $\left.\begin{matrix}LAI\\GCE\end{matrix}\right\}$ (*per conſtructionem*) latera LA, AI alterum alteri æqualia lateribus GC, CE, & ang. LAI (*per demonſtrata*) = ang. GCE: igitur (*per 4. pri.*) LI = GE.

In triangulis $\left.\begin{matrix}HLK\\DGF\end{matrix}\right\}$ latera HL, LK [*per demonſtrata, & per conſtructionem*) ſingula ſingulis æqualia lateribus DG, GF, & angulus HLK = ang. DGF: igitur (*per 4. pri.*) HK = DF.

In triangulis $\left.\begin{matrix}ALK\\CGF\end{matrix}\right\}$ [*per conſtructionem, & per demonſtrata*) latera AL, LK ſingula ſingulis æqualia lateribus CG, GF, atque angulus ALK = ang. CGF, quia (*per conſtructionem*) recti: ergo [*per 4. pri.*) AK = CF.

In triangulis $\left.\begin{matrix}HAK\\DCF\end{matrix}\right\}$ (*per conſtructionem, & per demonſtrata*) latera HA, AK ſingula ſingulis æqualia lateribus DC, CF, baſiſque HK = baſi DF. igitur (*per 8. pri.*) ang. HAK = ang. DCF.

In triangulis $\left.\begin{matrix}KLI\\FGE\end{matrix}\right\}$ [*per conſtructionem, & per demonſtrata,*) latera KL, LI ſingula ſingulis æqualia lateribus FG, GE, & angulus KLI = ang. FGE, quia (*per conſtruct.*, ambo recti: quare [*per 4. pri.*] IK = FE.

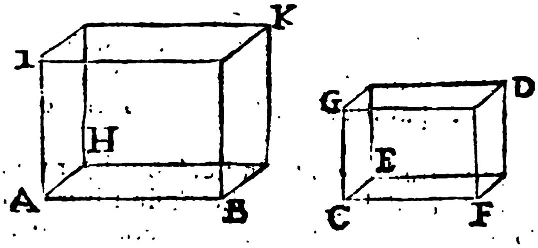
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In triangulis $\left.\begin{matrix}KAI\\FCE\end{matrix}\right\}$ (*per conſtruct. & per demonſtrata*) latera KA, AI ſingula ſingulis æqualia lateribus FC, CE, latuſque KI [*per demonſtrata.*] æquale lateri FE: ſequitur (*per* 8. *primi.*) ang. KAI = ang. FCE: quare ad datum punctum A, & ad datam rectam AB conſtitutus eſt ſolidus angulus A dato angulo ſolido C æqualis. q. e f.

Propoſ. XXVII. Probl. V.

Ad datam rectam AB parallelepipedum conſtituere ſimile, ſimiliterque poſitum dato parallelepipedo CD.



Conſtructio.

AD punctum A, & ad rectam AB [*per* 26. *hujus.*) ponatur ang. ſolidus A = ang. ſolido C, ſintque in his angulis ſolidis ſequentes plani anguli æquales.

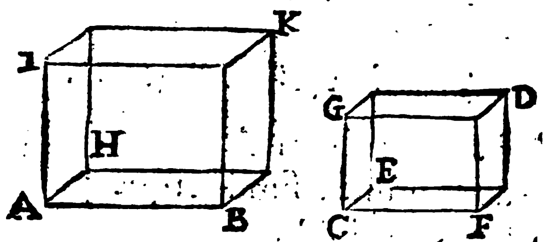
ang. planus BAI = angulo plano FCG.
ang. planus BAH = angulo plano FCE.
ang. planus IAH = ang. plano GCE.

Hoc poſito (*per* 12. *ſexti.*) fiat
FC : CG :: BA : AI. pariterque fiat
GC : CE :: IA : AH. hoc poſito
Ex æquo erit (*per* 22. *quinti.*)
FC : CE :: BA : AH
Compleantur parallelogramma BI, HI, BH.

Y Per

Per tria puncta B, I, H (*ut in ſchol. propoſ. 15. hujus.*] ducatur planum HK parallelum plano BI: planum BK parallelum plano HI, & planum BH parallelum plano KI: quo facto completum erit parallelepipedum AK, quod

Dico ſimile, & ſimiliter poſitum dato parallelepipedo CD.



Demonſtratio.

ANgulus BAH (*per conſtructionem.*] æqualis angulo FCE, atque latera circa iſtos angulos proportionalia: quare (*per def. 1. ſexti.*] parallelog. BH ſimile parallelog. FE, & ſimiliter poſitum.

Eadem ratione parallelog. *HI*, ſimile, & ſimiliter poſitum parallelog. EG; atque parallelog. *BI* ſimile, & ſimiliter poſitum parallelog. FG. Quo poſito in parallelep. *AK* erunt plana BH, HI, IB ſimilia, ſimiliterque poſita planis FE, EG, GF in ſolido CD. Cum autẽ (*per 24. hujus.*) in omni parallelepipedo tria plana ſint tribus oppoſitis planis ſimilia: inde ſequitur ſex plana parallelepipedi AK eſſe ſimilia, ſimiliterq; poſita ſex planis parallelepipedi CD: quare (*per def. 9. hujus.*) parallelepipedum AK ſimile erit, ſimiliterque poſitum parallelepipedo CD. q. e. f.

Propoſ. XXVIII. Theo. XXIII.

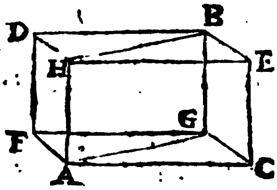
Si parallelepipedum AB a plano ducto per diagonios oppoſitorum planorum AG, HB ſectum fuerit.

Dico ab eodem plano biſariam ſecari.

De-

Demonſtratio.

QVia [*per 34. pri.*] rectæ AH, BG ſunt parallelæ, & æquales ipſi FD: (per *9. bujus*] AH, & GB inter ſe parallelæ, & æquales erunt: quare (*per 33. pri.*) HB parallela, & æqualis ipſi AG: ideoque diagonales AG, HB (*per def. 39. pri.*] ſunt in eodem plano; quod planum dico bifariam dividere parallelepipedum *AB*.



Oppoſita plana ED, CF (*per 24. bujus*] ſunt parallelogramma ſimilia, & æqualia: quare (*per 34. pri.*) triangulum BDH = triangulo GFA, paritérque triang. HEB = triang. ACG. Inſuper, cum in triangulis $\left.\begin{matrix}BDH\\GFA\end{matrix}\right\}$ angulus BDH (*per 10. bujus.*] ſit æqualis angulo CFA, atque latera circa æquales angulos proportionalia, quia [*per 34. pri.*] DB = FG, & DH = FA, triangula $\left.\begin{matrix}BDH\\GFA\end{matrix}\right\}$ (*per 6. ſexti*) erunt ſimilia: ob eamdem rationem ſimilia erunt triangula $\left.\begin{matrix}HEB\\ACG\end{matrix}\right\}$.

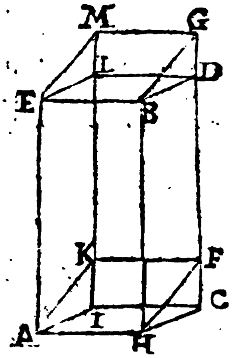
Vlterius quia (*per 24. bujus.*) parallelog. *AD* ſimile eſt, & æquale parallelog. CB: parallelog. CH ſimile, & æquale parallelog. GD: parallelog. vero *AB* commune: (*per def. 10. bujus.*] priſma *ADBG* æquale erit priſmati *ECHA*. q. e. d.

Propoſ. XXIX. Theor. XXIV.

Si parallelepipeda ACDE, AFGE ſuper eadem baſi AB, & in eadem altitudine fuerint conſtituta hac lege, ſcilicet quod parallelepipedorum inſiſtentes lineæ AI, AK: EL, EM: BD, BG: HC, HF in iiſdem collocentur rectis lineis CG, IM.

Dico hæc parallelepipeda eſſe æqualia.

Demonſtratio.



QVia parallelog. AL (*per 35. pri.*) = eſt parallelogrammo AM æquale, ablato communi trapetio AKLE; erit triang. AIK = triang. ELM.

Pari modo quia (*per 34. pri.*) IL, & KM (*per 34 pri.*) ſunt ipſi AE æquales (*per axio. 1.*) ablata communi parte KL, erit IK = LM, pertereа eſt (*per 34. pri.*) EL = AI.

Cum igitur duo triang. AIK, ELM } (*per. 29. pri.*) habeant angulum AIK = angulo ELM, atque latera circa æquales angulos proportionalia, propter demonſtratam antecedentium, & conſequentium æqualitatem: (*per 6. ſexti.*) dicta triangula erunt æquiangula, ideoque (*per 4. ſexti.*) ſimilia.

In triangulis AIK, HCF } quia (*per 34. pri.*) ſingula latera ſunt ſingulis lateribus æqualia, cumque (*per 10. hujus.*) ſinguli anguli ſint ſingulis angulis æquales: (*per 4. ſexti.*) dicta triangula erunt ſimilia, & æqualia.

Pari ratione ſimilia, & æqualia erunt triangula ELM, BDG: quatuor igitur triangula ELM, BDG, AIK, HCF ſimilia erunt, & æqualia.

Parallelog. AC (*per 24. hujus.*) ſimile, & æquale parallelog. ED.

Parallelog. AF ſimile, & æquale parallelog. EG.

Parallelog. IF ſimile, & æquale parallelog. LG.

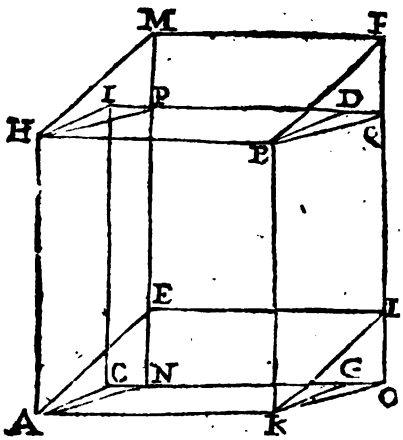
Quare (*per def. 10. hujus.*) priſma AHCK = priſmati EBDM addito communi ſolido AFDE: erit parallelepipedum ACDE = parallelepipedo AFGE ſuper eadem baſi, & in eadem altitudine conſtituto. q. e. d.

Pro-

Propoſ. XXX. *Theor.* XXV.

Si ſuper eadem baſi AB, & in eadem altitudine conſtituta fuerint parallelepipeda AIDK, AMFK, hac lege, quod ſcilicet rectæ AE, HM, BF, KL inſiſtentes parallelepipedi AMFK non cadant in lineas alterius parallelepipedi AIDK. Dico hæc parallelepipeda eſſe æqualia.

Conſtructio.



QVia (*per hyp.*) parallelepipeda AIDK, *AMFK*, ſunt in eodem altitudine: parallelogramma IG, ML erunt in eodem plano. Productis igitur lineis CG, ID, FL, ME hæ omnes lineæ erunt in plano parallelogrammorum IG; ML; atque CG concurret cum FL in O: & cum *ME* in N. ID vero concurret cnm FL in Q, & cum *ME* in P. Hoc poſito erit factum parallelogrammum PQON. Ducantur jam rectæ HP, BQ, KO, AN.

Demonſtratio.

PQ (*per* 34.*pri.*) = MF: MF = HB: ergo (*per axio.*1.) PQ = HB: ſunt autem (*per eamdem* 34. *pri.*) PQ, & *HB* parallelæ: quare (*per* 33. *pri.*) HP ipſi BQ æqualis, & parallela: ergo HQ erit parallelogrammum. Eadem ratione parallelogramma erunt AP, AO, *KQ*, NQ: quapropter ſolidum APQK (*per def.*21.*hujus.*) erit parallelepipedum. Cum autem (*per* 29. *hujus.*) parallelepipedum AIDK æquale ſit parallelepipedo APQ*K*; parallelepi-

pe-

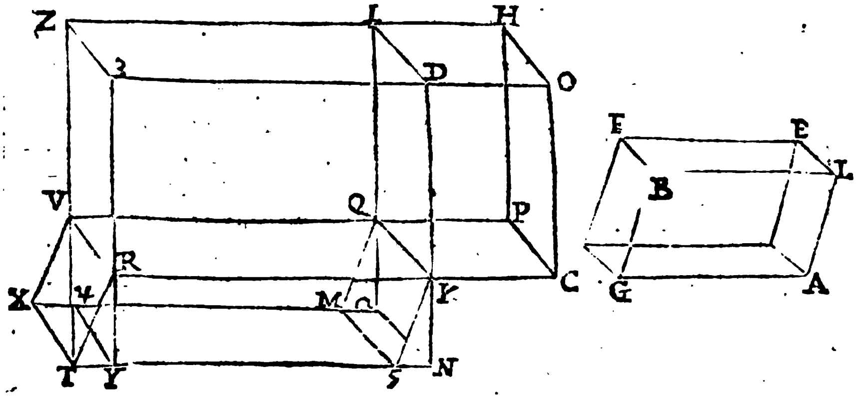
pedum autem APQK (*per eamdem* 29.] æquet parallelepipedum AMFK (*per axio*.1.) erit parallelepipedum AIDK = parallelepipedo AMFK. q. e d.

Propoſ. XXXI. Theor. XXVI.

Si parallelepipeda AEFG, CHIK ſuper æquales baſes AB, CD, & in eadem altitudine fuerint conſtituta.

Dico eſse æqualia.

In hoc theoremate parallelepipedorum inſiſtentes LE, BF, CP, OH &c. poſsunt eſse baſibus AB, CD rectæ, vel inclinatæ.



Conſtructio.

Producatur recta CK, fiatque KR = BL. In plano OK (*per* 23.*pri.*) fiat angulus RKS = ang. BLA, ſitque KS = LA. Compleatur parallelogrammum KT, Supra baſim KT in altitudine KQ conſtituatur parallelepipedum QSTV.

Producantur DK, & TS quouſque concurrant in N, perficiaturque parallelogrammum KY. Supra baſim KY conſtituatur pa-

parallelepipedum QNYV. Rectæ HI, & HV productæ coucurrant in Z: OD, & YR ſe tangant in 3: compleatur parallelepipedum IKRZ.

Demonſtratio quando inſiſtentes parallelepipedorum ſunt baſibus rectæ.

QVia rectæ KR, KS (*per conſtructionem.*) æquales rectis BL, LA, & angulus RKS = ang. BLA: (*per 29. & 34. primi.*) parallelog. KT, LG ſunt ſimilia, & æqualia. Pariter (*per conſtructionem.*) rectæ KQ, KS = rectis EL, LA, & angulus QKS = ang. ELA, quia QK, & EL ſupponuntur rectæ planis QT, LG; ergo parallelog. QS ſimile, & æquale parallelog. EA.

Eodem modo demonſtrabitur parallelog. KV ſimile, & æquale parallelog. LF.

Cum igitur in parallelepipedo QSTV (*per demonſtrata.*) tria plana KT, QS, KV æqualia ſint, & ſimilia tribus planis LG, EA LF in parallelepipedo AEFG, atque [*per 24. hujus.*] in parallelepipedo oppoſita plana ſint æqualia, & ſimilia: (*per def. 10. hujus.*) parallelepipedum QSTV = parallelepipedo AEFG. Quoniam vero (*per 29. hujus.*) parallelepipedum QSTV eſt æquale parallelepipedo QNYV. [*per axio. 1.*] parallelepipedum QNYV = parallelepipedo AEFG.

Parallelog. KT (*per 35. pri.*) = parallelog. KY, & parallelog. KT (*per conſtructionem*) = parallelog. LG; LG verò (*per hyp.*) = parallelog. CD: ergo parallelog. KY = parallelog. CD. Quare [*per 7. quinti.*)

Ut parallelog. CD, ad parallelog. DR; ita parallelog. KY ad parallelog. DR.

Sed [*per 25. hujus.*]

Ut parallelog. CD ad parallelog. DR; ita parallelepip. CHIK ad parallelepipedum KIZR.

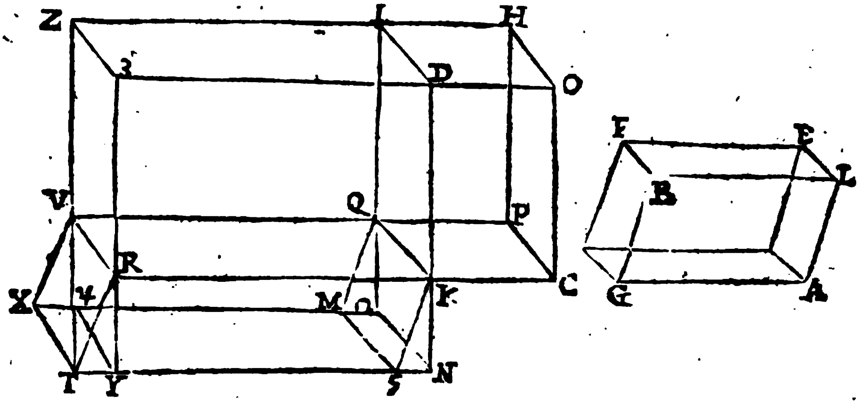
Pariterque (*per eamdem 25.*)

Ut parallelog. KY ad parallelog. DR; ita parallelepip. NQVY ad parallelepip. KIZR.

Ergo [*per 9. quinti.*]

Parallelepipedum CHIK = parallelepipedo QNYV.

Sed [*per demonſtrata.*)
Parallelepip. QNYV = parallelepip. AEFG : ergo erit etiam
Parallelepip. CHIK = paralleled. AEFG. q. e. 1. d.



Demonſtratio quando inſiſtentes ſunt baſibus inclinatæ.

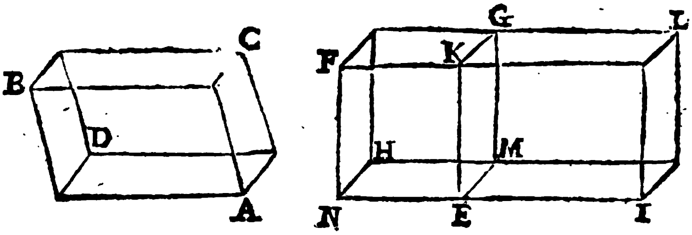
SUpra æquales baſes AB, CD in eadem altitudine intelligantur conſtituta parallelepipeda, quorum inſiſtentes ſint baſibus AB, CD inclinatæ. Hoc poſito [*per 29, vel 30. bujus.*) parallelepipeda cum inſiſtentibus inclinatis æqualia ſunt parallelepipedis cum inſiſtentibus rectis baſibus AB, CD: quare ſi) *per demonſtrata in prima parte*] parallelepipeda, quorum inſiſtentes ſunt baſibus rectæ, inter ſe ſunt æqualia; etiam æqualia inter ſe erunt illa parallelepipeda, quorum inſiſtentes ſunt baſibus inclinatæ. q. e. 2. d.

Propoſ. XXXII. *Theor. XXVII.*

Si parallelepipeda ABDC, EGFH ſupra baſes AB, EF conſtituta, eamdem habuerint altitudinem.

Dico inter ſe eſſe ut baſes.

Con-



Conſtructio.

AD rectam EK in angulo FNE (*per* 45. *pri.*) conſtituatur parallelogrammum IK = parallelog. AB. Parallelepipedi EGFH, hoc eſt plana GF, GH, EH, producantur ad partes I, & L. Per punctum I (*per Scholium propoſ.* 15. *bujus.*] ducatur planum IL plano EG parallelum; eritque completum parallelepipedum EL.

Demonſtratio.

QVia angulus KEI [*per conſtructionem.*) = ang. FNE [*per* 14. *primi.*) NEI erit linea recta, atque non ſolum IF erit unum parallelogrammum, verum etiam ſolidum ILFH erit unum parallelepipedum, cujus pars, nempè parallelepipedum ILGE, (*per* 37. *bujus.*) æquale erit parallelepipedo ABCD.

Quare [*per* 7. *quinti.*)
Ut parallelepi. ABCD ad parallelepi. EGFH;
Ita parallelepi. ILGE ad parallelepi. EGFH;
Sed (*per* 25. *bujus.*)
Ut parallelepi. ILGE ad parallelepi. EGFH,
Ita baſis IK ad baſim EF.
Ergo (*per* 11. *quinti.*]
Ut parallelepi. ABCD ad parallelepi. EGFH;
Ita baſis IK ad baſim EF;
Quia vero baſis IK (*per conſtructionem.*) = baſi AB: erit
Ut parallelepi. ABCD ad parallelepi. EGFH;
Ita baſis AB, ad baſim EF. q. e. d.

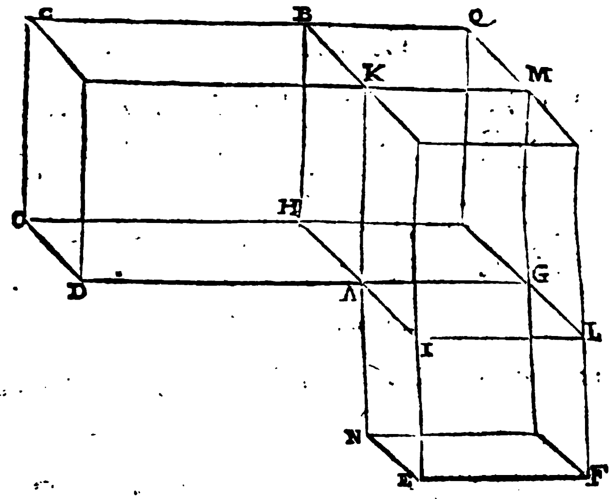
Pro-

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Propoſ. XXXIII. Theor. XXVIII.

Si fuerint ſimilia parallelepipeda ABCD, AEFG.

Dico inter ſe habere triplicatam rationem laterum homologorum DA; AG.



Conſtructio.

Homologa letera omnium ſimilium parallelogramm. taliter diſponantur ut ſint in rectum conſtituta; quodquidem fieri poteſt, quia ob ſuppoſitam parallelogrammorum DK, NG ſimilitudinem ang. DAK = NAG: quare (*per corol.3.prop.15.pri.*) Si DA,eſt in rectum cum AG, etiam AK erit in rectum cum AN. Pariter quia dummodo tamen parallelogramma DK, GN ſupponantur poſita in eodem plano: ang. DAH = ang. GAI, quando recta eſt DAG, etiam recta erit HAI.

Poſita hac homologorum laterum diſpoſitione compleantur parallelepipeda ABQC, IKML.

De-

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Demonſtratio.

QVia parallelog. AO (*per hyp.*) ſimile parallelog. AL,
DA : AH : : AG : AI.
Alternando [*per 16. quinti.*]
DA : *AG* : : *AH* : *AI*.
Pariter ob ſimilitudinem parallelog AB, AE.
HA : AK : : IA : AN :
Permutando [*per 16.quinti.*]
HA : IA : : AK : AN.
His manentibus parallelepipedum ABCD [*per 32. hujus*] ad parallelepipedum ABQG, ut baſis DH ad baſim HG;
Sed [*per 1. ſexti.*)
Ut baſis DH ad baſim HG: ita DA, ad AG: ergo
parallelepi. ABCD ad parallelepi. ABQG,
ut DA, ad AG.
Ob eamdem rationem, erit
Parallelip. ABQG ad parallelip. IKML,
ut HA ad AI:
Pariterque erit.
Parallelepi. IKML ad parallelepi. AEGF;
ut KA ad AN.
Cum autem (*per hyp.*] ſit
HA, ad AI, atque KA ad AN, ut DA AG,
Quatuor parallelepipeda ABCD: ABQG: IKML: AEGF,
inter ſe erunt in ratione DA, ad AG: quare
(*per def.* 10. *quinti.*)
Primum parallelepipedum ABCD, ad quartum ſibi ſimile AEGF rationem habebit triplicatam homologorum laterum DA, AG.
q. e. d.

Corollarium.

EX hoc theoremate fit manifeſtum, quod poſitis quatuor rectis lineis continue proportionalibus, ut prima ad quartam, ita parallelepipedum ſupra primam ad ſibi ſimile parallelepipedum ſupra ſecundam.

Pro-

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Propoſ. XXXIV. Probl. XXIX.

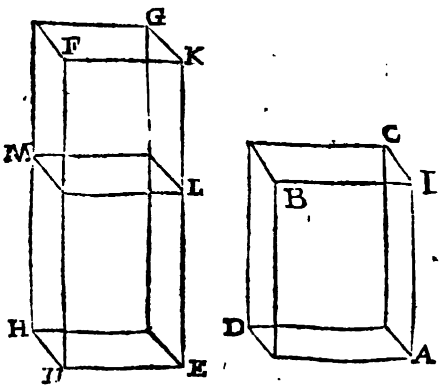
Si fuerint æqualia parallelepipeda ABCD, EFGH.

Dico baſes AD, EH, & altitudines AI, EK reciprocari, & viciſſim.

Si parallelepipedorum ABCD, EFGH reciprocentur baſes, & altitudines.

Dico parallelepipeda eſſe æqualia.

In hoc Theoremate hæc contingere poſſunt.



1. QVod parallalepipedorum inſiſtentes ſint baſibus rectæ.
2. Quod ſint baſibus inclinatæ.
3. Quod ſint æquales, vel inæquales.

Conſtructio quando inſiſtentes ſunt baſibus rectæ.

QVando inſiſtentes AI, EK &c. ſunt baſibus AD, EA rectæ, ſi EK major ſit AI [*per 3. pri.*] notetur EL = AI, atque (*per Scol. Propoſ.* 15. *hujus.*] ducatur planum LM baſi EH parallelum.

Demonſtratio.

QVia [*per hyp.*) inſiſtentes AI, EK ſunt baſibus AD, EH rectæ [*per def.* 4. *ſexti.* erunt menſura altitudinum parallelepipedorum ABCD, EFGH.

Hoc poſito, ſi altitudo AI ſit = altitudini EK [*per converſam prop.*

prop.31.hujus; erit etiam baſis AD = baſi EH: quare reciprocè, ut baſis AD ad baſim EH; ita altitudo EK ad altitudinem AI. q.e.1.d.

Demonſtratio, quando EK major quam AI.

QVia [*per conſtructionem.*] altitudo AI = altitudini EL, atque (*per hyp.*) parallelepi. ABCD = parallelepi. EFGH: (*per 7. quinti.*)

Ut parallelepi. ABCD ad parallelepi. EHLM;
Ita parallelepi. EFGH ad idem parallelepi. EHLM;
Sed (*per 32. hujus.*)
Ut parallelepi. ABCD ad parallelepi. EHLM;
Ita baſis AD ad baſim EH.

Eadem ratione, cum parallelepi. EFGH, EHLM ſint in eadem altitudine,

(*per eamdem 31. hujus.*)
Ut parallelepi. EFGH, ad parallelepi. EHLM.
Ita baſis KN, ad baſim LN:
Quare (*per 11. quinti.*)
Ut baſis AD, ad baſim EH; ita baſis KN ab baſim LN:
Sed (*per 1. ſexti.*)
Ut baſis KN, ad baſim LN; ita KE ad LE.
Et (*per conſtructionem.*) LE = AI, ergo erit
Ut baſis AD ad baſim EH; ita reciprocè altitudo EK
ad altitudinem AI. q. e. 2. d.

Demonſtratio, quando inſiſtentes ſunt baſibus inclinatæ.

SUpra baſes AD, EH intelligantur conſtituta parallelepipeda, quorum inſiſtentes ſint baſibus rectæ in eadem altitudine cum illis parallelepipedis, quorum inſiſtentes ſunt baſibus inclinatæ.

Hoc facto ſupra ſingulas baſes AD, EH conſtituta erunt duo parallelepipeda æque alta, quorum unum inſiſtentes habet baſi rectas, alterum vero inclinatas.

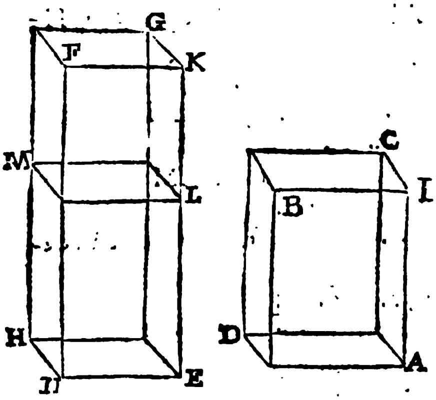
Parallelepipeda ſupra communes baſes AD, EH conſtituta (*per 29. vel 30. hujus.*) ſunt æqualia.

Quia

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Quia vero (*per demonſtrata.*) parallepipeda æqualia cum inſiſtentibus baſibus rectis, baſes, & altitudines habent reciprocas, etiam æqualia parallelepipeda cum inſiſtentibus baſibus inclinatis, habebunt baſes, & altitudines reciprocas. q. e. d.

Demonſtratio alterius partis.



Eſt (*per hyp.*) ut baſis AD ad baſim EH; ita altitudo EK ad altitudinem AI, ſiue LE.

Sed (*per 1. ſexti.*)

Ut EK ad LE, ita baſis KN ad baſim LN.

Ergo (*per 11. quinti.*)

Ut AD ad EH; ita KN ad LN

Sed (*per 25. hujus.*)

Ut KN ad LN; ita parallelep. EGFH ad parallelep. EHML,

Ergo (*per 11. quinti.*

Ut AD ad EH; ita parallelepi. EGFH ad parallelepi. EHML.

Cum autem (*per 32. hujus.*)

Ut AD ad EH; ita parallelepi. ABCD, ad parallelepi. EHML:

Erit (*per 11. quinti.*)

Ut parallelepi. ABCD ad parallelepi. EHML;

Ita parallelepi. EGFH ad idem parallelepi. EHML.

Quare (*per 9. quinti.*)

Parallelepi. ABCD = parallelepipedo EGFH. q. e. 2. d.

Propoſ. XXXV. *Theor.* XXX.

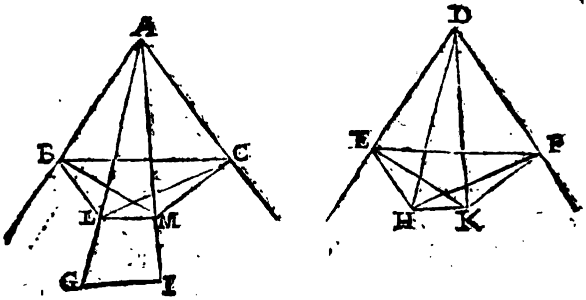
Si duo anguli plani BAC, EDF fuerint æquales, eorumque verticibus A, & D in ſublimi inſiſtant rectæ lineæ AG, DH, quarum AG cum lineis primo poſitis AB, AC contineat angulos BAG, GAC ſingulos ſingulis angulis EDH, HDF æqua-

æquales : atque in lineis *A*G, DH accepta fuerint quæcumque puncta G, & H, a quibus ad plana rectarum B*A*C, *EDF* ductæ ſint rectæ GI, HK additis lineis I*A*, KD.

Dico angulum G*A*I = angulo HDK.

Conſtructio.

SI AG major ſit quam DH, ex AG [*per* 3. *primi.*] auferatur AL = DH. A puncto L in plano trianguli GAI [*per* 31. *primi.*] ducatur LM parallela GI. Denuo a punctis M, & K ad rectas AB, AC, *DE*, *DF* perpendiculares ducantur MB, MC, KE KF, addanturque rectæ BC, BL, LC, *EF*, *EH*, *HF*.



Demonſtratio.

QVia (*per conſtructionem.*) LM parallela GI, atque GI recta ad planum per BA, AC ductum, etiam LM [*per* 8. *hujus.*] eidem plano recta erit: ideoque (*per def.* 3. *hujus.*) ang. LMA rectus.

Hoc poſito [*per* 47. *primi.*]

quad. AL = { quad. AM, quad. ML } quad. *A*M = { quad. *A*C, quad. CM }

Ergo (*per axio.* 1.)

quad. *A*L = { quad. *A*C, quad. CM, quad. ML }

Sed

Sed [*per 47. pri.*)

$$\left.\begin{array}{l}\text{quad. CM}\\ \text{quad. ML}\end{array}\right\} = \text{quad. CL: ergo quad. } AL = \left.\begin{array}{l}\text{quad. } AC\\ \text{quad. CL}\end{array}\right\}$$

Quare [*per 48. pri.*)

Angulus ACL eſt rectus.

Pariter (*per 47. primi.*)

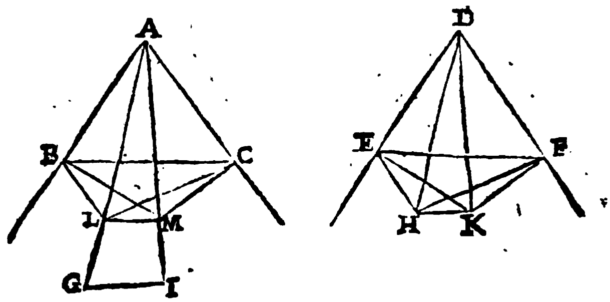
$$\text{quad. } AM = \left.\begin{array}{l}\text{quad. } AB\\ \text{quad. } BM\end{array}\right\} : \text{quare}$$

$$\text{quad. } AL = \left.\begin{array}{l}\text{quad. } AB\\ \text{quad. BM}\\ \text{quad. ML}\end{array}\right\} ;\& \text{ [per 47. pri.) q.} \left.\begin{array}{l}BM\\ ML\end{array}\right\} = \text{quad. BL:}$$

$$\text{Ergo quad. } AL = \left.\begin{array}{l}\text{quad. } AB\\ \text{quad. } BL\end{array}\right\} \text{ ideoque [per 48. pri.]}$$

Angulus ABL eſt rectus.

Eadem prorſus ratione demonſtratur angulos DFH, DEH eſſe rectos.



$$\text{His demonſtratis ang. } \left.\begin{array}{l}ABL\\ LAB\end{array}\right\} \text{ (per conſtructionem,}$$

& *per demonſtrata*) ſunt ſinguli æquales ſingulis angulis $\left.\begin{array}{l}DEH\\ HDE\end{array}\right\}$ latuſque AL = lateri DH: quare (*per 26. primi.*] AB = DE, & BL = EH.

Eadem ratione demonſtratur AC = D.F, & CL = FH.

In triangulis $\left.\begin{array}{l}BAC\\ EDF\end{array}\right\}$ [*per demonſtrata*] latera BA, AC ſingula ſingulis æqualia lateribus ED, DF, & ang. BAC = [*per hyp.*]

an-

angulo EDF: ergo (*per 4. pri.*) BC = EF, atque ang. ABC = ang. DEF, & ang. ACB = ang. DFE.

Sunt autem (*per conſtructionem.*) ang. $\left.\begin{matrix}ABM\\ ACM\end{matrix}\right\} = \text{ang.}\left.\begin{matrix}DEK\\ DFK\end{matrix}\right\}$ demptis æqualibus remanebit ang. CBM = ang. FEK, & ang. BCM = ang. EFK.

Cum autem [*per demonſtrata.*] ſit BC = EF:

(*per 26. primi.*)

Erit BM = EK, atque CM = FK.

In triangulis $\left.\begin{matrix}ACM\\ DFK\end{matrix}\right\}$ [*per demonſtrata.*) latera AC, CM ſingula ſingulis æqualia lateribus DF, FK, atque ang. ACM = (*per conſtructionem.*) ang. DFK: quare (*per 4. pri.*] AM = DK.

Cum autem (*per demonſtrata.*] ſit BL = EH [*per Schol. prop. 46. pri.*] quad. BL = quad. EH.

Sed [*per 47. pri.*)

$$\text{quad. } BL = \left.\begin{matrix}BM\\ ML\end{matrix}\right\} = \begin{matrix}\text{quad. } EK\\ \text{quad. } KH\end{matrix}$$

$$\text{Erunt } \left.\begin{matrix}\text{quad. } BM\\ \text{quad. } ML\end{matrix}\right\} \text{ \& q. } EH = \begin{matrix}EK\\ KH.\end{matrix}$$

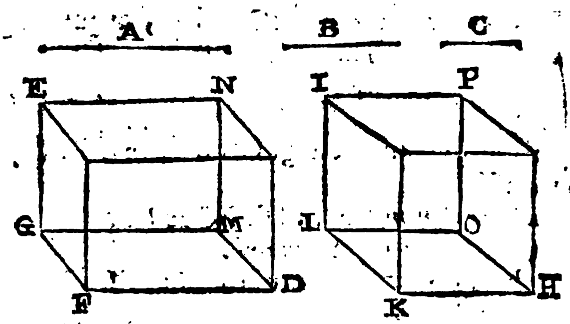
ablatis æqualibus quadratis rectarum æqualium BM, EK: remanebit quad. LM = quad. HK: ideoque (*per Schol prop. 46. pri.*) LM = HK.

Demum in triang. $\left.\begin{matrix}AML\\ DKH\end{matrix}\right\}$ [*per conſtructionem, & per demonſtrata.*] latera LA, AM ſingula ſingulis æqualia lateribus HD, DK, baſiſque LM = baſi HK: ergo [*per 8. pri.*] ang. LAM = ang. HDK. q. e. d.

Propoſ. XXXVI. Theor. XXXI.

Si tres rectæ lineæ A, B, C, proportionales fuerint.

Dico parallelepipedum a tribus datis lineis A, B, C deſcriptum æquale eſſe parallelepipedo æquilatero a media B deſcripto priori parallelepipedo æquiangulo.



Conſtructio.

EX tribus quibuſcumque angulis planis, FGM, FGE, MGE conſtituatur angulus ſolidus G, ſitque GM = A; GE = B; & GF, = C. Compleantur tria parallelogramma FM, FE, ME, atque totum parallelepipedum DNEF.

Pariter [*per* 26. *hujus*.] fiat ſolidus angulus L = ang. ſolido G, ſitque

ang. planus KLO = ang. FGM,
ang. planus KLI = ang. FGE,
ang. planus OLI = ang. MGE,

tres rectæ LO, LK, LI = ipſi B. Compleantur parallelogramma KO, KI, OI, atque totum parallelepipedum HPIK, quod á media proportionali B erit efformatum.

Demonſtratio.

QVia (*per conſtructionem*.) in parallelogrammis FM, *KO* ſunt æquales anguli FGM, *K*LO, inſuperque GM : LO :: LK : GF (*per* 14. *ſexti*.] erit parallelog. FM = parallelog. *K*O.

Pariterque quia (*per conſtruct*.) angulus ſolidus L æqualis eſt ſolido ang. G, ſi punctum L intelligatur in G, angulus planus *K*LI congruet cum angulo plano FGE, quia dicti anguli facti ſunt æquales: quare recta LI cadet ſupra GE.

Quia vero [*per conſtruct*.] LI, & GE ſingulæ ſunt æquales ipſi B, & inter ſe æquales erunt: ergo punctum I cadet in E. Quamobrem ſi ex E ad planum baſibus FM, KO commune [*per*

11.

11. *bujus*.] recta ducatur, hæc erit communis parallelepipedorum HI, DE altitudo.

Cum autem, & baſes FM, KO demonſtratæ ſint, & ipſæ æquales: erit [*per* 31. *bujus*.) parallelepi. DE = parallelepi. HI. q. e. d.

Propoſ. XXXVII. *Theor.* XXXII.

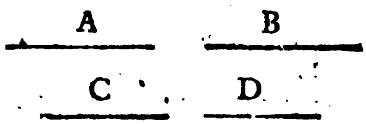
Si quatuor rectæ lineæ A, B, C, D fuerint proportionales.

Dico ſimilia parallepipeda ſuper has lineas deſcripta eſſe proportionalia, & viciſſim

Si ſimilia parallepipeda ſuper rectas lineas A, B, C, D deſcripta, fuerint proportionalia.

Dico rectas A, B, C, D eſſe proportionales.

Demonſtratio primæ partis.



Parallelepipedum ſupra A ad ſibi ſimile parallelepipedum ſupra B (*per* 33. *bujus*.] rationem habet triplicatam A, ad B. Eadem ratione parallelepi. ſupra C ad ſibi ſimile ſupra *D*, eſt in triplicata ratione C ad D: ſed [*per* 35. *quinti*.)ratio triplicata A ad B = triplicatæ C ad D: ergo ut parallelepip. ſupra A, ad prallelepi. ſimile ſupra B; ita parallelepi. ſupra C, ad parallelepi. ſimile ſupra D. q. e. 1. d.

Demonſtratio ſecundæ partis.

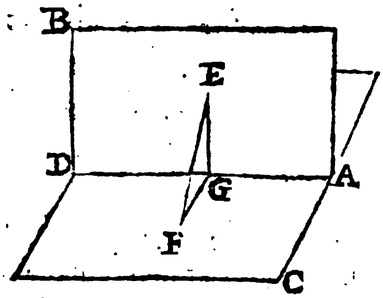
Recta A ad rectam B [*per* 33. *bujus*.) rationem habet ſubtriplicatā parallelepipedi ſupra A ad parallelepipedum ſupra *B*. Pariter recta C ad rectam D (*per eumdem* 33.) rationem habet ſubtriplicatam parallelepipedi ſupra C ad parallelepipedum ſupra D, ſed (*per* 35. *quinti*.) ſubtriplicata parallelepi. ſupra A ad parallelepip. ſupra B, eadem eſt ac ſubtriplicata parallepi. ſupra C, ad parallelepi. ſupra D: ergò eadem ratio A ad B, & C ad D. q. e. 2. d.

Propoſ. XXXVIII. *Theor.* XXXIII.

Si planum BA ſit plano DC rectum, & a puncto E in plano BA conſtituto ducta ſit EG plano DC recta.

Dico EG cadere in communem planorum ſectionem AD.

Conſtructio, & Demonſtratio.



SI ex puncto E quæ ducitur plano DC recta extra communem ſectionem AD cadere poteſt, cadat in F; à puncto E ad communem ſectionem DA perpendicularis ducatur EG: addatur FG.

Quia [*per hyp.*) planum BA rectum eſt plano DC, atque EG facta eſt perpendicularis communi ſectioni DA: (*per def. 4. hujus.*] EG recta erit plauo DC: quare (*per def. 3. hujus.*) ang. EGF erit rectus. Cum autem, & linea EF ſupponatur recta plano DC (*per eamdem def. 3.*) ang. EFG rectus erit: quare in triangulo EGF erunt duo anguli recti; (*quod eſt contra propoſ. 17. pri.*) ergo EF non eſt recta plano DC. Hoc abſurdum demonſtrari poteſt de quacumque alia linea extra communem ſectionem DA cadente: igitur linea quæ ex E recta ducitur plano DC in communem planorum ſectionem DA cadet. q. e. d.

Propoſ. XXXIX. *Theor.* XXXIV.

Si parallelepipedi *AB* latera oppoſitorum planorum BD, C*A* bifariam dividantur in punctis I, K, L, M, P, Q, N, O; per ſectiones autem

tem

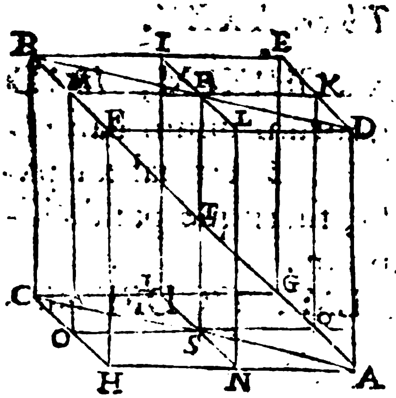
tem ducta ſint plana IN, KO, quorum communis ſectio ſit RS; ducta diametro AB.

Dico rectas AB, & RS; ſe mutuo bifariam ſecare in T.

Conſtructio.

Notentur rectæ SC, SA, RB, RD.

Demonſtratio.



In triangulis SQA, SOC (*per conſtructionem*) latera AQ, QS ſunt ſingula ſingulis æqualia lateribus CO, OS, & ang. AQS (*per 29. pri.*) = ang. COS: quare (*per 4. pri.*) SA = SC, & ang. ASQ = ang. CSO: addito communi ang. ASO; erunt ang. ASQ, ASO = ang. ASO, CSO

ſed ang. ASQ, ASO (*per 13. pri.*) = duobus rectis: ergo & ang. ASO, CSO duobus rectis æquales: ideoque (*per 14. pri.*) ASC erit linea recta. Eadem ratione demonſtratur DR, RB rectam lineam componere.

Rurſus quia utraque rectarum AD, BC (*per 34. pri.*) parallela, & æqualis ipſi FH: (*per 9. hujus*) erit AD parallela, & æqualis ipſi BC: quare (*per 33. pri.*) AC æqualis, & parallela ipſi DB atque medietas AS = medietati BR.

Denuo quia (*per demonſtrata*) AC eſt parallela DB (*per def. 39. pri.*) erunt in eodem plano. Sunt vero in plano rectarum AC, DB (*per 7. hujus*) etiam AB, RS; quare cum non ſint parallelæ, ſe ſecabunt; ſe igitur ſecent in T.

In triangulis $\left.\begin{matrix}AST\\BRT\end{matrix}\right\}$ (*per* 29. & *per* 15. *pri.*) anguli AST, ATS ſunt ſinguli ſingulis æquales angulis BRT, BTR; atque (*per demonſtrata*) AS = BR: ergo (*per* 26. *pri.*) erit non ſolum AT = BT, verum etiam ST = RT. q. e. d.

Corollarium.

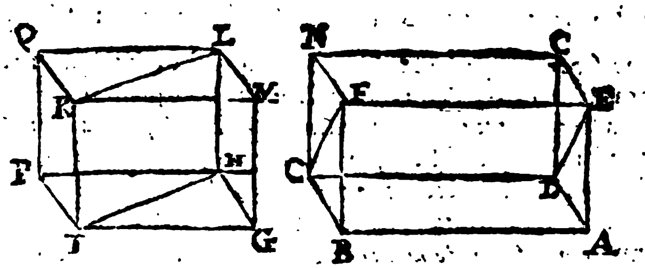
IN omni parallelepipedo diametri, ſeſe mutuo bifariam ſecant in uno puncto, ut in T, in quo diameter AB bifariam dividitur.

Propoſ. XXXX. Theor. XXXV.

Si fuerint duo priſmata GHIKLM, ABCDEF in eadem altudine, quorum unum baſim habeat triangulum GHI, aliud vero habeat parallelogrammum ABCD, fueritque parallelogrammum trianguli duplum.

Dico priſmata GHIKLM, ABCDEF eſſe æqualia.

Conſtructio.



PRoductis priſmatum planis efformentur parallelepipeda AN, GQ, quæ eamdem cum priſmatis habeant altitudinem: ideo que ſint æquè alta.

De-

Demonſtratio.

QUia [per 34. primi.] parallelog. GP eſt duplum trianguli GHI, atque [per hyp.] parallelogrammum AC ſit ejuſdem trianguli GHI duplū: [per axio.6.] erit parallelog. GP = parallelog. AC: quare parallelepipeda GQ, AN ſuper æquales baſes GP, AC, & in eadem altitudine erunt conſtituta: ideoque (per 31. hujus) parallelepip. GQ = parallelepipedo AN; ſed (per 32. hujus.] priſmata GHIKLM, ABCDEF ſunt medietates parallelepipedorum GQ, AN: ergo dicta priſmata erunt æqualia. q. e. d.

Corollarium.

EX hoc Theoremate colligitur, quod illa priſmata, in quibus unum ex planis oppoſitis accipitur tamquam baſis, eſſe ad invicem ut baſes, dummodo eandem habeant altitudinem: quia ſi priſmata æquè alta habeant baſes triangulares, ſunt ut baſes. Hujus veritatis ratio eſt manifeſta, nam completis parallelepipedis, hæc parallelepipeda erunt in eadem altitudine: ideoque (per 32. hujus] inter ſe erunt ut baſes; ſed priſmata cum baſibus triangularibus ſunt medietates parallelepipedorum, atque baſes triangulares medietates baſium parallelepipedorum: quare priſmata æquè alta, & triangulares baſes habentia, inter ſe erunt, ut baſes.

Quod demonſtratum eſt de priſmatis ſupra triangulares baſes conſtitutis, valet etiam circa priſmata polygonas baſes habentia; quia baſes polygonæ in triangula reſolvi poſſunt; ideoque omnia priſmata æqualis altitudinis, haud ſecus ac parallelepipeda inter ſe erunt ut baſes.

Elementi undecimi finis.

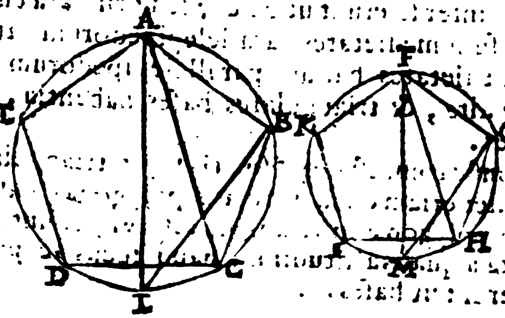
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EUCLIDIS ELEMENTUM DUODECIMUM, ET SOLIDORUM ELEMENTUM SECUNDUM.

Propoſ. I. Theor. I.

Si polygona ſimilia ABCDE, FGHIK in circulis ADC, FIH fuerint deſcripta.

Dico polygonum ABCDE ad polygonum FGHIK habere illam rationem, quam habet quadratum diametri AL, ad quadratum diametri FM.



Conſtructio.

Ductis circulorum diametris AL, FM, duobus ſimilium polygonorum æqualibus angulis ABC, FGH, ſubtendantur rectæ AC, FH, addanturque rectæ BL, GM.

Demonſtratio,

QVoniam (per ſuppoſitam polygonorum ſimilitudinem)
AB : BC :: FG : GH, & ang. ABC = ang. FGH;
erit (*per. 6. ſexti.*) triang. ABC æquiangulum triangulo FGH:
igitur ang. ACB = ang. FHG:

Sed (*per 21. tertij.*)

ang. ACB = ang. ALB: ob eandem cauſam ang. FHG = ang. FMG:
quare (*per axio. 1.*) ang. ALB = ang. FMG.

Cum igitur in triangulis $\left.\begin{matrix} ABL \\ FGM \end{matrix}\right\}$ (*per demonſt.*) ang. ALB = ang. FMG; atque (*per 31. tertij.*) ang. ABL = ang. FGM:
(*per corol. 2. prop. 32. pri.*) ang. BAL = ang. GFM quamobrem
(*per 4. ſexti.*) triangulum ABL ſimile triangulo FGM: ergo

LA : AB :: MF : FG,

Permutando (*per 16. quinti.*)

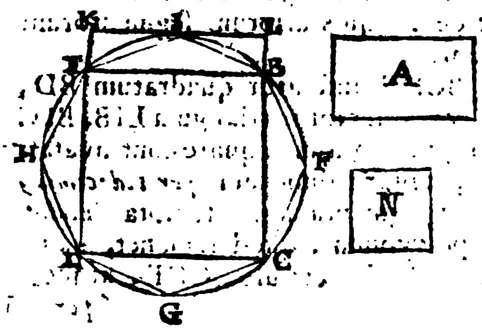
LA : MF :: AB : FG:

Igitur (*per 22. ſexti.*)

ut quad. diametri AL ad quad. diametri FM,
ita poligonum ABCDE, ad polygonum FGHIK: q. e. d.

LEMMA.

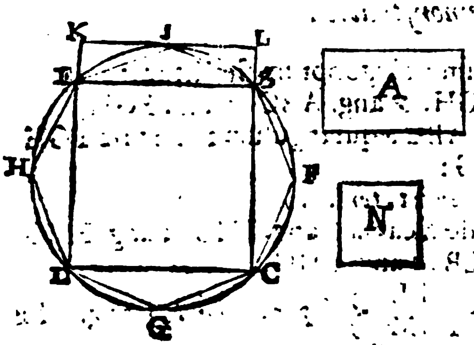
DAta magnitudine A minore circulo BCDE: in dato circulo deſcribere polygonum regulare, quod ſit majus quam A.



Conſtructio.

SIt magnitudo N defectus magnitudinis A, a circulo BCDE, hoc eſt A + N = circulo BCDE,

In dato circulo (*per 6. quarti.*) deſcribatur quadra-



dratum BCDE, quod ſi majus fuerit quam A factum erit q. e. f. Si poſtea idem quadratum BCDE minus fuerit magnitudine A (*per 30. tertii.*] arcus EB, BC &c. bifariam dividantur in I, F, G, H, ductiſque rectis EI, IB &c. conſtitutum eſt octogonum IBF &c. Si dictum octogonum fuerit etiam minus quam A, iterum bifariam dividantur arcus octogoni, ducanturque ſubtenſæ illis arcubus, quo facto compoſitum erit polygonum regulare à ſexdecim lateribus comprehenſum.

Hujuſmodi operatio continuetur quouſque omnia circuli ſegmenta EI, IB &c. ſimul accepta minora ſint quam N.

Dico polygonum EIBF &c. circulo BCDE inſcriptum majus eſſe magnitudine A. Per punctum I, idem de reliquis intelligendum, ducatur tangens KL, producantur rectæ CB, DE in K, & L.

Demonſtratio.

Quia quadratum BD circulo inſcriptum eſt medietas quadrati circa circulum deſcripti; erit quadratum BD majus dimidio circulo BCDE. Pariter quia triangulum EIB (*per 41. pri.*] eſt medietas parallelogrammi BK erit triangulum EIB majus medietate ſegmenti BIE: hoc de reliquis aliorum ſegmentorum triangulis venit intelligendum.

Hoc manente ſi a circulo BCDE auferatur quadratum BD, a ſegmentis vero EIB, BFC &c. auferantur triangula EIB, BFC &c. id, quod aufertur, erit ſupra dimidium; quare continuata tali diviſione, auferendo ſemper ſupra dimidium *per 1. decimi.*) tandem, quæ remanent circuli ſegmenta ſimul ſumpta debent eſſe minora quam N, quare polygonum, quod remanet, majus erit quam A, quia (*per hyp.*] A + N = circulo BCDE, atque

(*per*

(*per axio. 19.*) polygonum, atque ſegmenta ejdem circulo ſunt æqualia, q. e. ſ.

SCHOLIUM.

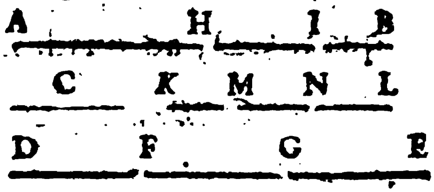
QVia in hoc loco, & in ſequentibus tyrones poſſent dubitare, an ſcilicet, quando ex majore magnitudine detrahitur ſupra dimidium, & ex reſiduo pariter auſertur ſupra dimidium, tandem quod reſtat in majore magnitudine debeat eſſe minus quacumque data quantitate; ideoque ſequens theorema, quod eſt primum Lib. 10. Euclidis, taliter demonſtratur.

Theorema I. Libri X. Elementorum Euclidis.

Si fuerint duæ magnitudines AB major, & C minor.

Dico ſi ex AB auferatur ſupra dimidium AH: pariterque ex reſiduo HB dematur pluſquam dimidium HI: hocque ſemper fiat: quod reſtat IB tandem minus erit quam C.

Conſtructio.



Fiat DE multiplex ipſius C proximè major quam AB. Dividatur DE in partes æquales ipſi C, ſintque DF, FG, GE.

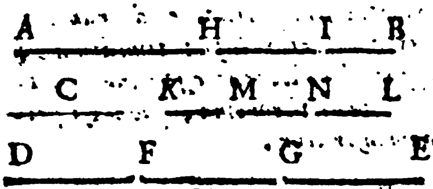
Quot ſunt partes in DE æquales ipſi C, tot fiant diviſiones in AB detrahendo ſupra dimidium ex tota AB, & ex reſiduo HB, ut tot ſint partes numero æquales in AB, quot ſunt in *DE*.

Denuo aſſumatur *K*L tam multiplex ipſius IB, quam multiplex eſt DE ipſius C. Si KL reſolvatur in partes *K*M, MN, NL æquales IB: erunt in *K*L tot partes æquales ipſi IB, quot ſunt in DE æquales ipſi C.

De-

Demonſtratio.

QVia (*per hyp.*) AH major quam HB, etiam dimidium AH majus dimidio HB: ſed (*per conſtructionem.*) dimidium HB majus quam IB: ergo (*per axio. 21.*) dimidium AH, & multo magis AH, major quam IB, ſeu KM.



Cum autem (*per hyp.*) HI major ſit quam IB, & (*per demonſtrata*) AH major quam KM: erit AI major KN: additis æqualibus IB, NL: (*per axio. 4.*) AB major quam KL: ſed (*per conſtruct.*) DE major quam AB: ergo DE major KL.

Quia (*per 15. quinti.*) DE: C:: KL: IB: (& *per demonſtrata*) & DE major quam KL (*per 14. quinti.*) etiam C erit major quam IB. q.e.d.

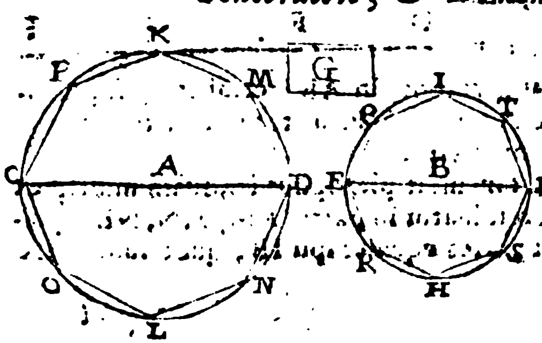
Si ex majore magnitudine auferatur ſola medietas; & ex reſiduo pariter ablata ſit medietas, hoc idem demonſtrabitur.

Propoſ. II. Theor. II.

Si fuerint circuli A, & B, quorum diametri CD, EF.

Dico circulum A ad circulum B eſſe, ut quad. CD, ad quad. EF.

Conſtructio, & Demonſtratio.



SI circulus A ad circulum B non habet illam rationem, quam obtinet quadratum diametri CD, ad quadratum diametri EF; erit verum dicere, quod circulus A ad cir-

circulum B majorem, vel minorem habeat rationem quam quad. CD ad quad. EF: hoc autem non eſſe poſſibile, ita demonſtratur.

Si circulus A ad circulum B majorem habeat, rationem, quam ſit ratio quad. CD,ad quad. EF; erit aſſignabilis magnitudo G, quæ ad circulũ B rationem habeat quam quad. CD ad quad. EF. Hoc poſito (*per 10. quinti.*) G erit minor quam circulus A.

In circulo A (*per Lemma 1.*) deſcribatur polygonum regulare KMD &c. quod ſit majus magnitudine G, cui ſimile deſcribatur in circulo B, nempe polygonum ITF &c.

Quia [*per 1. hujus.*]

Polyg. A, ad polig. B; ut quad. CD, ad quad. EF;

Atque (*per conſtructionem.*)

Magnitudo G, ad circulum B; ut quad. CD, ad quad. EF;

Erit (*per 11. quinti.*)

Polyg. A ad polig. B, ut magnitudo G ad circulum B.

Sed (*per conſtructionem.*)

Polyg. A majus quam magnitudo G:

Ergo (*per 14. quinti.*)

Polyg. B majus quam circulus B: quod fieri nequit, quia polyg. B eſt pars circuli B: quare circulus A ad circulum *B* nequit habere majorem rationem quam ſit ratio quadrati diametri CD ad quadratum diametri EF. q. e. d.

Si verò circulus A ad circulum B minorem obtineat rationem, quam quadratum diametri CD, ad quadratum diametri EF: erit convertendo (*per 26. quinti.*)

Major ratio circuli B ad circulũ A, quam quad. EF, ad quad. CD. Si ergo accipiatur magnitudo G, quæ ad circulum A ſit, ut quad. EF, ad quad. CD: (*per 10. quinti.*) erit G minor quam circulus B.

Hoc ſtante in circulo B [*per Lemma 1.*) deſcribatur polygonum regulare ITF &c. majus magnitudine C. Aliud ſimile polygonum inſcribatur circulo A, & ſit polygonum KMD &c.

Quia [*per conſtructionem.*]

Ut magnitudo G ad circulum A; ita quad. EF ad quad. CD;

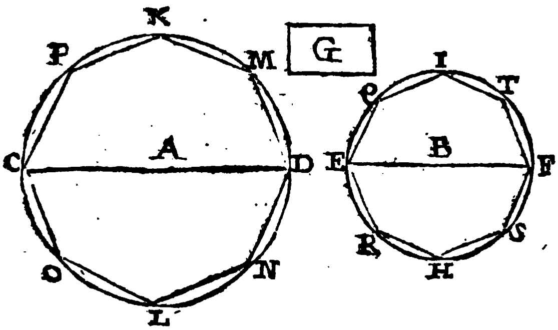
Atque (*per 1. hujus.*] ſit

Ut quad. EF ad quad. CD, ita polyg. ITF &c. ad polyg. KMD &c;

Erit (*per 11. quinti.*]

Magnitudo G ad circulum A, ut polyg. B ad polyg. A.

Cum autem [*per conſtructionem*] G minor ſit quam polyg.



(*per* 14. *quinti*.) etiam circulus A minor polyg. A : quod fieri nequit, quia polyg. eſt pars circuli. Si ergo circulus A ad circulum B rationem habere nequit majorem, vel minorem illa, quam habet quadratum diametri CD ad quadratum diametri EF, verum erit dicere, circulum A ad circulum B illam habere rationem, quam quadratum diametri circuli A ad quadratum diametri circuli B. q. e. 2. d.

Corollarium I.

EX hoc theoremate colligitur, circulos inter ſe duplicatam habere rationem diametrorum; quia diametrorum quadrata inter ſe [*per* 20. *ſexti*] duplicatam obtinent rationem laterum homologorum, hoc eſt eorumdem diametrorum: quare ratio circuli ad circulum erit duplicata diametri ad diametrum.

Corollarium II.

CIrculi inter ſe ſunt in ratione ſimilium polygonorum in circulis deſcriptorum; quia tam circulus ad circulum, quam polygonum ad ſimile polygonum rationem obtinent, quam quadrata diametrorum.

Corollarium III.

SEmicirculi, aliæque circulorum ſimiles partes inter ſe ſunt in ratione duplicata diametrorum, ſemidiametrorum, aliarumque partium aliquotarum ſimilium ipſarum diametrorum; & hoc quia (*per* 15. *quinti*) partes, cum pariter multiplicibus, in eadem ſunt ratione.

Pro-

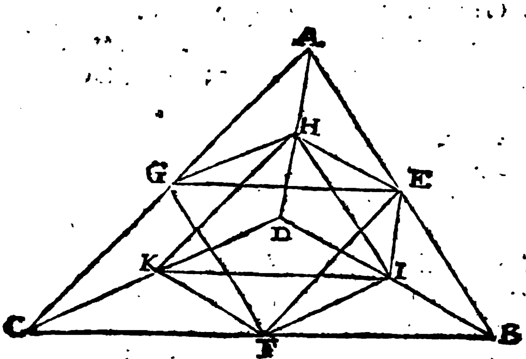
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Propoſ. III. Theor. III.

Si pyramis ABCD baſim habeat triangulum ABC.

Dico talem pyramidem partiri in duas pyramides triangulares baſes habentes inter ſe ſimiles, & æquales, atque ſimiles toti pyramidi: & in duo priſmata pariter inter ſe æqualia medietate totius pyramidis ABCD majora.

Conſtructio.



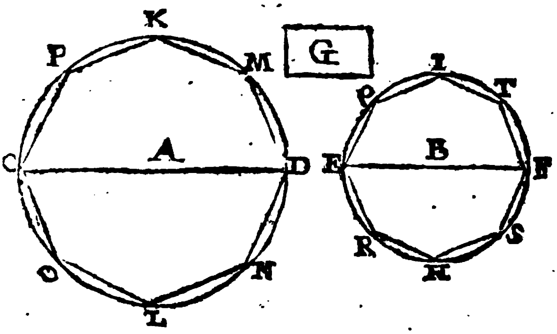
Omnia pyramidis latera AB, AC, AD &c. (*per* 10. *pri.*) biſariam dividantur in punctis E, F, G, H, I, *K*, junganturq; rectæ GE, GF, EF, HI, HK, KI, HG, HE, FK, FI, EI. Hac conſtructione poſita, ſi tota pyramis ABCD intelligatur ſecta a planis triangulorum HIK, EHG, EIF, atque a plano quadrilateri GHIF.

Dico datam pyramidem ABCD fore diviſam in duas pyramides HKID, AGEH inter ſe æquales, & ſimiles, atque ſimiles toti pyramidi ABCD; & in duo priſmata BEFIHK, CFEGHK, inter ſe æqualia, quæ ſimul accepta ſuperant dimidium pyramidis ABCD.

Demonſtratio primæ partis, videlicet quod pyramides AGEH, HKID ſint ſimiles.

Quia (*per conſtruct.*) rectæ DA, DB, ſunt biſariam ſectæ in punctis H, & I: unde (*per* 2. *ſexti.*) HI parallela ipſi AB: eadem ratione.

KI



KI parallela CB.
KH parallela CA.
EG parallela BC.
EF parallela AC.
FG parallela AB.
EH parallela BD.
HG parallela DC.
KF parallela BD.
FI parallela CD.

Vlterius (*per 9. undecimi*) FG parallela HI, atque GH parallela FI: quamobrem (*per def. 40. pri.*) quadrilatera *A*I: HB: DE: BG: HC: CI: & GI: erunt parallelogramma.

Duæ rectæ HE, HG [*per demonſtrata.*] parallelæ ſunt rectis *D*B, DC: quare (*per 10. undecimi*) ang. EHG = ang. BDC: eadem ratione

ang. HEG = ang. DBC, &
ang. HGE = ang. DCB.

Quare triangula GHE, CDB ſunt æquiangula: ideoque (*per 4. ſexti.*) ſimilia. Ob eamdem cauſam

triang. *A*HE ſimile triang. *A*DB.
triang. *A*HG ſimile triang. *A*DC.
triang. *A*EG ſimile triang. *A*BC.

Ergo (*per def. 9. undec.*) pyramis *A*EGH ſimilis pyramidi *A*BCD.

Pariter quia [*per demonſtrata*] HI parallela *A*B, & HK parallela *A*C (*per 10. undec.*] ang. KHI = ang. C*AB*.

ob eamdem cauſam { ang. HIK = ang. *A*BC:
ang. HKI = ang. *ACB*:

Quare (*per 4. ſexti*)

triang. HIK ſimile triang. *ABC*.
triang. D*HI* ſimile triang. *DAB*.
triang. *DI*K ſimile triang. DBC.
triang. DK*H* ſimile triang. D*A*C.

Ergo pyramis HIKD (*per def. 9. undec.* ſimilis pyramidi *ABCD*: quare (*per 21. ſexti*) pyramis *HI*KD ſimilis pyramidi *A*EGH. q. e. d.

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Demonſtratio ſecundæ partis, nempe quod pyramis HIKD = ſit pyramidi AEGH.

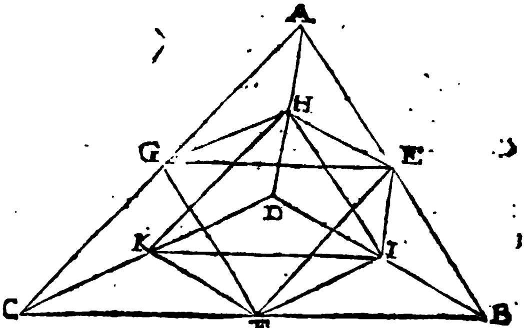
(*per 38. primi.*) triang. AGH = triang. HKD
triang. AEH = triang. HID.
triang. AGE = triang. HKI.
triang. HGE = triang. DKI.

quare (*per def. 10. undec.*) pyramis AEGH = pyramidi HIKD. q. e. 2. d.

Demonſtratio III. partis, videlicet quod ſolida EBFHIK, EFCGHK ſint duo priſmata æqualia.

Quod ſint priſmata, ita demonſtratur.

IN ſolido EBFHIK oppoſita plana hoc eſt triangula BEF, IHK [*per demonſtrata*] ſunt ſimilia; pariter quia (*per 34. pri.*) BE =: IH: BF = IK: EF = HK; dicta triangula BEF, IHK ſimilia erunt, & æqualia. Inſuper quia (*per demonſtrata*) horum triangulorum latera ſunt ſingula ſingulis parallela: (*per 15. undec.*) talium triangulorum plana erunt parallela.



Ulterius quia (*per demonſtrata*) plana EI, BK, EK ſunt paparallelogramma [*per def. 13. undec.*] ſolidum EBFHIK ab his planis comprehenſum; erit priſma.

In ſolido EFCGHK plana oppoſita, nempe triangula EHG, FKC [*per demonſtrata*] ſunt ſimilia: rurſus ſunt æqualia, quia (*per 34. pri.*) EH = FK: HG = KC: EG = FC: inſuper quia (*per demonſtrata*) latera trianguli EHG ſingula ſunt ſingulis lateribus trianguli FKC parallela [*per 15. undec.*] horum triangulorum plana erunt etiam parallela: cum autem reliqua plana

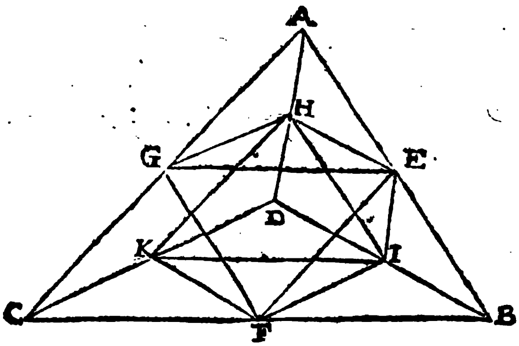
EK, EC, HC hoc ſolidum comprehendentia (*per demonſtrata*) ſint parallelogramma: (*per* 13. *undec:*) ſolidum EFCGHK erit priſma.

Quod poſtea dicta priſmata ſint æqualia demonſtratur, quia ambo ſunt in eadem altitudine, unumque habet baſim triangulum EBF, alterum vero parallelogrammum EC [*per* 41.*pri.*] duplum trianguli EBF: quare [*per* 40. *undecimi.*] hæc priſmata erunt æqualia. q. e. 3. d.

Demonſtratio IV. partis, nempe quod dicta priſmata ſint dimidio totius pyramidis majora.

Priſma EBFHIK (*per axio.9.*) majus eſt pyramide EBFI; ſed pyramis EBFI [*per demonſtrata*] = pyramidi HIKD, ergo etiam priſma EBFHIK maius pyramide HIKD.

Inſuper, quia [*per demonſtrata*) pyramis HIKD = pyramidi AEGH, atque priſma EBFHIK = priſmati EFCGHK: erit etiam priſma EFCGHK majus quam pyramis AEGH: quare duo priſmata erunt duobus pyramidibus majora. Quia vero tota pyramis ABCD in dictas duas pyramides, & in dicta duo priſmata fuit diviſa: duo priſmata erunt dimidio totius pyramidis majora. q. e. 4. d.



LEMMA II.

Si pyramidis *ABCD* omnia latera a baſe *ABC*, ad verticem D ducta fuerint bifariam diviſa in punctis *H*, *I*, *K*.

Dico planum per puncta *H*, *I*, K ductum bifariam dividere altitudinem pyramidis *ABCD*.

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Demonſtratio.

Rectæ HI, HK [*per 2. ſexti*) parallelæ ſunt rectis AB, AC: quare [*per 15. undec.*] planum per *HI*, *HK* ductum erit parallelum plano per AB, AC ducto: ideoque [*per 17. undec.*] omnes rectæ a pyramidis vertice D ad baſim ABC ductæ, erunt proportionaliter ſectæ; ſed DB [*per conſtruct.*] a plano *HI*K eſt bifariam diviſa in I: ergo etiam altitudo pyramidis ab eodem plano *HI*K erit bifariam diviſa. q. e. d.

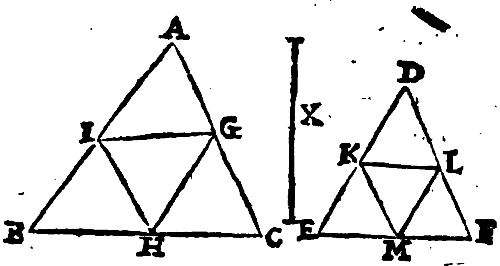
Propoſ. IV. Theor. IV.

Si duæ pyramides eamdem altitudinem, & pro baſibus triangula habētes in duas pyramides æquales, & ſimiles toti pyramidi, & in duo priſmata æqualia dividantur, rurſuſque factæ piramides ſimiliter fuerint diviſæ.

Dico ut baſis ABC ad baſim DEF, ita eſſe omnia priſmata unius pyramidis ad alia priſmata alterius pyramidis numero æqualia.

Conſtructio.

Duo triangula ABC, *DEF* intelligantur baſes duarum pyramidum cum eadem altitudine X. Iſtarum baſium ABC, *DEF* omnia latera bifariam dividantur in punctis *IG HK*LM: ducanturque rectæ IG, GH, *HI*, KL, LM, MK.



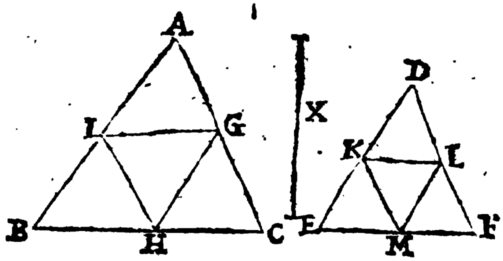
Hoc facto in baſe ABC triangula AIG, IG*H* erunt baſes ſimilium, & æqualium pyramidum; uti pariter triangulum G*H*C,

 atque

atque parallelogrammum GB erunt æqualium priſmatum l [ſuppoſita tamen pyramidum diviſione, ut in præcedente p factum fuit.] Quod de baſe ABC dictum fuit, etiam circa ba DEF eſt intelligendum.

Nam quot diviſiones in pyramide, cujus baſis triangulu ABC, factæ ſunt, tot etiam fiant in pyramide ſupra baſim D conſtituta, ut numero æqualia priſmata in utraque pyramid inveniatur.

Demonſtratio.



QVia (*per conſtruc* BC, *EF* in *H*, & M fuerunt biſariam di viſa: erit

BC : HC : : EF : MF

& (*per corol.4.ſexti.*) Trang. *BAC* ſimile triang. *HGC*; pariterque triang. *EDF* ſimile triang. MLF:

Quare [*per 22. ſexti.*]

Ut triang. ABC ad triang. *HGC*; ita triang. DEF, ad triang. *MLF*: permutando [*per 16. quinti.*)

Ut triang. ABC ad triang. *EDF*; ita triang. *HGC* ad triang. MLF: Sed (*per corol. prop. 40. undec.*)

Ut triang. *HGC* ad triang. MLF; ita priſma ſuper baſim *HGC* ad priſma ſuper baſim MLF.

Cum autem priſmata ſuper baſes *HGC*, MLF [*per 3. bujus.*) æqualia ſint priſmatis ſuper baſes parallelogrammas *BG*, *EL* conſtitutis:

erunt [*per 15. quinti.*)

Duo priſmata ſuper baſes *HGC*, & *BG* conſtituta ad duo priſmata ſuper baſes MLF, & *EL* efformata;

Ut baſis *HGC* ad baſim MLF: quia [*per lemma 2. bujus*) hæc priſmata ſunt in eadem altitudine.

Quia vero [*per demonſtrata.*]

Ut baſis *HGC* ad baſim MLF; ita baſis *BAC*, ad baſim *EDF*, erit

Ut

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erit (*per 11. quinti.*)

Ut baſis BAC ad baſim EDF; ita duo priſmata ſupra baſes HGC, & BG, ad duo priſmata ſupra baſes MLF, & EL.

Conſimili modo ſi intelligamus pyramides ejuſdem altitudinis ſupra baſes triangulares AIG, DKL in duas pyramides, & in duo priſmata, uti ſupra, diviſas; eadem ratione demonſtrabitur, ut baſis AIG ad baſim DKL; ita priſmata unius pyramidis eſſe ad priſmata alterius pyramidis: Quod pariter eſt dicendum de priſmatis factis in pyramidibus ſupra baſes IHG, KML conſtitutis, nempe quod baſes ſint ad baſes, ut priſmata ad priſmata: hocque ſemper verificabitur quouſque pyramidum diviſio fuerit continvata; ſed omnes baſes, ad omnes baſes ſunt ut baſis AIG ad baſim DKL;

Ut autem baſis AIG ad baſim DKL; ita baſis ABC, ad baſim DEF:

Ergo (*per 11. quinti.*)

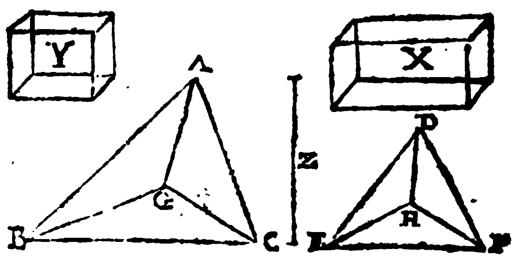
Ut baſis ABC ad baſim DEF; ita omnia priſmata unius pyramidis, ad omnia priſmata alterius pyramidis. q. e. d.

Propoſ. V. Theor. V.

Si Pyramides ABCG, DEFH ſupra triangulares baſes ABC, DEF, & in eadem altitudine Z fuerint conſtitutæ.

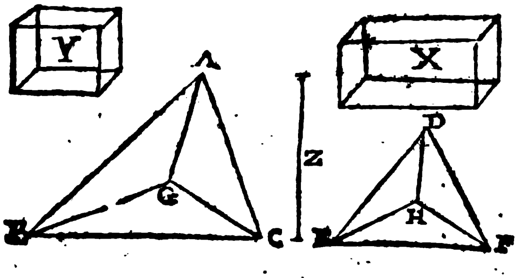
Dico pyramides eſſe ad inuicem, ut baſes.

Conſtructio, & Demonſtratio.



SI pyramis ABCG in altitudine Z non eſt ad pyramidem DEFH ut baſis ABC ad baſim DEF; neceſſariò ſequitur, quod pyramis ABCG ad pyramidē DEFH

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majorem, vel minorem habeat rationé, quam baſis ABC, ad baſim DEF: quod autem hoc eſſe nequeat ita demonſtratur.

Si pyramis ABCG ad pyramidé DEFH majorem habeat rationem quam baſis ABC ad baſim DEF; intelligatur magnitudo X, quæ ad pyramidem DEFH habeat rationem baſis ABC, ad baſim DEF: hoc ſtante (*per* 10. *quinti.*] magnitudo X erit minor quam pyramis ABCG: ſit igitur X minor pyramide ABCG quantitate Y.

Hoc poſito pyramis ABCG intelligatur diviſa quemadmodum fuit factum in 3. prop. hujus,] continvataque talis diviſio in illis pyramidibus, quæ per hanc diviſionem reſultant, quouſque omnes pyramides ablatis priſmatis, ſint minores quam Y.

Jiſdem numero diviſionibus, quibus partita fuit pyramis ABCG, pariter dividatur pyramis DEFH: eruntque tot pryſmata in pyramide ABCG, quot ſunt in pyramipe DEFH.

His taliter peractis (*per* 4. *bujus*] erunt.

Omnia priſmata pyramidis ABCG, ad omnia priſmata pyramidis DEFH; ut baſis ABC, ad baſim DEF;

Sed (*per hyp.*]

baſis ABC ad baſim DEF eſt ut X ad pyramidem DEFH:

Quare (*per* 11. *quinti*)

Ut magnitudo X ad pyramidem DEFH;

Ita omnia priſmata pyramidis ABCD, ad omnia priſmata pyramidis GEFH.

Cum autem [*per conſtruct.*] magnitudo X minor ſit quam omnia priſmata pyramidis ABCG: [*per* 14. *quinti*) erit etiam pyramis DEFH minor, quam omnia priſmata ejuſdem pyramidis DEFH: quod repugnat, quia pyramis eſt totum, & priſmata ſunt pars pyramidis: ergo pyramis ad pyramidem nequit habere majorem rationem, quam babeat baſis ad baſim DEF. q. e. i. d.

Quod poſtea pyramis ABCG ad pyramidem DEFH nequeat habere minorem rationem, quam ſit ratio baſis ABC ad baſim DEF, ita euincitur. Si

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Si ratio pyramidis ABCG ad pyramidem DEFH, minor eſt, quam ratio baſis ABC, ad baſim DEF.

Convertendo [*per 26. quinti*]

Pyramis DEFH ad pyramidem ABCG majorem habebit rationem, quam baſis DEF ad baſim ABC.

Aſſumatur magnitudo X quæ ſit ad pyramidem ABCG; Ut baſis DEF ad baſim ABC: eritque (*per 10. quinti*) magnitudo X minor pyramide DEFH.

Ut in prima parte factum fuit diuidatur pyramis DEFH in tot priſmata, quæ omnia ſimul ſumpta majora ſint, quam X. ſimilis diviſio pariter fiat in pyramide ABCG.

Hoc facto [*per 4. hujus*.)

Ut omnia priſmata pyramidis DEFH, ad omnia priſmata pyramidis ABCG numero æqualia;

Ita baſis DEF, ab baſim ABC:

Sed [*per conſtructionem*)

Ut baſis DEF, ad baſim ABC; ita magnitudo X ad pyramidem ABCG:

Ergo (*per 11. quinti*]

Ut magnitudo X ad pyramidem ABCG;

Ita omnia priſmata pyramidis DEFH, ad omnia priſmata numero æqualia pyramidis ABCG.

Cum autem magnitudo X [*per conſtruct.*) minor ſit quam priſmata pyramidis DEFH: [*per 14. quinti.*] erit etiam piramis ABCG minor omnibus priſmatis ejuſdem pyramidis ABCG: quod eſſe non poteſt, quia priſmata ſunt partes pyramidis: ergo pyramis ABCG ad pyramidem DEFH nequit habere minorem rationem, quam baſis ABC, ad baſim DEF. q. e. 2. d.

Cum igitur (*per demonſtrata*) pyramis ad pyramidem non poſſit habere majorem, vel minorem rationem, quam baſis ad baſim; ſequitur quod pyramides triangulares baſes habentes in eadem altitudine, inter ſe ſint ut baſes. q. e. d.

Corollarium.

ÆQuales Pyramides in eadem altitudine, æquales habent baſes, vel eamdem baſim, & viciſſim:

Æquales pyramides ſuper æquales baſes, vel ſuper eamdem

 baſim

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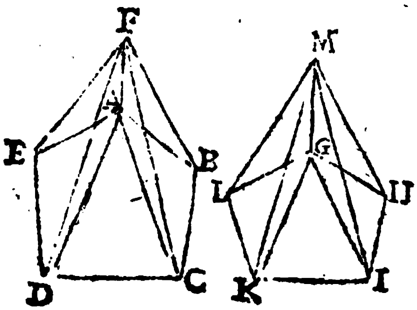
baſim conſtitutæ, æquales habent altitudines, vel eamdem altitudinem.

Propoſ. VI. Theor. VI.

Si pyramides ABCDEF, GHIKLM eamdem habuerint altitudinem, atque latera numero æqualia.

Dico tales pyramides inter ſe habere rationem baſis ABCDE, ad baſim GHIKL.

Conſtructio.



Pyramidum baſes reſolvantur in triangula AED, DAC &c. Super dicta triangula intelligatur efformatæ pyramides in eadem altitudine: quo facto datæ pyramides ex tribus pyramidibus triangulares baſes habentibus, & in eadem altitudine erunt compoſitæ.

Demonſtratio.

Quia duæ pyramides ABCF, ACDF ſunt æque altæ; (*per* 5. *hujus*.)
Ut baſis ABC, ad baſim CAD; ita pyramis ABCF ad pyramidem CADF.

Eadem de cauſa,
Ut baſis CAD ad baſim DAE; ita pyramis CADF ad pyramidem DAEF.

Igitur componendo (*per* 18. quinti.)
Ut baſis ABCDE ad baſim ADE; ita pyramis ACDEF ad pyramidem ADEF.

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Simili ratione demonſtrabitur

Ut baſis GHIKL ad baſim KGL; ita pyramis GHIKLM ad pyramidem KGLM,

Convertendo (*per corol. prop. 4. quinti.*)

Ut baſis GKL ad baſim GHIKL; ita pyramis KGLM ad pyramidem GHIKLM;

Sed (*per 5. hujus.*)

Ut baſis ADE ad baſim KGL; ita piramis ADEF ad pyramidem KGLM: quamobrem baſes erunt inſiſtentibus ſuis pyramidibus proportionales, hoc eſt

baſis ABCDE ad pyramidem ABCDEF.
baſis ADE — ad pyramidem ADEF.
baſis GKL — ad pyramidem GKLM.
baſis GHIKL ad pyramidem GHIKLM:

Ergo ex æquo (*per 22. quinti.*)

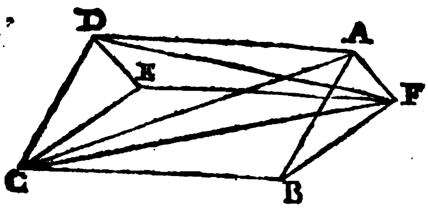
Ut baſis ABCDE ad baſim GHIKL;
Ita pyramis ABCDEF ad pyramidem GHIKLM. q. e. d.

Propoſ. VII. Theor. VII.

Si priſma ABCDEF baſes AFB, DEC habeat triangulares.

Dico hoc priſma partiri in tres pyramides æquales triangulares baſes habentes.

Conſtructio.

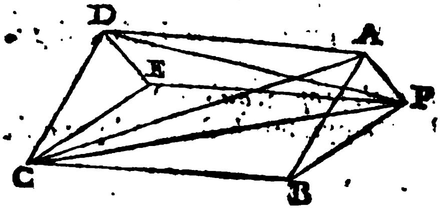


Trium parallelogrammorum AE, BE, BD ducantur diametri DF, CF, AC.

Demonſtratio.

Triangula ABC, ADC (*per 34. pri.*) ſunt æqualia: Quia verò pyramides ABCF, ADCF ſunt in eadem altitudine;

(*per*



[per 5. hujus.]
Ut baſis ABC ad baſim ADC; ita pyramis ABCF ad pyramidem ADCF: ſed baſes, [per 34. pri.] ſunt æquales: ergo etiam
pyramis ABCF = pyramidi ADCF.

eadem ratione
pyramis ADFC = pyramidi DEFC;
Cum ergo pyramis ADCF eadem ſit ac pyramis ADFC; tres pyramides ABCF, ADCF, DEFC erunt æquales. q. e. d.

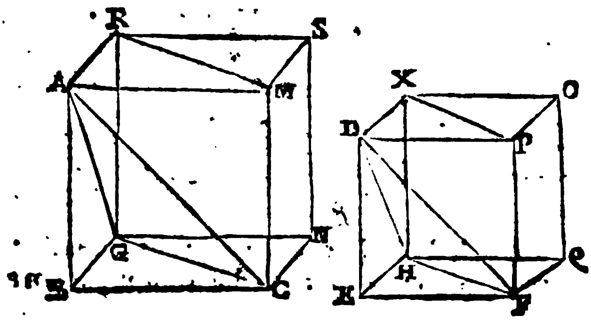
Corrularium.

Quæcumque pyramis eſt tertia pars priſmatis eamdem baſim, & altitudinem habentis, & viciſſim.

Quodcumque priſma eſt triplum pyramidis eamdem baſim, & altitudinem habentis.

Propoſ. VIII. Theor. VIII.

Si fuerint ſimiles pyramides ABCG, DEFH. Dico hujuſmodi pyramides inter ſe habere rationem triplicatam homologorum laterum BC, EF.



Con-

Conſtructio.

Quando pyramides baſes habent triangulares.

PRoducantur plana pyramides comprehendentia, compleanturque parallelogramma *BN*, BM : BR : EQ : EP : EX, necnon etiam parallelepipeda *BS*, EO.

Notentur rectæ RM : GC : XP : HF, ducanturque plana RC, XF.

Demonſtratio.

QVia (*per hyp.*) triangulum A*BC* eſt ſimile triangulo DEF, [*per def. 1. ſexti.*)
ang. *ABC* = ang. DEF : inſuperque *AB* : *BC* : : *DE* : *EF* :
Quare parallelog. *BM* ſimile erit parallelog. EP.
Eadem ratione demonſtrabitur
parallelog. *BR* ſimile parallelog. *EX*, &
parallelog. *B*N ſimile parallelog. EQ.

Quia vero (*per 24. undec.*] in parallelepipedo tria plana ſunt ſingula ſingulis ſimilia tribus planis oppoſitis : inde ſequitur, ſex plana parallelepipedi *BS* ſimilia eſſe ſex planis parallelepipedi EO :
Quare [*per def. 9. undec.*) erit
parallelepipedum *BS* ſimile parallelepipedo EO.
ergo (*per 33. undec.*)
dicta parallelepipeda inter ſe erunt in triplicata ratione laterum homologorum *BC*, *EF*.

Cum autem plana RC, XF [*per 28. undec.*) biſariam dividant parallelepipeda *BS*, EO :
igitur [*per 15. quinti.*]
parallelepipedorum medietates, hoc eſt priſmata ARMCG*B*, DXPFHE, inter ſe erunt in triplicata ratione laterum homologorum *BC*, *EF* :
Sed [*per corol. prop. 7. hujus.*)
dicta priſmata ſunt pyramidum *BGCA*, *EHFD* tripla :
(*per 15. quinti.*)
etiam pyramides *BGCA*, *EHFD* inter ſe erunt in triplicata ratione laterum homologorum *BC*, *EF*. q. e. d.

Con-

Conſtructio

Quando pyramides ſimiles baſes habent polygonas.

SImilium pyramidum baſes polygonæ reſolvantur in triangula numero æqualia, atque datæ pyramides dividantur in pyramides numero æquales baſibus correſpondentes, & triangulares baſes habentes.

Demonſtratio.

QVia (per primam partem hujus prop. ſimiles pyramides triangulares baſes habentes, ſunt in triplicata ratione laterum homologorum: quemadmodum ſingulę piramides ſunt ſingulis pyramidibus in triplicata ratione laterum homologorum; ita (*per* 12. *quinti*) erunt omnes pyramides ad omnes pyramides in triplicata ratione eorumdem laterum homologorum. q. e. d.

Propoſ. IX. Theor. IX.

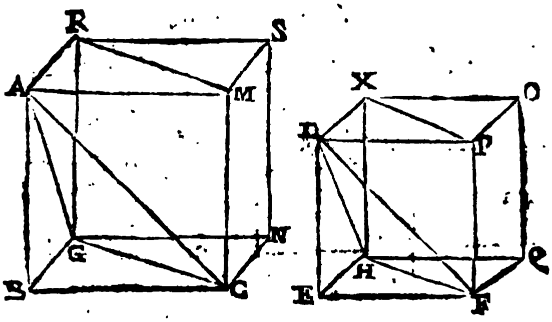
Si fuerint æquales pyramides BGCA, EHFD.
Dico talium pyramidum baſes BGC, EHF, & altitudines BA, ED eſſe reciprocas, & viciſſim.
Si pyramidum baſes, & altitudines fuerint reciprocæ.
Dico pyramides eſſe æquales.

Conſtructio

Quando pyramides ſunt triangulares.

QVemadmodum in ſuperiore propoſitione producantur pyramidum plana, fiantque parallelogramma BM: BR: BN: EP: EX. EQ; atque compleantur parallelepipeda BS, EO: adanturque rectæ RM, XP.

De-



Demonſtratio primæ partis.

QVia (*per hyp.*) pyramis *B*GCA = eſt pyramidi EHFD: erit (*per corol. 7. hujus.*]
priſma *B*GCARM = priſmati EHFDXP,
Sunt autem (*per* 28. undec.)
parallelepipeda *B*S, EO priſmatum dupla: ergo
parallelepipedum *B*S = parallelepipedo EO:
Quare (*per* 34. undec.]
Ut baſis *B*N ad baſim EQ; ita altitudo ED ad altitudinem *B*A;
Sed (*per* 15. *quinti.*
Ut *B*N, ad EQ; ita triang. *B*GC ad triang. EHF:
Igitur [*per* 11. *quinti.*]
Ut triangulum *B*GC baſis pyramidis *B*GCA, ad triang. EHF baſis pyramidis EHFD;
ita altitudo ED, ad altitudinem *B*A: q. e. d.

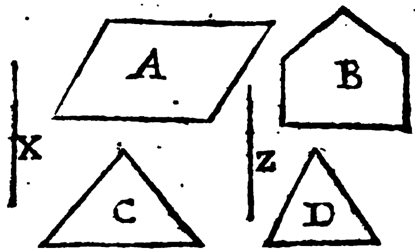
Demonſtratio ſecundæ partis.

QVia [*per hyp.*] baſis *B*GC, ad ſiaſim *E*HF, ut altitudo *E*D ad altitudinem *B*A:
erit (*per* 15. *quinti.*)
parallelog.*B*N ad parallelog.*E*Q ut altitudo *E*D,ad altitudinem *B*A: Eritque (*per* 34. *undec.*)
parallelepipedum *B*S = parallelepipedo *E*O.
Cum autem (*per* 28. *undec.*] priſmata *B*GCARM., *E*HFDXP ſint medietates parallelepipedorum *B*S, *E*O: pariterq;[*per* 7.*hujus*) pyramides *B*GCA, *E*HFD ſint priſmatum tertiæ partes: erit pyramis *B*GCA = pyramidi *E*HFD. q.e.d. Cu-

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Conſtructio.

Quando æquales pyramides habent baſes polygonas.



ÆQualium pyramidum polygonis baſibus A, & *B* (*per* 45. *pri.*) æqualia conſtituantur parellelogrã-ma: pariter dictis parallelogrammis (*per* 41. *pri.*]æqualia ponantur triangula C, & D. Supra C in altitudine pyramidis A, hoc eſt in altudine X, & ſupra D in altitudine pyramidis ſupra B hoc eſt Z intelligantur conſtitutæ duæ pyramides triangulares, quarum illa, quæ baſim habet C in eadem erit altitudine cum illa, quæ baſim habet A, atque pyramis ſupra D ejuſdem altitudinis erit cum pyramide ſupra *B*.

Demonſtratio.

PYramis ſupra baſim C, conſtituta [*per* 6. *hujus.*] æqualis eſt pyramidi ſupra A efformatæ, atq; pyramis ſupra D æqualis pyramidi ſupra B.

Cum autem (*per hyp.*) pyramides ſupra A, & *B* ſint æquales, etiam pyramides ſupra C, & D æquales erunt.

Sed (per primam partem hujus propoſitionis.)

Ut baſis C, ad baſim D; ita reciprocè altitudo Z ad altitudinẽ X; ergo etiam ut baſis A, ad baſim B; ita reciprocè altitudo pyramidis B hoc eſt Z, ad altitudinem pyramidis A hoc eſt X. q. e. d.

Si vero in pyramidibus cum baſibus polygonis A, & B fuerit reciprocè, ut baſis ad baſim, ita altitudo ad altitudinem.

Dico pyramidem ſupra A = pyramidi ſupra B.

Demonſtratio.

Manente conſtructione ſuperius facta

ut baſis C ad baſim D; ita baſis A ad baſim B: atq; pyramis ſupra C = pyramidi ſupra A, & pyramis ſupra D = pyramidi ſupra B.

Quia

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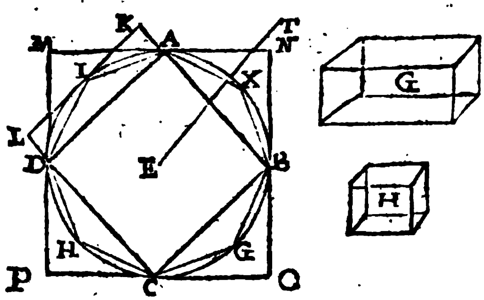
Quia verò [per primam partem hujus propoſitionis.] ſi baſis C ad baſim D; ita reciprocè altitudo Z ad altitudinem X. ∴ Piramis ſupra C eſt æqualis piramidi ſupra D, ergo ſi A ad B, ut Z ad X, erit piramis ſuper A æqualis piramidi ſuper B. q. e. d.

LEMMA III.

Dato cylindro, cujus baſis circulus ABCD, & altitudo EF; atque data magnitudine G cylindro minore quantitate H.

In cylindro deſcribi poteſt priſma data magnitudine G majus.

Conſtructio.



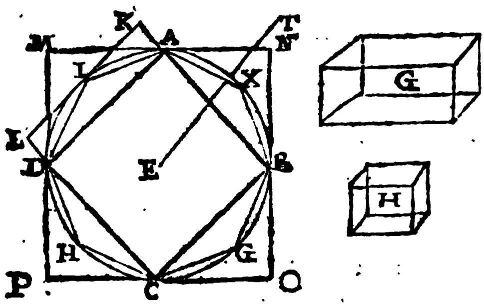
IN dato circulo [*per 6. quarti.*] deſcribatur quadratum ABOD, & circa circulum notetur quad. MNOP. Arcus AB, BC, CD, DA (*per* 30. *tertii.*] biſariam dividantur in X, G, H, I, junganturque rectæ AX, XB, BG &c. quo facto in dato circulo deſcriptum erit octogonum AXBG &c. Ducatur L*K*, quæ circulum tangat in I, producanturque CD, BA uſque ad L, & *K*.

Tandem ſupra circulum ABCD tamquam baſim in altitudine EF debemus concipere cylindrum: pariterque ſupra quadrata MO, & AC debemus intelligere deſcripta priſmata in altitudine EF.

De-

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Demonſtratio.



QVia quadratum MO majus eſt circulo ABCD, & eſt duplum quadrati AC: erit quad. AC majus quam medietas circuli ABCD.

Sed (*per corol. prop. 40. undecimi.*) priſmata æquealta inter ſe ſunt, ut baſes: ergo priſma ſupra quadratum MO duplum priſmatis ſupra quadratum AC.

Cum autem priſma ſupra quad. MO(*per axio.9.*) majus ſit cylindro ſupra circulum ABCD conſtituto: ſequitur quod priſma ſupra quad. AC ſit majus medietate cylindri ſupra circulum ABCD.

Pariter quia [*per 41. primi.*] parallelogrammum KD eſt duplum trianguli AID, etiam priſma ſupra parallelog. KD [*per corol. prop. 40. undecimi.*] erit duplum priſmatis ſupra triangulum AID; ſed (*per axio. 9.*) priſma ſuper parallelog. KD majus eſt ſegmento cylindri ſupra ſegmentum circuli AID conſtituto: ergo priſma ſupra triangulum AID majus medietate ſegmenti cylindri ſupra ſegmentum circuli AID efformato.

Hoc demonſtrato ſi a cylindro, cujus baſis circulus ABCD auferatur priſma cujus baſis quadratum ABCD: pariterque ſi a ſegmentis cylindri ſupra circuli ſegmenta AID, DHC &c. conſtitutis demantur priſmata ſuper triangula AID, DHC &c. efformata, ablatum (*per demonſtrata.*] ſuperabit reſidui dimidium: quare continvata tali diviſione (*per ſcol. lemmatis 1. hujus.*) tamdem remanebunt cylindri ſegmenta, quorum baſes ſunt ſegmenta circuli AI, ID, DH &c. minora magnitudine H; ſed (*per hyp.*] H, & G æquant cylindrum ſupra circulum ABCD: ergo priſma ſupra polygonum AKBG &c. ſuperabit magnitudinem G. q. e. f.

Co-

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Coorollarium.

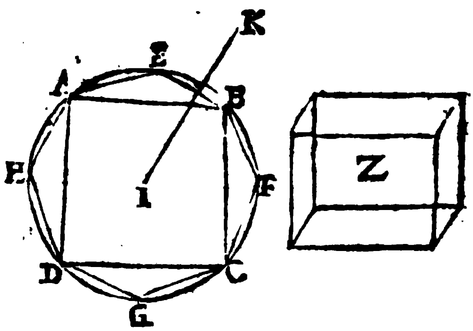
QVod de cylindro dictum eſt, etiam cono poterit applicari; ſiquidem & in cono deſcribi poterit pyramis major quacumque data magnitudine cono minore.

Propoſ. X. Theor. X.

Si conus, & cylindrus fuerint ſuper eadem baſi ABCD, & in eadem altitudine IK. Dico conum eſſe tertiam cylindri partem.

Conſtructio, & Demonſtratio.

SI conus non eſt tertia pars cylindri, neque cylindrus erit coni triplus, at maior, vel minor triplo coni.



Sit, ſi fieri poteſt, cylindrus major triplo coni. Accipiatur magnitudo Z, quæ ſit coni tripla; eritq; magnitudo Z [*per 10. quinti*] cylindro minor.

In cylindro (*per lemma antecedens*] in altitudine IK deſcribatur priſma, quod ſit majus quam Z, ſitque tale priſma conſtitutum ſupra baſim polygonam AEBF &c. Super hanc eamdem baſim polygonam AEBF &c. intelligatur etiam deſcripta pyramis in altitudine IK, quæ minor erit quam conus ſupra circulum ABCD deſcriptus. His poſitis ſatis conſtat circulum ABCD baſim eſſe cylindri, & coni; polygonum vero AEBF &c. eſſe baſim priſmatis, atque pyramidis in altitudine IK.

Priſma, cujus baſis polygonum AEBF &c. (*per corol. prop. 7. hujus*) eſt triplum pyramidis eamdem baſim, & altitudinem cum pryſmate habentis; cum autem [*per hyp.*] & magnitudo Z ſit

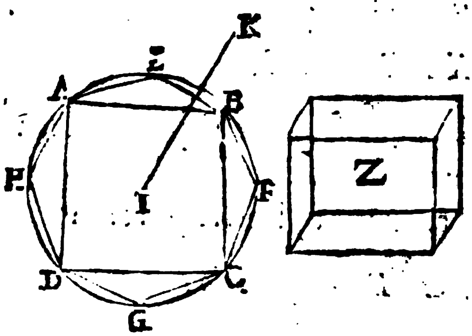
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tripla coni ſupra circulum ABCD deſcripti.

Erit (*per* 11. *quinti.*)

Ut priſma ad pyramidem; ita magnitudo Z ad conum.

Sed [*per conſtructionem.*) priſma majus eſt quam magnitudo Z: ergo (*per* 14. *quin.*] etiam pyramis major erit cono: quod repugnat, quia pyramis eſt pars coni: quare cylindrus non erit major triplo coni.



Si poſtea cylindrus ponatur minor triplo coni, convertendo [*per* 26. *quinti*) conus major erit ſubtriplo cylindri.

Accipiatur magnitudo Z, quæ ſit ſubtripla cylindri, & hæc magnitudo Z (*per* 10. *quinti*) erit minor cono.

In cylindro (*per lemma* 3.] deſcribatur pyramis major quam Z, habeatque hæc pyramis baſim poligonam AEBF &c. in altidine IK.

Quia pyramis ſupra baſim AEBF &c. (*per corol. prop.* 7. *hujus.*] eſt ſubtripla priſmatis ſuper eadem baſi AEBF &c. & in eadem altitudine IK conſtituti: pariterque magnitudo Z (*per hyp.*) ſubtripla cylindri: erit

Ut pyramis ad priſma; ita Z ad cylindrum.

Quia vero (*per conſtructionem*] pyramis ſuperat Z: (*per* 14. *quinti.*] etiam priſma cylindrum ſuperabit: quod cum fieri nequeat, quia priſma eſt pars cylindri: ſequitur igitur cylindrum ad conum ſuper eadem baſi, & in eadem altitudine conſtitutum non poſſe habere rationem majorem, vel minorem tripla: ergo cylindrus erit coni triplus. q. e. d.

Pro-

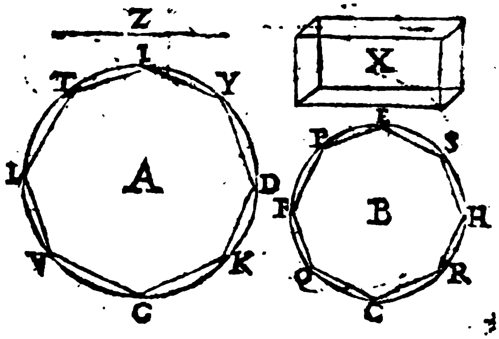
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Propoſ. XI. Theor. XI.

Si coni, vel cylindri ſupra baſes, ſeu circulos A, & B in altitudine Z fuerint conſtituti.

Dico inter ſe habere rationem baſis A ad baſim B,

Conſtructio, & Demonſtratio.



SI cylindrus ſupra circulum A ad cylindrum ſupra circulum B non eſt, ut circulus A ad circulum B: neceſſario cylindrus ſupra A ad cylindrum ſupra B majorem, vel minorem, habebit rationem, quam ſit ratio circuli A ad circulum B. Quod autem neutrum eſſe poſſit taliter evincitur.

Quo ad primum. Si cylindrus ſupra circulum A ad cylindrum ſupra circulum B majorem habeat rationem, quam circulus A ad circulum B; aſſumatur magnitudo X, quæ ad cylindrum ſupra circulum B, habeat rationem circuli A ad circulum B: Hoc facto (*per* 10. *quinti*) magnitudo X minor erit cylindro ſupra circulum A deſcripto.

In cylindro, cujus baſis eſt circulus A (*per lemma* 3.] in altitudine Z inſcribatur priſma, quod ſit majus, quam X, habeatque tale priſma baſim poligonam ITLV &c.

In circulo EFGH inſcribatur polygonum EPFQ &c. ſimile polygono ITLV &c.: quod quidem facillimè fiet, ſi in tot partes æquales dividatur circumferentia EFGH, in quot æquales partes diviſa fuit circumferentia ILCD.

Supra polygonum EPFQ &c. intelligatur conſtitutum priſma in altitudine Z.

His manentibus, quia (*per conſtructionem*) magnitudo X ad cylindrum ſupra circulum B eſt, ut circulus A ad circulum B:

atque (*per corol. 2. propoſ. 2. hujus.*)

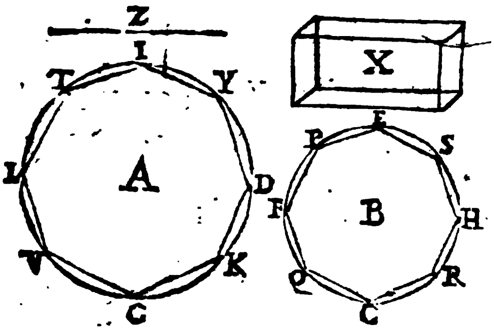
Ut circulus A ad circulum B; ita poly. ITLV &c. ad poly. EPFQ.

Pariter [*per corol. prop. 40. undec.*]

Ut polyg. ITLV &c. ad poly. EPFQ &c. ita priſma ſupra poly. ITLV &c. ad priſma ſupra poly. EPFQ &c.

Erit [*per* 11. quinti.]

Ut magnitudo X ad cylindrum ſupra circulum B;
Ita priſma ſupra poly. ITLV &c. ad priſma ſupra poly. EPFQ &c.



Quia verò (*per conſtructionem.*) magnitudo X minor eſt quam priſma ſupra polyg. ITLV &c. (*per* 14. *quinti.*) & cylindrus ſupra circulum B minor erit quam priſma ſupra poly. EPFQ &c. conſtitutum: quod repugnat, quia priſma eſt pars cylindri: ergo cylindrus ad cylindrum nequit majorem habere rationem, quam baſis ad baſim.

Quo ad ſecundum. Si cylindrus ſupra circulum A ad cylindrum ſupra circulum B minorem habuerit rationem, quam circulus A ad circulum B.

Convertendo [*per 26. quinti.*]

Major erit ratio cylindri ſupra circulum B ad cylindrum ſupra circulum A, quam circuli B ad circulum A.

Sit magnitudo X ad cylindrum ſupra circulum A;
uti circulus B ad circulum A.

Erit [*per 19. quinti.*)

Magnitudo X minor cylindro ſupra circulum B conſtituto.

Quemadmodum factum fuit in prima parte hujus theorematis, in cylindro conſtituto ſupra circulum B, inſcribatur priſma, majus quam X, ejuſque baſis ſit polygonum EPFQ &c.: pariterque in circulo A inſcribatur aliud polygonum ITLV &c. ſimile poly-

gono E P F Q &c. His poſitis.

Eſt [*per conſtructionem*] magnitudo X ad cylindrum ſupra circulum A;

Ut circulus B ad circulum A;

Sed (*per corol. 2. prop. 2. hujus*.)

Ut circulus B ad circulum A, ita poly. EPFQ &c. ad poly. ITLV &c.

Pariter (*per corol. prop. 10. undec.*)

Ut poly. EPFQ &c. ad ſimile poly. ITLV &c.;

Ita priſma ſupra poly. EPFQ. ad priſma ſupra poly. ITLV &c.

Ergo [*per* 11. *quinti*.)

Ut magnitudo X ad cylindrum ſupra circulum A;

Ita priſma ſupra Poly. EPFQ &c. ad priſma ſupra poly. ITLV &c.

Quia vero [*per conſtructionem*] magnitudo X minor eſt quam priſma ſupra poly. E PFQ &c. conſtitutum:

Erit (*per* 14. *quinti*.)

Cylindrus ſupra circulum A minor priſmate ſupra polygonum ITLV &c. conſtituto: quod fieri nequit, quia priſma eſt pars cylindri: ergo cylindrus ad cylindrum non poteſt habere rationem minorem, quam ſit ratio baſis ad baſim. Cum autem [*per demonſtrata*) neque majorem habere poſſit; erit cylindrus ad cylindrum in eadem altitudine, ut baſis ad baſim. q. e. d.

Quod pariter coni ejuſdem altitudinis ad invicem ſint, ut baſes, demonſtratur, quia [*per* 10. *hujus*) coni ſunt tertiæ partes cylindrorum: quare (*per* 15. *quinti*) ut cylindrus ad cylindrum ita conus ad conum: ſed [*per demonſt.*) cylindrus ad cylindrum eſt, ut baſis ad baſim: ergo [*per* 11. *quinti*.) ut conus ad conum, ita baſis ad baſim. q. e d.

Corollarium.

Coni, & cylindri ſuper æquales baſes, & in eadem altitudine, ſunt æquales, quia ex viſis ſunt in ratione baſium.

Æquales coni, & cylindri ſupra æquales baſes conſtituti, ſunt in eadem altitudine.

Si coni, & cylindri ſint æquales, & in eadem altitudine, baſes habent æquales.

Si coni, & cylindri baſes habeant æquales inter ſe ſunt, ut altitudines.

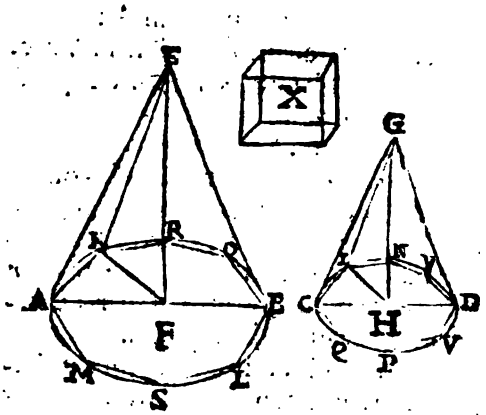
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Propoſ. XII. Theor. XII.

Si coni, & cylindri ſimiles AEB, CGD ſupra baſes AB, CD fuerint deſcipti.

Dico inter ſe habere rationem triplicatam diametrorum baſium AB, CD.

Conſtructio, & Demonſtratio.



SI conus AEB ad conum CGD non habet triplicatam rationem diametri AB ad diametrum CD, ponamus primo habere majorem.

Accipiatur magnitudo X, quæ ad conum CGD rationem habeat triplicatam diametri AB ad diametrum CD: quo facto (*per* 10. *quinti.*) X erit minor cono AEB.

In cono AEB, & in altitudine EF (*per Lemma* 3.) inſcribatur pyramis major quam X ſupra baſim polygonam AKR &c.

In circulo CD inſcribatur polygonum CIN &c. ſimile polygono AKR &c. Supra hoc polygonum in altitudine GH inſcribatur pyramis, quam

Dico ſimilem pyramidi deſcriptæ in cono AEB.

Ducantur rectæ EK, HI, atque KE, IG,

Quia (*per hyp.*) ſimiles ſunt coni AEB, CGD; erit (*per def.* 20. *undecimi.*)

AB: CD:: FE: HG.

Pariterque (*per* 15. *quinti.*)

AF: CH:: FE: HG.

Permutando (*per* 16. *quinti.*)

AF: FE:: CH: HG.

In

In triangulis $\left.\begin{matrix}AFE\\CHG\end{matrix}\right\}$ [*per ſchol. def.* 19. *undec.*) anguli AFE, CHG ſunt æquales, quia recti, & [*per demonſtrata.*] circa æquales angulos latera ſunt proportionalia:

Quare (*per* 6. *ſexti.*)

Triangulum AFE ſimile triangulo CHG: quare

EA: AF:: GC: CH.

In triangulis $\left.\begin{matrix}AFK\\CHI\end{matrix}\right\}$ anguli $\left.\begin{matrix}AFK\\CHI\end{matrix}\right\}$ ſunt æquales, utpote inſiſtentes ſimilibus arcubus AK, CI; atq; eſt AF: FK:: CH: HI, quia (*per def. circuli.*) ſunt antecedentia antecedentibus, & conſequentia conſequentibus æqualia quare (*per* 6. *ſexti.*] triang. AFK ſimile triangulo CHI.

Unde [*per* 4. *ſexti.*]

FA: AK:: HC: CI:

Ergo ex æqualitate [*per* 32. *quinti.*]

EA: AK:: GC: CI.

In triangulis $\left.\begin{matrix}EFK\\GHI\end{matrix}\right\}$ (*per hyp.*) EF: FK:: GH: HI;

Sunt autem, & comprehenſi anguli EFK, GHI æquales, quia recti; ergo (*per* 6. *ſexti.*]

EK: KF:: GI: IH.

Cum autem (*per demonſtrata*) ſimilia ſint triangula FKA, HIC;

(*per* 4. *ſexti.*]

FK: KA:: HI: IC:

Quare ex æquo (*per* 22. *ſexti.*)

EK: KA:: GI: IC:

Ergo [*per* 5. *ſexti.*)

Triangulum AEK ſimile triangulo CGI.

Eadem ratione reliqua triangula comprehendentia pyramidem ſupra polygonum AKRO &c. conſtitutā, ſimilia erunt triangulis continentibus pyramidē ſupra polygonū CINY &c. efformatam.

Si ergo harum pyramidum plana ſunt ſimilia, & numero æqualia (*per def.* 9. *undecimi.*) ſimiles erunt pyramides. q. e. d.

Demonſtrata hac pyramidum ſimilitudine; quia (*per conſtructionem*) magnitudo X eſt ad conum CGD in triplicata ratione diametri AB ad diametrum CD: & pariter, quia pyramis deſcripta in cono AEB [*per* 8. *bujus*) ad pyramidem factam in co-

no CGD rationem habet triplicatam homologorum laterum AK, CI; ratio autem AK, ad CI (*per 4. ſexti.* eſt ratio AF ad CH, ſeu AB ad CD:

Ergo [*per 11. quinti.*)

Ratio AK ad CI erit ratio AB ad CD: ergo pyramis deſcripta in cono AEB (*per 35. quinti.*] ad pyramidem efformatam in cono CGD erit in triplicata ratione diametri AB ad diametrum CD:

Ergo (*per 11. quinti.*]

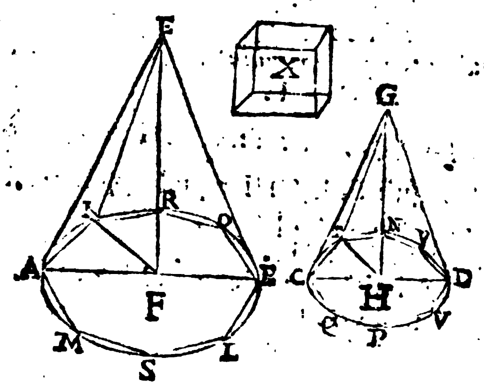
Ut magnitudo X ad conum CGD, ita pyramis facta in cono AEB ad pyramidem efformatam in cono CGD.

Sed (*per conſtructionem.*)

Magnitudo X minor quam pyramis coni AEB:

Ergo (*per 14. quinti.*].

Conus CGD minor erit quam pyramis ejuſdem coni CGD: quod cum repugnet, quia pyramis eſt ſemper pars coni: ergo conus ad conum non poteſt habere majorem rationem, quam ſit triplicata diametrorũ baſium. q. e. i. d.



Pariter quod ratio coni AEB ad conum CGD non poſſit eſſe minor quam triplicata diametrorum baſium AB, CD, taliter demonſtrabitur.

Si ratio coni AEB ad conum CGD minor ſit quam triplicata diametrorum AB, CD; invertendo (*per 26. quinti.*) ratio coni CGD ad conum AEB erit major quam triplicata diametri CD ad diametrum AB.

Ponatur magnitudo X ad conum AEB in triplicata ratione diametri DC ad diametrum AB: quo ſtante (*per 10. quinti.*) X minor erit quam conus CGD.

In cono CGD (*per lemma 3. hujus*) inſcribatur pyramis, cujus baſis polygonum CINY &c. quæ ſit major quam X. Pariter in baſe coni AEB inſcribatur aliud polygonum AKRO &c. ſimile polygono CINY &c. Supra polygonum AKRO &c. in altitu-

dine

dine coni efformetur alia pyramis, quæ (*per demonſtrata in prima parte*) erit ſimilis pyramidi alterius coni CGD.

Quia magnitudo X (*per conſtructionem*) eſt ad conum AEB in ratione triplicata diametri CD ad diametrum AB: pariter quia (*per 8. hujus.*) pyramis coni CGD ad pyramidem ſimilem coni AEB eſt in triplicata ratione CD ad AB:

Erit [*per 11. quinti.*)

Magnitudo X ad conum AEB, ut pyramis coni CGD ad pyramidem coni AEB.

Cum autem [*per conſtructionem*) magnitudo X. minor ſit quam pyramis coni CGD: *per 14. quinti.*) erit & conus AEB minor pyramide ejuſdem coni AEB; quod eſſe nequit, quia pyramis eſt pars coni: ergo conus ad ſimilem conum nequit habere rationem minorem quam ſit triplicata diametrorum. q. e. 2. d.

Hoc ſtante dicendum erit ſimiles conos inter ſe habere triplicatam rationem diametrorum baſium q. e. d.

Quod de conis ſimilibus fuit demonſtratum, valet etiam de cylindris pariter ſimilibus, quia & cylindri ſimiles ſunt in triplicata ratione diametrorum baſium, cum (*per 10. hujus,*) cylindri ſint conorum tripli, ideoque (*per 15. quinti.*)

Ut conus ad conum, ita cylindrus ad cylindrum;

Sed (*per demonſtrata.* (coni ſimiles inter ſe ſunt in triplicata ratione diametrorum baſium: ergo & cylindri in eadem ratione erunt. q. e. d,

SCHOLIUM.

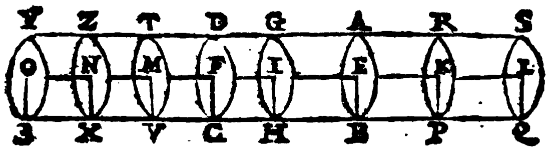
Quod de ſolis conis, atque cylindris rectis ſimilibus in ſuperiore Propoſitione ab Euclide fuit demonſtratum, valet etiam de ſcalenis ſimilibus (juxta def. 24. hujus) & ſcholium ejuſdem definitionis, in quo Scholio oſtenſum fuit, conos, atque cylindros ſimiles ſcalenos correſpondentes conos, atque cylindros rectos ſimiles habere, dummodo coni, & cylindri recti, & ſcaleni eaſdem habeant baſes, & altitudines.

Pro-

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Propoſ. XIII. *Probl.* XIII.

Si cylindrus ABCD, ejuſque axis FE ſecentur plano GH oppoſitis planis AB, CD parallelo. Dico cylindrum ABHG, eſſe ad cylindrum GHCD, uti axis EI, ad axem IF.



Conſtructio.

Cylindrus ABCD ad utramque partem una cum axe EF producatur. In axe producto verſus O accipiantur portiones FM, MN, NO ſingulæ æquales FI. Pariter ex alia parte in axe producto verſus L notentur EK, KL æquales ipſi EI.

Per puncta M,N,O,K,L rectæ notentur MV,NX,O3, KP, LQ parallelæ ipſi FC, & æquales.

Hoc facto, ſi cylindrus ABCD ſupra axem FE revolvatur, rectæ MV, NX &c. deſignabunt circulos TV, ZX &c. parallelos, & æquales circulis DC, AB.

Demonſtratio.

Quia cylindri GB, AP, RQ (*per conſtructionem*) baſes, & altitudines habent æquales (*per* 11. *hujus.*) inter ſe æquales erunt: eadem de cauſa æquales erunt cylindri HD, CT, VZ, XY: ergo cylindrus HS erit tam multiplex cylindri HA, quam multiplex eſt axis IL axis IE: pariterque cylindrus HY erit tam multiplex cylindri HD, quam axis IO multiplex axis IF.

Demum quia cylindri HS, HY eamdem habent baſim HG; (*per corol. prop*: 11. *hujus*) inter ſe erunt ut altitudines: quare

Quando axis IL = IO, etiam cylind. HS = cylin. HY,

Quan-

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Quando IL major IO, etiam cylin. HS major cylin. HY;
Quando IL minor IO, etiam cylin. HS minor cylin. HY.
Ergo [*per def. 6. quinti*:]
Cylin. HA ad cylin. HD; ut axis IE, ad axem IF. q. e. d.

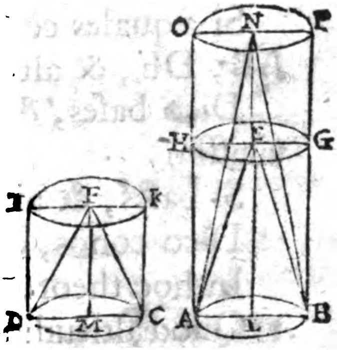
Propoſ. XIV. Theor. XIV.

Si coni, uel cylindri baſes AB, CD habuerint æquales,

Dico inter ſe eſſe, ut altitudines LE, MF.

Conſtructio.

CYlindrus AG verſus E producatur una cum Axe LE, abſcindatur EN = MF: per N ducatur planum PO parallelum plano GH; eritque efformatus circulus PO parallelus circulo GH.



Demonſtratio.

QVia (*per conſtruct.*) cylindri DK, HP non ſolum altitudines MF, EN habent æquales, verum etiam (*per hyp.*) baſes DC, HG (*per corol. 11. hujus*) cylin. DK = cylin. HP.

Quare (*per 7. quinti.*)

Ut cylin. AG ad cylin. HP; ita cylin. AG ad cylin. CI.

Sed (*per 13. hujus.*)

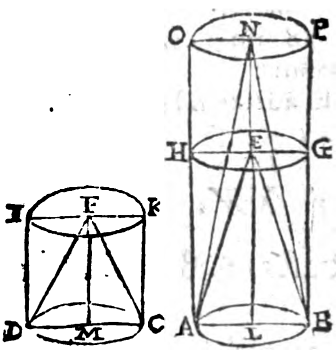
Ut cylin. AG ad cylin. HP; ita axis LE, ad axem EN.

Eſt vero (*per conſtructionem*) EN = MF: ergo

Ut cylin. AG ad cylin. CI; ita axis LE ad axem MF.

Sed axes LE, MF ſunt cylindrorum altitudines: ergo

cylin-



cylindrus AG ad cylindrum CI, ut altitudo LE ad altitudinem MF. q. e. d.

Pariter quod conus AEB ſit ad conum DFC, ut axis LE ad axem MF; demonſtratur, quia [*per* 10. *bujus.*] coni ſunt tertia pars cylindrorum, ideoq; (*per* 15. *quinti.*] ut conus ad conum, ita cylindrus ad cylindrum: cum autem [*per demonſtrata*] cylindrus ad cylindrum ſit ut axis ad axem.

Erit (*per* 11. *quinti.*]

Ut conus AEB ad conum DFC; ita axis LE ad axem MF. q. e. d.

Propoſ. XV. Theo. XV.

Si æquales coni, vel cylindri baſes habeant AB, DE, & altitudines LC, MF.

Dico baſes, & altitudines eſſe reciprocas, & viciſſim.

Si baſes, & altitudines ſint reciprocæ.

Dico conos, & cylindros, eſſe æquales.

In hoc theoremate duo poſſunt contingere. 1: Quod altitudines ſint æquales. 2. Quod ſint inæquales.

Conſtructio, & Demonſtratio.

1 SI conorum, & cylindrorum altitudines MF, LC fuerint æquales (*per corol. prop.* 11. *buius.*] etiam baſes erunt æquales: quare ut baſis DE ad baſim AB; ita reciprocè altitudo LC ad altitudinem MF. q. e. d.

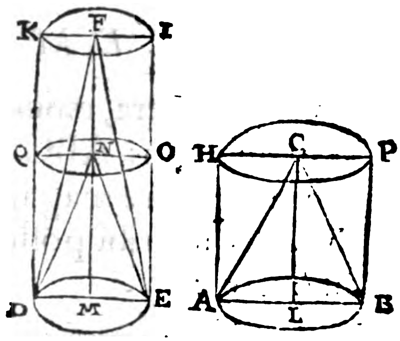
2. Si altitudo MF major fuerit quam altitudo LC: ex MF (*per* 3. *pri.*) accipiatur MN = LC: Per N ducatur planum QO plano DE parallelum.

Quo-

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Quoniam (*per hyp.*] cylindrus DI = cylindro AP:
Erit *per 7. quinti.*)
Ut cylin. DI ad cylin. DO; ita cylin. AP ad cylin. DO:
Sed (*per 11. bujus.*)
Ut cylin. AP ad cylin. DO; ita baſis AB ad Baſim DE;
Pariterque [*per 14. hujus.*]
Ut cylin. DI ad cylin. DO; ita MF ad MN:
Ergo *per 11. quinti.*)
Ut baſis AB ad baſim DE; ita MF ad MN, ſeu LC:

Quamobrem æqualium cylindrorum baſes, & altitudines erunt reciprocè proportionales. q. e. d.



Eadem veritas demonſtrabitur in conis æqualibus, quia (*per 10. bujus.*) coni ſunt tertiæ partes cylindrorum: quare ſi cylindri æquales baſes, & altitudines habent reciprocas, uti demonſtratum fuit, etiam ratio conorum [*per 15. quinti.*] eadem erit. q. e. d.

Viciſſim poſtea ſuppoſita conorum, atque cylindrorum baſium, & altitudinum reciprocatione, ſi altitudines fuerint æquales, etiam baſes æquales erunt. quare [*per corol. propoſ. 11. bujus.*) etiam coni, & cylindri inter ſe æquales erunt.

Si verò altitudines fuerint inæquales, nempe MF major, & LC minor accipiatur, ut prius, MN = LC, ducaturque planum OQ plano DE parallelum.

Cum igitur [*per hyp.*] ſit, ut baſis AB ad baſim DE;
Ita altitudo MF ad altitudinem LC, ſiuè MN
atque [*per 11. bujus.*]
Ut baſis AB ad baſim DE; ita cylin. AP ad cylin. DO:
erit (*per 11. quinti.*]
Ut cylin. AP ad cylin. DO; ita altit. MF ad altit. MN.
Sed (*per 14. hujus.*)
Ut MF, ad MN; ita cylin. DI ad cylin. DO:

Qua-

Quare (*per 11. quinti*)
Ut cylin. AP ad cylin. DO; ita cylin. DI ad cylin. DO:
Igitur (*per 9. quinti.*)
cylindrus AP = cylindro DI. q. e. d.

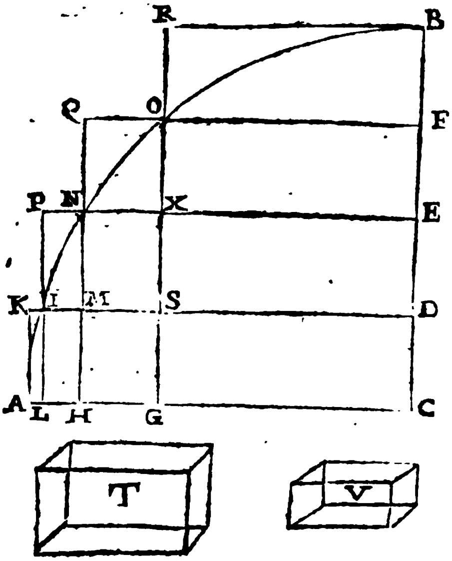
Quod de cylindris fuit demonſtratum, etiam conis applicari poteſt, cum coni ſint tertiæ partes cylindrorum. q. e. d.

SCHOLIUM.

DUæ ſequentes Euclidis propoſitiones, cum nimis diffuſæ, atque tyronibus difficiles, ommittuntur ſubſtituendo loco talium propoſ. ſequentia lemmata.

LEMMA IV.

Data ſphæra, cujus maximi circuli pars quarta ſit quadrans ABC, dataque magnitudine T ſphæra minore magnitudine V: in data ſphæra cylindri inſcribi poſſunt, quorũ ſumma major ſit quam T.



Conſtructio.

REcta CB in quaſcumque partes æquales dividatur, uti ſunt partes CD, DE, EF, FB. Per puncta D, E, F, B, [*per 31. pri.*] rectæ notentur DK, EP, FQ, BR parallelæ ipſi AC. Denuò per puncta A, I, N, O ducantur rectæ AK, IP; NQ, OR ipſi BC parallelæ, quæ concurrant cum rectis DI, EN &c. in punctis K, P, Q, R. Demum producantur PI, in L: QN, in H: & RO in G. facta

Facta hac conſtructione non ſolum completa erunt rectangula quadranti ABC inſcripta, uti ſunt rectangula CI, DN, EO, verum etiam circumſcripta CK, DP, EQ, FR.

Demonſtratio.

Parallelogramma CK, DP, EQ, FR quadranti ABC circumſcripta excedunt parallelogramma CI, DN, EO eidem quadranti inſcripta, parallelogrammis, LK, MP, XQ, *FR*, per quæ tranſit circumferentia quadrantis ABC.

Quia vero parallelogramma LK, MP, XQ, FR, ſimul accepta æquant parallelogrammum CK, & hoc quia FR = CS: XQ = GM: MP = HI; LK verò ſpatium commune: erit parallelogrammum CK exceſſus parallelogrammorum circumſcriptorum ſupra parallelogramma inſcripta quadranti ABC.

Hac conſtructione facta, ſi intelligamus quod quadrans ABC ſuper rectam BC immobilem revolvatur revolutione completa quodlibet parallelogrammum deſignabit cylindros hæmiſphærio inſcriptos, & circumſciptos; prout ſcilicet, parallelogramma cylindros efformantia ſunt in ſphæræ quadrante ABC inſcripta, vel circumſcripta.

Quia verò (*per demonſtrata*) exceſſus rectangulorum circumſcriptorum ſupra rectangula inſcripta, eſt rectangulum CK, etiam exceſſus cylindrorum ſphæræ circumſcriptorum ſupra cylindros ſphæræ inſcriptos, erit cylindrus a rectangulo CK efformatus; ſed ſphæra minor eſt quam omnes cylindri a rectangulis circumſcriptis efformati: ergo exceſſus ſphæræ ſupra cylindros inſcriptos minor erit cylindro a rectangulo CK efformato: quare ſphæra major erit quam cylindri ſphæræ inſcripti quantitate minore, quam ſit cylindrus a rectangulo CK deſcriptus.

Cum autem rectangulum C*K* ſemper ſupra dimidium poſſit imminui, accipiendo minus quam dimidium altitudinis CD; quod facillimè fieri poterit taliter multiplicãdo partes ſemidiametri CB, ut earum numerus excedat duplum numeri partium præexiſtentium: quo facto cylindrus à tali rectangulo efformatus (*per prop. 1. decimi*), minor erit magnitudine V: tandem exceſſus ſphæræ ſupra inſcriptos cylindros minor erit quam V: ergo omnes inſcripti cylindri majores quam T: quia (*per hyp.*) T, & V ſphæ-

V ſunt ſphæræ æquales. Quod autem de hemisphærio dictum eſt, etiam de tota ſphæra eſt intelligendum. q. e. f.

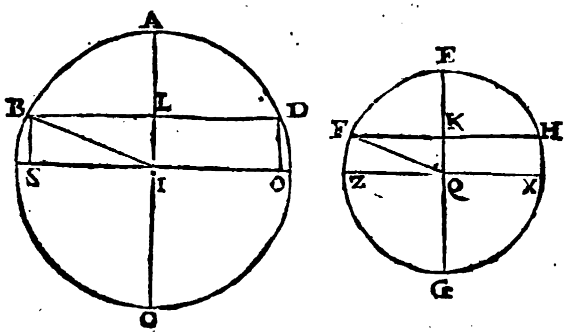
LEMMA V.

Si in duabus ſphæris ABCD, EFGH cylindri ſimiles fuerint inſcripti.

Dico tales cylindros inter ſe habere triplicatam rationem diametri AC ad diametrum EG.

Conſtructio.

IN ſphæris ABCD, EFGH ſint deſcripti ſimiles cylindri recti geniti ex revolutione ſimilium rectangulorum BO, XF ſupra diametro immobiles LI, KQ facta; ducantur rectæ BI, FQ.



Demonſtratio.

QVia [*per hyp.*) ſimilia ſunt rectangula BO, FX erit, (*per def.* 1. *ſexti.*)
Ut DB, ad BS; ita HF, FZ.
Sed [*per* 34. *pri.*)
BS = LI, atque FZ = KQ: ergo
Ut DB : LI; : HF: KQ:

atque

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atque (*per 15. quinti.*]
BL: LI : : FK: KQ,
Quia vero, & angulus BLI = ang. FKQ, quia recti;
(*per 6. ſexti.*]
erit triangulum BLI ſimile triangulo FKQ:
Igitur LB : BI : : KF : FQ.
Sed (*per def. circuli.*)
BI = IA, & FQ = QE: ergo
BL: IA : : FK: QE,
Talium rationum antecedentibus, & conſequentibus duplicatis:
ergo erit (*per 15. quinti.*)
BD: AC : : FH: EG.

Cum autem ſimiles cylindri BO, FX geniti ex revolutione rectangulorum BO, FX baſium diametros habeant BD, FH: (*per 12. hujus.*) cylindri BO, FX erunt in triplicata ratione BD, FH.

Sed [*per demonſtrata.*)
BD: FH : : AC: EG: quare
cylindrus BO ad cyli. FX erit in triplicata ratione AC, ad EG.

Si vero cylindri numero æquales in ſphæris ABCD, EFGH fuerint inſcripti ſinguli ſingulis ſimiles,
(*per 12. quinti.*)
Ut unus cylindus unius ſphæræ, ad alium cylindrum ſimilem in altera ſphæra conſtitutum; ita omnes cylindri unius ſphæræ, ad omnes cylindros alterius ſphæræ;

Sed (*per demonſtrata*].cylindrus in una ſphæra deſcriptus ad cylindrum alteri ſphæræ inſcriptum eſt in triplicata ratione diametrorum: ergo omnes cylindri ad omnes cylindros rationem habebunt triplicatam diametri unius ſphæræ ad diametrum alterius ſphæræ. q. e. d.

Propoſ. XVIII. Theor. XVIII.

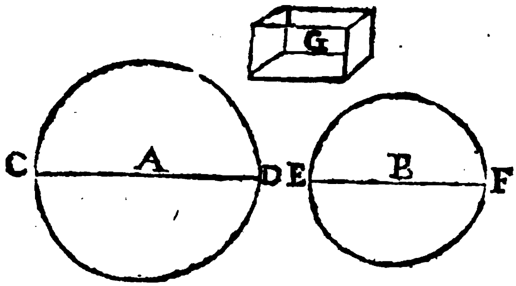
Si fuerint Sphæræ A, & B.

Dico inter ſe habere rationem triplicatam diametrorum CD, EF.

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Demonſtratio.

SI sphæra A ad ſphæram B non habet triplicatam rationem diametri CD ad diametrum EF, neceſſario habebit ſphæra A ad ſphæram B majorẽ, vel minorem rationem quam ſit triplicata diametri CD ad diametrum EF: quod neutrum eſſe poſſit ita demonſtratur.



Primo ſi ſphæra A ad ſphæram B majorem habeat rationem quam ſit triplicata CD ad EF; erit aſſignabilis magnitudo G, quæ ad ſphæram B rationem habeat triplicatam CD, ad EF; eritque (*per* 10. *quinti*) G minor quam ſphæra A: quare in ſphæra A (*per lemma* 4. *hujus*] poterunt inſcribi cylindri, qui omnes ſimul accepti, ſint majores quam G.

In ſphæra B inſcribantur cylindri ſimiles, & numero æquales cylindris ſphæræ A:

His peractis, quia [*per conſtructionem*) magnitudo G ad ſphæram B rationẽ habet triplicatam CD ad EF: ulterius, quia cylindri ſphæræ A ad cylindros ſimiles, & numero æquales ſphæræ B (*per lemma* 5. *hujus*.] rationem habent triplicatam CD, ad EF:

[*per* 11. *quinti*]

Ut magnitudo G ad ſphæram B; ita cylindri ſphæræ A ad cylindros ſphæræ B;

Sed (*per hyp.*)

Magnitudo G minor quam cylindri ſphæræ A: ergo

(*per* 14. *quinti*]

Sphæra B minor quam cylindri in ſphæra B deſcripti: quod repugnat quia cylindri ſunt pars ſphæræ: igitur ſphæra A ad ſphæram B nequit rationem habere majorem quam ſit triplicata diametrorum CD, EF. q. e. i. d.

2. Si ſphæra A ad ſphæram B rationem habeat minorem, quam

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quam ſit triplicata diametri CD ad diametrum EF;

Convertendo (*per 26. quinti*)

Sphæra B ad ſphæram A majorem habebit rationem, quam ſit triplicata diametrorum EF, CD.

Si ergo magnitudo G ad ſphæram A rationem habuerit triplicatam diametri EF ad diametrum CD; magnitudo G [*per 10. quinti*] minor erit quam ſphæra B.

Quemadmodum ſuperius factum fuit, in ſphæra B deſcribantur cylindri, qui omnes ſimul accepti majores ſint quam G: & in ſphæra A deſcribantur cylindri ſimiles, & numero æquales cylindris ſphæræ B.

His peractis, quia magnitudo G ad ſphæram A (*per conſtruct.*) rationem habet triplicatam EF ad CD, pariterque (*per lemma 5. bujus*) cylindri ſphæræ B ad cylindros ſimiles, & numero æquales ſphæræ A rationem habent triplicatam EF, ad CD:

erunt [*per 11. quinti.*)

Ut G ad ſphæram A; ita cylindri ſphæræ B ad cylin. Sphæræ A.

Quia vero [*per conſtructionem*] magnitudo G minor eſt quam cylindri ſphæræ B:

(*per 14. quinti.*]

Sphæra A minor erit quam cylindri ejuſdem ſphæræ A: quod eſſe nequit, quia ſphæra eſt totum, & cylindri inſcripti pars: quamobrem ſphæra A ad ſphærã B non poterit habere rationem minorem, quam ſit triplicata diametrorum CD, EF: igitur ſphæra ad ſphæram ſolam rationem triplicatam habebit. q. e. d.

Corollarium.

EX hoc theoremate colligitur ſphæras inter ſe habere illam rationem, quam obtinent Cubi a diametris deſcripti: ſiquidem cubi ſunt parallelepipeda ſimilia, quæ [*per 33. undec.*) habent triplicatam rationem laterum homologorum; hoc eſt diametrorum:

Quare (*per 11. quinti.*)

Ut ſphæra ad ſphæram; ita cubus unius diametri ad cubum alterius diametri.

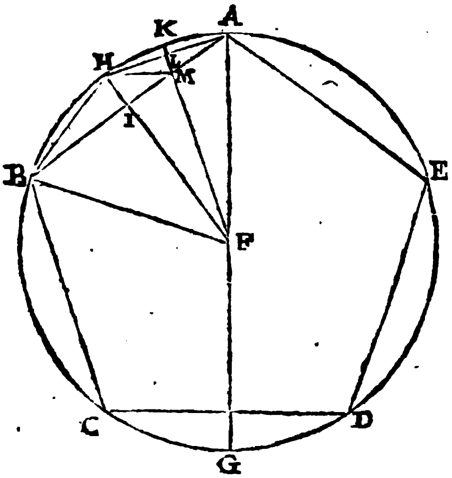
Theoremata duo Elementaria ex Decimo Tertio Euclidis Elemento excerpta.

Propoſ. X. *Theor.* X.

Si in circulo deſcriptum fuerit pentagonum æquilaterum ABCDE.

Dico potentiam lateris pentagoni AB æqualem eſſe potentiæ lateris exagoni, & decagoni in eodem circulo deſcriptorum.

Conſtructio.



Ducatur diameter AFG, atque ſemidiameter FB. Arcus BA (*per* 30. *tertij.*) bifariam dividatur in H, & arcus HA in *K*. Notentur rectæ FH, F*K*, HB, HA, HM.

Hoc facto HA erit latus decagoni; BF vero (*per corol. propoſ.* 15. *quarti.*) erit latus exagoni.

Demonſtratio.

Quia (*per conſtructionem*) arcus AH = arcui HB: (*per* 27. *tertii.*) ang. AFH = ang. HFB.

In triangulis AII / BIF } latera AF, FI ſingula ſingulis æqualia lateribus BF, FI, & [*per demonſtrata*] ang. AFI = ang. BFI: [*per* 4. *pri.*] AI = BI; atque ang. AIF = BIF: quare (*per def.* 10. *pri.*) ambo recti. Eadem ratione AL = FL, & ang. AFL, HFL æquales, quia [*per hyp.*]: arcus AK = arcui KH.

Cum

Cum autem (*per hyp.*) pentagonum ABCDE ſit æquilaterum : (*per* 28. *tertij*) & arcus AB, BC &c. inter ſe æquales: Et [*per conſtructionem*) AH eſt medietas arcus AB; erunt omnes arcus AB, CD &c. arcus AH dupli, atque talium arcuum medietates ipſi AH æquales, & ipſius AK duplæ.

Ulterius ſemicirculus ABCG = ſemicirculo AEDG, ablatis æqualibus Arcubus ABC, AED; erit CG = DG: igitur arcus CD eſt bifariam diviſus in G; ideoque arcus CG = AH, atque duplus AK, vel KH.

His manentibus cum arcus BC duplus ſit arcus HA, & CG, duplus HK: erit arcus BCG duplus arcus BHK: ergo (*per* 33. *ſexti*) ang. BFG duplus ang. BFK: ſed (*per* 20. *tertij.*] idem angulus BFG duplus anguli BAF: igitur ang. BFM = ang. BAF: quamobrem triangulum ABF æquiangulum triangulo FBM: igitur (*per* 4. *ſexti.*

AB : BF :: BF : BM:

Ideoque [*per* 17. *ſexti*]

AB × BM = quad. BF.

Rurſus in triangulis ALM, HLM } latera AL, LM (*per demonſtrata*] ſingula ſingulis æqualia lateribus HL, LM, & angulus ALM = ang. HLM, quia ambo recti: ergo (*per* 4. *pri.*] AM = HM, & angulus MAL = angulo MHL.

Quia vero in triangulo iſoſcele BHA (*per* 5. *pri.*) ang. MAL = ang. MBH: [*per axio.* 1.] erit ang. AHM = ABH: quapropter triang. AHM ſimile triangulo ABH; ergo (*per* 4. *ſexti.*]

AB : AH : AH : AM,

Unde [*per* 17. *ſexti.*]

AB × AM = quad. AH.

Sed (*per demonſtrata.*)

AB × BM = quad. BF:

Ergo

AB × AM + AB × BM = quad. AH + quad. BF:

Sed (*per* 2. *ſecundi.*]

AB × AM + AB × BM = quad. AB: ergo

quad. AB = quad. AH + quad. BF:

Sed quad. AB eſt potentia lateris pentagoni: quad. AH potentia lateris decagoni: quad. BF potentia exagoni in eodem circulo

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culo inſcripti: ergo potentia lateris pentagoni = potentiæ lateris exagoni, & decagoni. q. e. d.

Propoſ. XII. *Theor.* XII.

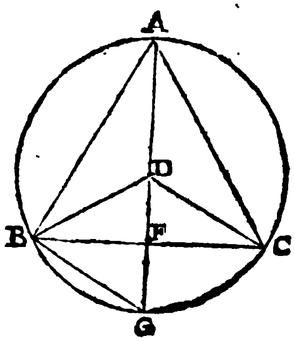
Si in circulo inſcriptum fuerit triangulum æquilaterum ABC.

Dico potentiam lateris trianguli AB triplam eſſe potentiæ ſemidiametri DA.

Conſtructio.

Ducatur diameter AG, quæ ſecet BC in F: addanturque rectæ DB, BG.

Demonſtratio.



Quia ſemicirculus ABG = ſemicirculo ACG, atque [*per 29. tertij.*) arcus AB = arcui AC: erit arcus BG = arcui CG:

Cum autem [*per 29. tertij*) arcus BGC ſit tertia pars totius periphæriæ ABCA, & BG ſit dimium arcus BGC erit arcus BG pars ſexta ejuſdem periphæriæ ABCA: quare recta BG erit latus exagoni (*per corol. prop.* 15. *quarti*] = ſemidiametro DB.

Quoniam verò in triang. ABG (*per* 31. *tertij*) angulus ABG eſt rectus.

[*per* 47. *pri.* quad. AG = quad. *AB* + quad. *BG*

Sed (*per* 4. *ſecundi*]

quad. AG quadruplum quad. *BG*: ergo

quad. *AB* + quad. *BG* quadrupla quad. *BG*: igitur

quad.

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quad. *AB* triplum quad. *BG* : quare
potentia *AB* lateris trianguli eſt tripla potentiæ *BG* lateris exagoni. q.e.d.

Corollarium I.

EX demonſtratis in hoc Theoremate colligitur, potentiam diametri circuli ſeſquitertiam eſſe potentiæ lateris trianguli æquilateri in eodem circulo inſcripti: cum enim [*per demonſtrata*) quad. AG quadruplum ſit quad. *BG*;& quad. *AB* triplum ejuſdem quad. *BG*: erit quad. AG ſeſquitertium quad. *AB*. q. e. d.

Corollarium II.

PAriter colligitur *BC* latus trianguli æquilateri in circulo inſcripti, biſariam dividere ſemidiametrum DG in F : quia (*per demonſtrata*]

quad. *AB* triplum quad. *BG*:

qualium igitur partium quad. *AB* eſt 12. talium partium quad. *BG* erit 4.

Sed [*per 47. primi.*)

quad. *BG* = quad. *BF* + quad. FG: ergo

quad. *BF* + quad. FG erit earumdem partium 4.

Cum autem latus *BC* [*per corol. 1.*] ſit biſariam diviſum in F; ſitque [*per hyp.*] *AB* = BC : erit *AB* duplum *BF* : quare (*per 4. ſecundi*.] quad. AB quadruplum quad. *BF* : igitur qualium partium quadratum *AB* eſt 12, talium partium quad. *BF* erit 3;

Sed (*per demonſtrata*)

quad. *BF* + quad. FG earumdem partium erat 4: igitur quad. FG erit 1: ergo quad. *BG*, ſeu quad. DG quadruplum quad. FG. quare DG dupla FG. q.e.d.

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